Coordination of supply chain after demand disruptions when retailers compete

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Abstract

This paper investigates the coordination mechanism for a supply chain with one manufacturer and two competing retailers when the demands are disrupted. This differs from conventional supply chain coordination models under a static case. We will consider different scenarios of the problem: the production deviation cost may be either incurred to the manufacturer or to the retailers; the supply chain is to be coordinated by either a linear quantity discount schedule or an all-unit quantity discount schedule. In each case, we discuss how the supply chain is coordinated, and how it differs from the original planned scheme. We obtain some interesting managerial observations; for example, the discount slope is unchanged when the manufacturer bears the production deviation cost, but may change when the retailers bear the deviation cost. Some numerical examples are given to illustrate the theoretical results.

Keywords: Coordination mechanism; Supply chain management; Disruption management; Game theory

1. Introduction

Coordination has been a major research issue in the study of supply chain management. Very often, a supply chain can be coordinated by using an appropriate quantity discount schedule. The conventional research on supply chain coordination focuses on the decision-making under normal environment, i.e., how to plan an optimal coordination scheme in order to maximize the channel profit. When such a plan is being executed, however, various disruptions may occur, raising concerns on whether and how the originally planned coordination scheme is still valid in the new disrupted environment. While a well-designed plan is the first step to achieve an efficient supply chain management, it is at least of the same importance to ensure the effective execution of the plan in real time, especially when a disruption occurs. In this paper, we will discuss a supply chain coordination problem after the occurrence of a disruption to the market demand. In other words, we focus on how to adjust

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the original coordination mechanism (quantity-discount policies) to coordinate the supply chain again after the demand was disrupted.

Unexpected changes of the market demand are very common in practice. For example, the outbreak of SARS caused a large sudden demand for respirators and disinfectors; the epidemic of mad cow disease affected a large degree of the demand for beef consumption. It also occurs very frequently that the report of a defective part will lower the demand of a car, and a particular event may make a book to be a best seller. Such demand disruptions have impacts on consumers, retailers, wholesalers, manufacturers and suppliers in the entire supply chain. New models and effective coordination schemes for the supply chain are needed to handle the disruptions (Yu and Qi, 2004). For example, the manufacturer who uses an all-unit quantity discount mechanism to coordinate the supply chain often lowers quantity breakpoints and corresponding wholesale prices when the market demand falls suddenly.

One main difference between the coordination under disruptions and the coordination under normal environment is that the sudden change of demand will cause certain deviation costs that do not exist before. The deviation costs may be caused by the over-time production and the expedited delivery for an increased demand, the extra inventory holding and possible disposal for a decreased demand. The cost may be incurred to either the manufacturer or to the retailers. To achieve an effective supply chain management, such deviation costs must be appropriately taken into account.

Given a disruption to the demand, we will consider how to coordinate a supply chain consisting of one manufacturer and two competing retailers. In the system, the manufacturer produces and sells the products to the retailers, and the retailers, who may also add some value to the products, sell the final products to customers. The manufacturer will try to use a quantity discount policy to coordinate the supply chain. We consider the case where the demands for the two retailers are price-sensitive and partially substitutable between them. This implies that the decision of one retailer also has an impact on the demand of the other because some customers may switch between the two retailers. While in a supply chain there are usually many retailers in reality, the simplified modeling of two retailers facilitates us to perform an analytical investigation to the system and observe some managerial insights that will potentially help us to approach more generalized cases with multiple competing retailers.

Demand disruption is an event independent of the coordination scheme (specifically, quantity discount). In this paper, our main interest is to study the impact of disruption on quantity discount. To provide a better understanding of the impact of demand disruption, we will also discuss the case of an uncoordinated supply chain in which the manufacturer simply charges a constant wholesale price to the retailers. We show how the manufacturer needs to change the wholesale price and production plan under such an uncoordinated supply chain. Our results can also been applied to the normal case without demand disruption, i.e., $\Delta q_1 = \Delta q_2 = 0$.

Comparing with the recent literature on disruption management for supply chain coordination, this paper is new in the following two aspects. First, the model considers two competing retailers where their demands are mutually affected by their retail prices, as well as the disruptions. Second, we address the case that the deviation cost is incurred to the retailers, which has not been discussed before. We will elaborate these new features in details in the literature review.

The rest of the paper is organized as follows. Section 2 provides a review for related literature. Section 3 introduces the basic coordination model when two retailers compete in a Bertrand market. Section 4 studies the coordination of supply chain with a linear quantity discount schedule when the demand for one retailer is disrupted and the manufacturer bears the deviation costs. Section 5 extends the results to the case in which the demands for both retailers are disrupted. Section 6 studies the case where the retailers bear the deviation costs. Section 7 analyzes an all-unit quantity discount schedule for the supply chain with demand disruptions. Section 8 develops a Stackelberg game model for an uncoordinated supply chain without quantity discount. These analytical results are illustrated by numerical examples in Section 9. Section 10 summarizes this paper.

2. Literature review

We start with the literature review on supply chain coordination under normal environments. Along the line of the coordination management of supply chain, there are the intra-firm models and the
inter-firms models. In the intra-firm models, a firm (or integrated channel) has multiple divisions that are organized as profit centers and the purpose is to maximize the total channel profits (for example, Jeuland and Shugan, 1983). In such cases, the channel is usually coordinated by transfer pricing with a portion of profit transferred to the system headquarters, in which the transferred profits may be different across retailers. For the inter-firms model, however, the Federal Robinson-Patman Act forbids different transferred profits across different retailers (Ingene and Parry, 1995a, b; Chen et al., 2001; Cachon and Harker, 2002), implying that all retailers have to be treated equally.

The main body of the supply chain coordination literature belongs to the inter-firms models. The very basic supply chain contains two members, one supplier and one retailer. For such a model, Jeuland and Shugan (1983) discussed some coordination mechanisms and showed that the coordination can bring higher profits to all channel members. The results are further extended by Moorthy (1987), Jeuland and Shugan (1988), Schneeweiss et al. (2004) and Qin et al. (2007).

For a supply chain with multiple members, there are two types of models: the vertical channel model (for example, Corbett and Karmarkar, 2001) and the multi-retailer model. We discuss the latter in more detail because our problem belongs to the multi-retailer model. Very often, it is assumed that the retailers serve geographically dispersed heterogeneous markets, and the market demand for a retailer only depends on its own retail price. Ingene and Parry (1995a) introduced a two-part tariff coordination mechanism into a two-level vertical channel consisting of a manufacturer and multiple retailers. Boyaci and Gallego (2002) studied coordination issues in a supply chain consisting of one wholesaler and one or more geographically dispersed retailers. They pointed out that the channel members could improve the channel as a whole and distribute the gains of coordination through bargaining, but they did not give a specific coordination mechanism. Chen et al. (2001) considered a two-echelon system in which a supplier distributes a single product to multiple retailers. They showed that the traditional quantity discount schemes alone do not guarantee perfect coordination for the systems with non-identical retailers when the inventory cost is considered. They gave an order-frequency-based discount schedule that can fully coordinate the supply chain. Cachon (2001) studied the competitive and cooperative selection of inventory policies in a two-echelon supply chain with one supplier and multiple identical retailers. In his model, the retailers incurred the inventory holding and backorder penalty costs and each retailer experienced independent Poisson demand.

Much research has been conducted on competing retailers, but only a few papers study the coordination of supply chain when retailers compete. Ingene and Parry (1995b) analyzed the case for a single manufacturer selling through two competing retailers who have overlapping market areas rather than exclusive territories. They have shown that the channel can be coordinated by a linear quantity discount schedule, but two-part tariff schedules, in general, cannot fully coordinate the channel. Recently, Bernstein and Federgruen (2003, 2004) generalized the problem to models with multiple retailers.

This paper is also closely related to incentive mechanism design. Villadsen (1995) introduced a three-level hierarchy model with a principal (owner), a supervisor (manager), and two productive agents and gave some incentive mechanisms. Wang (2000) studied an incentive mechanism between the IS manager and the central management. The central management designed an incentive compatible mechanism that can motivate the IS manager to reveal private information truthfully. Corbett and DeCroix (2001) examined the impact of supply contracts on the effort that reduces consumption of indirect materials. They found that it is always possible to increase channel profits with a shared savings contract, but the shared savings contract cannot achieve full channel coordination. However, the above models did not consider the impact of some possible demand change on the contract and the manufacturer’s optimal production quantity, an important issue we mainly focus on in this paper. Xiao et al. (2005) pointed out some detailed explanation on the difference between the models of the coordination management of the supply chain with disruptions and the models of conventional incentive mechanism design.

Supply chain disruption management specifically incorporating deviation costs is relatively new. Xiao et al. (2004) considered disruption management for a two-stage production and inventory system. Xiao and Yu (2006) studied the effects of supply chain disruptions on the evolution of retailers’ behavior, where retailers with bounded rationality play a
duopoly quantity competition with homogeneous goods. All the above work assumed a centralized system without considering coordination schemes. Qi et al. (2006) explored how to update a machine schedule after a machine disruption or a job disruption.

The earliest work on supply chain coordination for a demand disruption is Qi et al. (2004). They studied a model with one manufacturer and one retailer. Xu et al. (2006) studied the case of production cost being disrupted. Xiao et al. (2005) and Xiao and Qi (2006) extended the models to the case with two competing retailers. In Xiao et al. (2005), the two competing retailers have the options of investment on sales promotion and thus the demands are affected by the retail prices. In Xiao and Qi (2006), a different scenario, a production cost disruption to the manufacturer, was studied, while our interest in this paper is the demand disruption.

While the above papers on disruption management all assumed that the manufacturer bears the production deviation cost, our model considers two cases: the production deviation cost is borne by either the manufacturer or the retailers, and compares the effects of demand disruption for these two cases.

3. The basic model

We consider a basic supply chain consisting of one manufacturer and two competing retailers, a model similar to Ingene and Parry (1995b). After purchasing the product from the manufacturer with a wholesale price, retailer $i$ adds some value to the product with a unit cost $c_i$. Then the retailer sells the final product with the retail price $p_i$, $i = 1, 2$. The unit production cost of manufacturer is a constant $c_0$. Using common assumptions in the literature, we assume that the market demand for retailer $i$ is

$$q_i(p_1, p_2) = a_i - p_i + dp_j, \quad 0 < d < 1, \quad i, j = 1, 2, \quad j \neq i,$$

where $a_i$ is the market scale for retailer $i$ (i.e., the maximum possible demand) and $d$ is the substitutability coefficient. The substitutability coefficient is a measure of the sensitivity of the $i$th retailer’s sales to the change of $j$th retailer’s price. In general, economists assume that the market scales are equal, i.e., $a_1 = a_2 = a$ (Cachon and Harker, 2002). When the suffixes $i$ and $j$ simultaneously emerge in an equation, we always mean $i, j = 1, 2, i \neq j$. For convenience, we will omit it in the rest of the paper.

Such a supply chain can be coordinated by different schemes. In this paper, we mainly investigate a linear quantity-discount wholesale schedule proposed in Ingene and Parry (1995b), i.e., if the order quantity of retailer $i$ is $q_i$, he will be charged by the manufacturer with $(w - bq_i)q_i + T$, where $w$ is the maximum variable wholesale price, $b \geq 0$, is the discount slope of the per-unit wholesale pricing schedule, and $T$ is the fixed fee paid by a retailer to the manufacturer.

According to the above descriptions, we know that the profit function of retailer $i$ is

$$\pi_i = (p_i - c_i - w)(a_i - p_i + dp_j) + b(a_i - p_i + dp_j)^2 - T,$$

where $p_i > c_i + w$, i.e., the profit of unit product should be positive. Then the profit function of the manufacturer is

$$\pi_0 = (w - c_0)\sum_{j \neq i}(a_i - p_i + dp_j) - b\sum_{j \neq i}(a_i - p_i + dp_j)^2 + T.$$

The total channel profit is

$$\Pi(p_1, p_2) = \sum_{j \neq i}(p_i - c_i - c_0)(a_i - p_i + dp_j).$$

Assuming the reservation utilities of the retailers are 0, Ingene and Parry (1995b) shown that the supply chain is fully coordinated when the manufacturer adopts a linear quantity-discount schedule with following parameters:

$$w^* = \frac{d(q_1^* + q_2^*)}{1 - d^2} + c_0,$$

$$b^* = \frac{d}{2(1 + d)},$$

$$0 \leq T \leq \min\{(1 - b^*)q_1^2, (1 - b^*)q_2^2\},$$

where $q_i^* = \frac{d}{2d}(a_i - c_i + dc_j - (1 - d)c_0) > 0$ is the optimal order quantity and

$$p_i^* = \frac{a_i + da_j + (1 - d^2)(c_0 + c_i)}{2(1 - d^2)} > 0$$

is the optimal retail price for retailer $i$, respectively.

In the following, we will still use this linear quantity-discount schedule (mechanism) to coordinate the supply chain with demand disruptions. We
will see that the coordination mechanisms will be different in the case with demand disruptions. In other words, \( w^* \), \( b^* \) and the interval of \( T \) are different in the two cases. We also find an all-unit quantity-discount schedule to coordinate the supply chain with two identical competing retailers.

4. Coordination of supply chain with one demand disruption

4.1. Modeling of demand disruptions

The above model studies the static case of supply chain coordination. Particularly, the demand function is assumed to be deterministic and known. In practice, the demand is often disrupted unexpectedly due to the change of market environment, such as terrorist attack, quality reasons, new government laws or policy, new industry regulations, new technology, and so on. When a disruption occurs, the coordination scheme designed under the static case may become invalid. Thus, the supply chain needs to be re-coordinated. In the following, we use the notation with a tilde (or \( \sim \)) to denote the case under demand disruptions. In this section, we first consider the case in which only one retailer faces a demand disruption. This may happen due to the change of service quality and reputation of a specific retailer. For example, the construction of the infrastructure about the retailer often reduces its market demand, and a special promotion results in a higher demand.

Without loss of generality, we assume that the market demand for retailer 1 after the disruption is

\[
\tilde{q}_1(\tilde{p}_1, \tilde{p}_2) = a_1 + \Delta a - \tilde{p}_1 + d \tilde{p}_2.
\]

Thus, the total production deviation quantity is

\[
\tilde{q}_1 + \tilde{q}_2 - q_1^* - q_2^* = a_1 + a_2 + \Delta a - (1 - d)(\tilde{p}_1 + \tilde{p}_2) - q_1^* - q_2^*.
\]

If the realized total demand \( \tilde{q}_1 + \tilde{q}_2 \) is higher than the planned total demand \( q_1^* + q_2^* \), more products have to be produced to meet the unplanned increased demand, extra machines, labor overtime or increased raw material price will induce a higher unit production cost. If the realized total demand is lower than the planned total demand, there will be some leftover inventory, which has to be disposed of or sold at a lower price. With a decreased demand, there may be some extra holding costs for the unused raw material and product. Trimarchi (2003) suggested that, irrespective of any consideration of subjective utility, unexpected and substantial wealth transfers lead to disruption cost. For the above reasons, we assume a unit penalty cost \( c_u \) for the increased production and a unit penalty cost \( c_s \) for the decreased production, \( c_u > 0 \) and \( c_s > 0 \). Based on the above descriptions, we know that the total profit of the centralized supply chain is

\[
\tilde{\Pi} = \sum_{i=1}^{2}(\tilde{p}_i - c_0 - c_i)(a_i - \tilde{p}_i + d \tilde{p}_j) + \Delta a(\tilde{p}_1 - c_0 - c_1)
\]

\[
- c_u[a_1 + a_2 + \Delta a - (1 - d)(\tilde{p}_1 + \tilde{p}_2) - q_1^* - q_2^*]^{+}
\]

\[
- c_s[q_1^* + q_2^* - a_1 - a_2 - \Delta a + (1 - d)(\tilde{p}_1 + \tilde{p}_2)]^{+},
\]

where \((x)^{+} = \max\{x,0\}\). The first term in Eq. (5) is the total profit of the channel without demand disruption, the second term is the revenue change incurred by the changed market scale, the third term is the total deviation cost incurred by an increased
production, and the fourth term is the total deviation cost incurred by a decreased production.

Define \((\tilde{\mathbf{p}}^*_1, \tilde{\mathbf{p}}^*_2) = \arg\max_{\mathbf{p}_1 \geq 0, \mathbf{p}_2 \geq 0} \mathbf{H}\). For convenience, we differentiate the channel profit function (5) into two cases, and then combine both cases to give the optimal solution. We see that if \(\Delta a \geq q^*_1 + q^*_2 + (1 - d)(\tilde{\mathbf{p}}_1 + \tilde{\mathbf{p}}_2) - a_1 - a_2\), Eq. (5) becomes

\[
\tilde{\mathbf{H}}_1 = \sum_{j=1}^2 (\tilde{p}_j - c_0 - c_j)(a_j - \tilde{p}_j + d\tilde{p}_j) + \Delta a(\tilde{p}_1 - c_0 - c_1)
- c_u[a_1 + a_2 + \Delta a - (1 - d)(\tilde{p}_1 + \tilde{p}_2) - q^*_1 - q^*_2]
\]

and if \(\Delta a \leq q^*_1 + q^*_2 + (1 - d)(\tilde{p}_1 + \tilde{p}_2) - a_1 - a_2\), Eq. (5) becomes

\[
\tilde{\mathbf{H}}_2 = \sum_{j=1}^2 (\tilde{p}_j - c_0 - c_j)(a_j - \tilde{p}_j + d\tilde{p}_j) + \Delta a(\tilde{p}_1 - c_0 - c_1)
+ c_u[a_1 + a_2 + \Delta a - (1 - d)(\tilde{p}_1 + \tilde{p}_2) - q^*_1 - q^*_2].
\]

Furthermore, we can obtain from the Kuhn-Tucker condition that the optimal solution is

\[
\tilde{\mathbf{p}}^*_2 = \frac{\Delta a + a_1 + a_2 - q^*_1 - q^*_2}{2(1 - d)} + \frac{1}{4(1 + d)}(a_1 - a_1 - \Delta a) + \frac{c_2 - c_1}{4}.
\]

\[
= \frac{(1 + 3d)\Delta a}{4(1 - d^2)} + \tilde{p}^*_2.
\]

(11)

\[
\tilde{\mathbf{p}}^*_1 = \frac{(3 + d)\Delta a}{4(1 - d^2)} + \tilde{p}^*_1.
\]

(12)

The corresponding optimal order quantities for the two retailers are

\[
\tilde{q}^*_1 = \tilde{q}^*_1 + \frac{1}{4}\Delta a \quad \text{and} \quad \tilde{q}^*_2 = \tilde{q}^*_2 - \frac{1}{4}\Delta a.
\]

Solving the Kuhn-Tucker condition of Eq. (7), we have that if \(-a_1 < \Delta a \leq -2(1 - d)c_u\),

\[
\tilde{\mathbf{p}}^*_1 = \tilde{\mathbf{p}}^*_1 + \frac{1}{2}c_u + \frac{\Delta a}{2(1 - d^2)},
\]

(13)

\[
\tilde{\mathbf{p}}^*_2 = \tilde{\mathbf{p}}^*_2 + \frac{1}{2}c_u + \frac{d\Delta a}{2(1 - d^2)}.
\]

(14)

and the corresponding optimal order quantities for the two retailers are

\[
\tilde{q}^*_1 = \tilde{q}^*_1 + \frac{1}{2}(1 - d)c_u + \frac{1}{2}\Delta a \quad \text{and} \quad \tilde{q}^*_2 = \tilde{q}^*_2 + \frac{1}{2}(1 - d)c_u.
\]

If \(\Delta a > -2(1 - d)c_u\), the optimal solution of Eq. (7) is given by Eqs. (11) and (12). According to above analysis and Xiao et al. (2005), we have

**Theorem 1.** The optimal solution for Eq. (5) is given by Eqs. (9) and (10) for \(\Delta a \leq 2(1 - d)c_u\), by Eqs. (11) and (12) for \(-2(1 - d)c_u < \Delta a < 2(1 - d)c_u\), by Eqs. (13) and (14) for \(-a_1 < \Delta a \leq -2(1 - d)c_u\).

From Theorem 1, we see that a single demand disruption to retailer 1 has the following impact on the entire channel.

- If \(\Delta a > 0\), both retailers should increase the retail prices, and if \(\Delta a < 0\), they should decrease the retail prices, even though only the demand of retailer 1 is disrupted.
- When there is a large demand increase, i.e., \(\Delta a \geq 2(1 - d)c_u\), the entire channel should increase the total production quantity \((\tilde{q}^*_1 + \tilde{q}^*_2 > \tilde{q}^*_1 + \tilde{q}^*_2)\).

While retailer 1 should increase the order quantity, retailer 2 should reduce his orders.
market. The profit function of retailer offers both retailers an identical linear quantity-
when a single demand disruption occurs. A discount schedule can coordinate the supply chain
making case when the demand is disrupted. In
quantity shift between the retailers. This phenom-
model tells that we should make some order
quantity. However, the current competing-retailers
model similarly
for a demand change with a small scale, the
order quantity, retailer 2 should increase his
order quantity, retailer 1 should decrease the original production quantity
~
~
~
~

We find that there are similarities as well as differences between the above observations and the results obtained in a single-retailer model (Qi et al., 2004). For the disrupted retailer, the retail price changes in the same way as the single retailer-model. The single-retailer model similarly suggests that the order quantity be changed for a large demand increase or decrease, but for a demand change with a small scale, the single-retailer model suggests keeping the order quantity. However, the current competing-retailers model tells that we should make some order quantity shift between the retailers. This phenomenon reveals the relationship when multiple retailers compete.

4.3. Decision-making for a decentralized supply chain and supply chain coordination

Now we consider the decentralized decision-making case when the demand is disrupted. In Section 3, we point out that a linear quantity-discount schedule can coordinate the supply chain without disruption. We will investigate how to use the same scheme to coordinate the supply chain when a single demand disruption occurs.

Consider a mechanism in which the manufacturer offers both retailers an identical linear quantity-discount schedule \((\hat{\nu} - \hat{b}\hat{q}_i)\hat{q}_i + \hat{T}\). Given the mechanism, the retailers play a static game with complete information with each other in a Bertrand market. The profit function of retailer \(i\) is

\[
\pi_i = (\hat{p}_i - c_i - \hat{\nu})(\hat{a}_i - \hat{p}_i + d\hat{p}_j) + \hat{b}(\hat{a}_i - \hat{p}_i + d\hat{p}_j)^2 - \hat{T},
\]

where \(\hat{a}_i = a_i + \Delta a\) and \(\hat{a}_2 = a_2\). Solving the first-order conditions of Eq. (15), we find the Nash equilibrium retail prices are

\[
\hat{p}^*_i = \frac{2(1 - \hat{b})(1 - 2\hat{b})\hat{a}_i + d(1 - 2\hat{b})^2\hat{a}_j}{4(1 - \hat{b})^2 - d^2(1 - 2\hat{b})^2} + \frac{2(1 - \hat{b})c_i + d(1 - 2\hat{b})c_j + (2 + d - 2\hat{b} - 2d\hat{b})\hat{w}}{4(1 - \hat{b})^2 - d^2(1 - 2\hat{b})^2}.
\]

(16)

Now the manufacturer needs to see whether it is possible to choose appropriate parameters in the above policy so that the channel is coordinated. Similar to Theorem 1, we differentiate the coordination problem into three cases based on the degree of the disruption. We know that the decentralized supply chain is coordinated when the two Nash equilibrium retail prices are equivalent to the two optimal retail prices of the centralized channel. By solving \(\hat{\nu}^*\) and \(\hat{b}^*\) from \(\hat{p}^*_1 = \hat{p}^*_1\) and \(\hat{p}^*_2 = \hat{p}^*_2\), we can derive the following.

Theorem 2. The supply chain is coordinated by a linear quantity discount schedule with

(1) For \(\Delta a \geq 2(1 - d)c_u\), \(\hat{\nu}^* = (d(\hat{q}^*_1 + \hat{q}^*_2))/(1 - d^2) + c_0 + c_u, \hat{b}^* = d/(2(1 + d)), 0 < \hat{T} \leq \min((1 - \hat{b}^*\hat{q}^*_1, (1 - \hat{b}^*\hat{q}^*_2)\), where \(\hat{q}^*_1 = q^*_1 + \frac{1}{2}\Delta a - \frac{\hat{b}}{\hat{b}_u}(1 - d)\) and \(\hat{q}^*_2 = q^*_2 - \frac{\hat{b}}{\hat{b}_u}(1 - d);

(2) For \(-2(1 - d)c_s < \Delta a < 2(1 - d)c_u\), \(\hat{\nu}^* = (d(\hat{q}^*_1 + \hat{q}^*_2))/(1 - d^2) + \Delta a/(2(1 - d)) + c_0, \hat{b}^* = d/(2(1 + d)), 0 < \hat{T} \leq \min((1 - \hat{b}^*\hat{q}^*_1, (1 - \hat{b}^*\hat{q}^*_2)\), where \(\hat{q}^*_1 = q^*_1 + \frac{1}{2}\Delta a - \frac{\hat{b}}{\hat{b}_u}(1 - d)\) and \(\hat{q}^*_2 = q^*_2 - \frac{\hat{b}}{\hat{b}_u}(1 - d);

(3) For \(-a_1 < \Delta a \leq -2(1 - d)c_s\), \(\hat{\nu}^* = \frac{d(\hat{q}^*_1 + \hat{q}^*_2)}{1 - d^2} + c_0 - c_s, \hat{b}^* = \frac{d}{2(1 + d)}, 0 < \hat{T} \leq \min((1 - \hat{b}^*\hat{q}^*_1, (1 - \hat{b}^*\hat{q}^*_2)\), where \(\hat{q}^*_1 = q^*_1 + \frac{1}{2}(1 - d)c_s + \frac{1}{2}\Delta a\) and \(\hat{q}^*_2 = q^*_2 + \frac{1}{2}(1 - d)c_s\).

Proof. Part (1) Recall that when \(\Delta a \geq 2(1 - d)c_u\), the optimal retail prices of the centralized channel are given by Eqs. (9) and (10). If the channel is coordinated, then we have \(\hat{p}^*_1 = \hat{p}^*_1, i = 1, 2\), where \(\hat{p}^*_i\) are given by Eqs. (9) and (10). Solving the Equations with respect to \(\hat{\nu}\) and \(\hat{b}\), we have \(\hat{\nu}^* = (d(\hat{q}^*_1 + \hat{q}^*_2))/(1 - d^2) + c_0 + c_u, \hat{b}^* = d/(2(1 + d))\).

According to Eq. (15), we get \(\hat{\nu}^*(\hat{q}^*_1, \hat{q}^*_2) = (1 - \hat{b}^*\hat{q}^*_1 - \hat{T}, i = 1, 2\). Note that the reservation
utilities of retailers are 0. Hence, we have \( 0 \leq T \leq \min\{(1 - b^*)q_1^{*2}, (1 - b^*)q_2^{*2}\} \).

Similar to Part (1), Parts (2) and (3) follow. \( \square \)

In Theorem 2, we assume that the reservation utilities of retailers are 0, which means that the retailers would like to participate as long as they can gain a nonnegative profit. From Theorem 2, according to the coordination mechanism, each retailer will gain a nonnegative profit, and the larger the \( \Delta a \), the higher the maximum variable wholesale price \( \tilde{w}^* \). An observation from Theorem 2 is that in the coordination scheme, \( b^* \) is a constant equal to the original value in the coordination scheme regardless of the disruption degree \( \Delta a \). Note that \( b^* \) indicates the discount slope the manufacturer offers to the retailers. The constant \( b^* \) implies that it is not necessary to adjust the discount slope for a demand disruption. This is interesting in that (1) it simplifies the channel coordination administration by reducing one parameter adjustment, and (2) it helps maintain a good and stable supplier–retailer relationship because the fixed discount slope indicates a goodwill shown by the supplier.

5. Coordination of supply chain with two demand disruptions

Section 4 considers the case in which there is only one retailer with single demand disruption. Sometimes, we also encounter cases where the two markets are simultaneously disrupted, such as by the change of the quality of product, the emergence of new technology, and new policy. To model the two demand disruptions, we can assume that \( \tilde{a}_1 = a_1 + \Delta a_1 \), and \( \tilde{a}_2 = a_2 + \Delta a_2 \).

We start with the case of centralized decision making. Similar to Eq. (4), we know that the demand function of retailer \( i \) is

\[
\tilde{q}_i(\tilde{p}_1, \tilde{p}_2) = \tilde{a}_i - \tilde{p}_i + d\tilde{p}_j
\]

and the total profit of the centralized channel is

\[
\tilde{\Pi} = \sum_{j=1}^{2} (\tilde{p}_j - c_0 - c_j)(\tilde{a}_j - \tilde{p}_j + d\tilde{p}_j)
\]

\[
- c_0[\tilde{a}_1 + \tilde{a}_2 - (1 - d)(\tilde{p}_1 + \tilde{p}_2) - q_1^{*2} - q_2^{*2}] +
\]

\[
- c_j[(1 - d)(\tilde{p}_1 + \tilde{p}_2) + q_1^{*2} + q_2^{*2} - \tilde{a}_1 - \tilde{a}_2]^+.
\]

Similar to Theorem 1, we can show the following theorem.

**Theorem 3.** The optimal solution of Eq. (18) satisfies:

(1) if \( \Delta a_1 + \Delta a_2 \geq 2(1 - d)c_u \), then

\[
\tilde{p}_1^* = \frac{p_1^* + \frac{1}{2}c_u + \frac{\Delta a_1 + d\Delta a_2}{2(1 - d^2)}}{2(1 - d^2)}
\]

and

\[
\tilde{p}_2^* = \tilde{p}_2^* + \frac{1}{2}c_u + \frac{\Delta a_2 + d\Delta a_1}{2(1 - d^2)};
\]

(2) if \(-2(1 - d)c_u < \Delta a_1 + \Delta a_2 < 2(1 - d)c_u \), then

\[
\tilde{p}_1^* = \frac{(3 + d)\Delta a_1 + (1 + 3d)\Delta a_2 + p_1^*}{4(1 - d^2)}
\]

and

\[
\tilde{p}_2^* = \frac{(1 + 3d)\Delta a_1 + (3 + d)\Delta a_2 + p_2^*}{4(1 - d^2)};
\]

(3) if \( \Delta a_1 + \Delta a_2 \leq -2(1 - d)c_u \), then

\[
\tilde{p}_1^* = p_1^* - \frac{1}{2}c_u + \frac{\Delta a_1 + d\Delta a_2}{2(1 - d^2)}
\]

and

\[
\tilde{p}_2^* = p_2^* - \frac{1}{2}c_u + \frac{\Delta a_2 + d\Delta a_1}{2(1 - d^2)}.
\]

The proof of Theorem 3 is similar to that of Theorem 1, and is omitted. Compare Theorems 1 and 3, we find that the two cases have the same thresholds, though in Theorem 1 it is measured by the disruption for a single retailer and in Theorem 3 it is measured by the sum of disruptions for both retailers. According to Theorem 3 and Eq. (17), we can also give the optimal order quantities, respectively.

In a decentralized supply chain, the Nash equilibrium retail prices are given by Eq. (16) with \( \tilde{a}_i = a_i + \Delta a_i \). Similar to Theorem 2, we can derive the following results for the case of two demand disruptions.

**Theorem 4.** For the case with two demand disruptions, the supply chain is coordinated by a linear quantity discount schedule with

\[ 0 \leq \tilde{T} \leq \min\{(1 - b^*)q_1^{*2}, (1 - b^*)q_2^{*2}\}, \quad b^* = \frac{d}{2(1 + d)} \]

(1) if \( \Delta a_1 + \Delta a_2 \geq 2(1 - d)c_u \), then
\[ w^* = \frac{d(q_i^* + q_j^*)}{1 - d^2} + c_0 + c_u, \]

where \( q_i^* = q_i^* + \frac{k}{2} \Delta a_i - \frac{k}{2} c_u (1 - d); \)

(2) if \(-2(1 - d)c_u < \Delta a_1 + \Delta a_2 < 2(1 - d)c_u\), then

\[ w^* = \frac{d(q_i^* + q_j^*)}{1 - d^2} + \frac{\Delta a_1 + \Delta a_2}{2(1 - d)} + c_0, \]

where \( q_i^* = q_i^* + \frac{k}{2} \Delta a_i - \frac{k}{2} c_u (1 - d); \)

(3) if \(\Delta a_1 + \Delta a_2 \leq -2(1 - d)c_u, \Delta a_1 > -a_1 and \Delta a_2 > -a_2\), then

\[ w^* = \frac{d(q_i^* + q_j^*)}{1 - d^2} + c_0 - c_u, \]

where \( q_i^* = q_i^* + \frac{k}{2} \Delta a_i + \frac{k}{2} c_u (1 - d). \)

Theorem 4 gives the coordination mechanism of the decentralized supply chain with two market scale changes. It also reveals some interesting impacts of the disruption on the entire supply chain.

When the overall effect of the disruptions, \(|\Delta a_1 + \Delta a_2|\), is large, i.e., for Cases 1 and 3, the order quantity for each retailer solely depends on his own disruption, to increase order quantity with an increased demand, and to decrease order quantity with a decreased demand.

When the overall effect of the disruptions is small, i.e., for Case 2, the order quantity for each retailer depends on the disruption to both of them. Even if a retailer has an increased demand disruption, he may also need to reduce the order quantity if his competitor has a larger demand increase, say when \(0 < \Delta a_1 < \Delta a_2\) and \(\Delta a_1 + \Delta a_2 < 2(1 - d)c_u\). Similarly, even if a retailer has a decreased demand disruption, he may have the opportunity to increase the order quantity. The only case for the retailers to keep their original order quantity is that \(\Delta a_1 = \Delta a_2\), i.e., they have the same demand disruptions.

Similar to the case of one demand disruption, there is no need to adjust the discount slope \(b^*\) to respond to the demand disruptions.

6. Coordination of supply chain when retailers bear deviation costs

Up to this point, we have assumed that the manufacturer bears the production deviation cost. In some other cases, the retailers may need to bear the deviation cost rather than the manufacturer. For example, in channels without a return policy such as most products in the food industry, the quality of the intermediate product can be kept in a long time, but the quality of the final product can only be kept in a short time. Thus, the retailers have to dispose of their products if the products are overdue or sell the products in a secondary market at a price less than the unit cost. When the market demand increases, they have to order again and pay more wages for workers, which is often associated with more marginal cost. One major difference for the case of retailer bearing deviation cost is that the order quantity change with any retailer will cause a deviation cost.

Let a unit deviation cost of the increased order quantity be \(c_w\) and a unit penalty cost of the decreased order quantity be \(c_d\) for retailer \(i, i = 1, 2\). For simplicity, we assume that \(\text{sgn}(\Delta a_1) = \text{sgn}(\Delta a_2)\), i.e., the market scales for two retailers either all increase or all decrease. The case in which one market scale increases and the other market scale decreases can be analyzed by employing a similar analysis.

6.1. Decision-making for a centralized supply chain

According to the above descriptions, the total profit of the centralized supply chain is

\[ \tilde{\Pi} = \sum_{j=1}^{2} \left[ (\tilde{p}_j - c_0 - c_j)(\tilde{a}_j - \tilde{p}_j + d\tilde{p}_j) - c_w(\tilde{a}_j - \tilde{p}_j + d\tilde{p}_j - q_j^*)^+ - c_d(q_j^* + \tilde{p}_j - d\tilde{p}_j - \tilde{a}_j)^+ \right]. \]

(19)

Since each retailer bears a unique deviation cost, there are four deviation cost terms in Eq. (19), which is more complicated than the case in Eq. (5) when the manufacturer bears the deviation costs. For a centralized supply chain, we have the following characteristic.

Theorem 5. Assume \(\text{sgn}(\Delta a_1) = \text{sgn}(\Delta a_2)\). The optimal solution of Eq. (19) has the following characteristic: Two retailers order more or less than the planned quantities simultaneously. In other words, it is impossible that one retailer orders strictly more and the other orders strictly less.

Proof. Suppose that retailer 1 orders strictly more than the planned quantity and retailer 2 orders...
strictly less. Thus, Eq. (19) is equivalent to the following:

\[
\tilde{F} = \sum_{j \neq i}^{2} (\tilde{p}_j - c_0 - c_i)(\tilde{a}_j - \tilde{p}_i + d\tilde{p}_j)
\]

\[
= c_{u1}(\tilde{a}_1 - \tilde{p}_1 + d\tilde{p}_2 - q^*_1)
- c_{u2}(q^*_1 + \tilde{p}_2 - d\tilde{p}_1 - \tilde{a}_2) .
\]

The optimal solution of Eq. (20) is

\[
\tilde{p}^*_1 = p^*_1 + \frac{1}{2} c_{u1} + \frac{d\Delta a_2 + \Delta a_1}{2(1 - d^2)}
\]

and

\[
\tilde{p}^*_2 = p^*_2 - \frac{1}{2} c_{u2} + \frac{d\Delta a_1 + \Delta a_2}{2(1 - d^2)} .
\]

Furthermore, we can show from the above assumption that \(\Delta a_1 > c_{u1} + d c_{u2}\) and \(\Delta a_2 < -c_{u2} - d c_{u1}\), which contradicts \(\sgn(\Delta a_1) = \sgn(\Delta a_2)\). Hence, it is impossible that one retailer orders strictly more and the other orders strictly less. \&

According to Theorem 5, we only need to consider the case in which two retailers order more or less simultaneously, i.e., two retailers simultaneously bear the first penalty term or the second penalty term. For the centralized supply chain, we have the following conclusion.

**Theorem 6.** Assume \(\sgn(\Delta a_1) = \sgn(\Delta a_2)\). The optimal solution of Eq. (19) is

(1) if \(\Delta a_1 \geq \max\{c_{u1} - d c_{u2}, \ 0\}\) and \(\Delta a_2 \geq \max\{-c_{u2} - d c_{u1}, \ 0\}\),

\[
\tilde{p}_{11}^* = p_{11}^* + \frac{1}{2} c_{u1} + \frac{d\Delta a_2 + \Delta a_1}{2(1 - d^2)} ;
\]

(2) if \(0 \leq \Delta a_1 < c_{u1} - d c_{u2}, \ \Delta a_2 + d\Delta a_1 \geq c_{u2}(1 - d^2)\) and \(\Delta a_2 \geq 0\),

\[
\tilde{p}_{21}^* = p_{21}^* + \frac{1}{2} d c_{u2} + \frac{(2 - d^2)\Delta a_1 + d\Delta a_2}{2(1 - d^2)} \quad \text{and}
\]

\[
\tilde{p}_{22}^* = p_{22}^* + \frac{1}{2} c_{u2} + \frac{d\Delta a_1 + \Delta a_2}{2(1 - d^2)} ;
\]

(3) if \(\Delta a_1 + d\Delta a_2 \geq c_{u1}(1 - d^2), \ \Delta a_1 \geq 0\) and \(0 \leq \Delta a_2 < c_{u2} - d c_{u1}\),

\[
\tilde{p}_{31}^* = p_{31}^* + \frac{1}{2} c_{u1} + \frac{d\Delta a_2 + \Delta a_1}{2(1 - d^2)} ,
\]

\[
\tilde{p}_{32}^* = p_{32}^* + \frac{1}{2} d c_{u2} + \frac{(2 - d^2)\Delta a_1 + d\Delta a_2}{2(1 - d^2)} ;
\]

(4) if \(- (1 - d^2)c_{u1} < \Delta a_1 + d\Delta a_j < (1 - d^2)c_{u1}, \ i = 1,2, j \neq i,\)

\[
\tilde{p}_{4i}^* = p_{4i}^* + \frac{\Delta a_i + d\Delta a_j}{1 - d^2} ;
\]

(5) if \(\Delta a_1 \leq \min\{d c_{u2} - c_{s1}, 0\}\) and \(\Delta a_2 \leq \min\{d c_{u1} - c_{s2}, 0\},\)

\[
\tilde{p}_{5i}^* = p_{5i}^* + \frac{\Delta a_i + d\Delta a_j}{2(1 - d^2)} ;
\]

(6) if \(d c_{u2} - c_{s1} < \Delta a_1 \leq 0, \ \Delta a_2 \leq 0\) and \(\Delta a_2 + d\Delta a_1 \leq -(1 - d^2)c_{u2},\)

\[
\tilde{p}_{61}^* = p_{61}^* - \frac{1}{2} d c_{u2} + \frac{(2 - d^2)\Delta a_1 + d\Delta a_2}{2(1 - d^2)} ,
\]

\[
\tilde{p}_{62}^* = p_{62}^* + \frac{1}{2} c_{u2} + \frac{d\Delta a_1 + \Delta a_2}{2(1 - d^2)} ;
\]

(7) if \(\Delta a_1 + d\Delta a_2 \leq - (1 - d^2)c_{s1}, \ \Delta a_1 \leq 0\) and \(d c_{s1} - c_{s2} < \Delta a_2 \leq 0,\)

\[
\tilde{p}_{71}^* = p_{71}^* - \frac{1}{2} c_{s1} + \frac{d\Delta a_2 + \Delta a_1}{2(1 - d^2)} ,
\]

\[
\tilde{p}_{72}^* = p_{72}^* + \frac{1}{2} d c_{s2} + \frac{(2 - d^2)\Delta a_2 + d\Delta a_1}{2(1 - d^2)} .
\]

Proof of Theorem 6 is given in Appendix A. In particular, the above seven cases are mutually exclusive and have covered all possibilities. Theorem 6 implies that the optimal retail price is an increasing function of the incremental market scales \(\Delta a_i\) and \(\Delta a_j\). From Theorem 6 and Eq. (1), we have the following.

**Corollary 1.** Assume \(\sgn(\Delta a_1) = \sgn(\Delta a_2)\). For the centralized channel, the optimal order quantities are:

(1) if \(\Delta a_1 \geq \max\{c_{u1} - d c_{u2}, 0\}\) and \(\Delta a_2 \geq \max\{c_{u2} - d c_{u1}, 0\},\)

\[
\bar{q}_i^* = q_i^* + \frac{1}{2}(\Delta a_i - c_{w} + d c_{w});
\]
(2) if $0 \leq \Delta a_1 < c_{u1} - dc_{u2}$, $\Delta a_2 + d\Delta a_1 \geq c_{u2}(1 - d^2)$ and $\Delta a_2 \geq 0$, 
\[ \tilde{q}_1^* = q_{1}^*, \quad \tilde{q}_2^* = q_{2}^* + \frac{1}{2}(d\Delta a_1 + \Delta a_2) - \frac{1}{2}(1 - d^2)c_{u2}; \]
(3) if $\Delta a_1 + d\Delta a_2 \geq (1 - d^2)c_{u1}$, $\Delta a_1 \geq 0$ and $0 \leq \Delta a_2 < c_{u2} - dc_{u1}$, 
\[ \tilde{q}_1^* = q_{1}^* + \frac{1}{2}(\Delta a_1 + d\Delta a_2) - \frac{1}{2}(1 - d^2)c_{u1}, \quad \tilde{q}_2^* = q_{2}^*; \]
(4) if $-(1 - d^2)c_{u2} < \Delta a_i + d\Delta a_j < (1 - d^2)c_{u_i}, \quad i = 1, 2, j \neq i$, 
\[ \tilde{q}_i^* = q_{i}^*; \]
(5) if $\Delta a_1 \leq \min\{dc_{s2} - c_{s1}, 0\}$ and $\Delta a_2 \leq \min\{dc_{s1} - c_{s2}, 0\}$, 
\[ \tilde{q}_1^* = q_{1}^* + \frac{1}{2}(\Delta a_1 + c_{s1} - dc_{s2}); \]
(6) if $dc_{s2} - c_{s1} < \Delta a_1 \leq 0$, $\Delta a_2 \leq 0$ and $\Delta a_2 + d\Delta a_1 \leq -(1 - d^2)c_{s2}$, 
\[ \tilde{q}_1^* = q_{1}^* \text{ and } \tilde{q}_2^* = q_{2}^* + \frac{1}{2}(d\Delta a_1 + \Delta a_2) + \frac{1}{2}(1 - d^2)c_{s2}; \]
(7) if $\Delta a_1 + d\Delta a_2 \leq -(1 - d^2)c_{s1}$, $\Delta a_1 \leq 0$ and $dc_{s1} - c_{s2} < \Delta a_2 \leq 0$, 
\[ \tilde{q}_1^* = q_{1}^* + \frac{1}{2}(\Delta a_1 + d\Delta a_2) + \frac{1}{2}c_{s1}(1 - d^2) \text{ and } \tilde{q}_2^* = q_{2}^*; \]

From Corollary 1, we find that for each retailer the optimal order quantities for the centralized channel are more (less) than the planned quantities if the market scales are sufficiently larger (smaller) than the original market scales. If the demand disruption is small for a retailer, then he may need to keep the original order quantity, though the other retailer will order a different quantity. This is quite different from the case of manufacturer bearing the deviation cost where the only possibility of keeping the original order quantity is $\Delta a_1 = \Delta a_2$.

6.2. Decision-making for decentralized supply chain

In this subsection, we will give the Nash equilibrium of a decentralized supply chain. Because the retailers bear the deviation costs, the profit function for retailer $i$ is given by
\[ \tilde{p}_i = (\tilde{p}_i - c_i - \bar{w}(\bar{a}_i - \tilde{p}_i + d\tilde{p}_j) + \tilde{b}(\bar{a}_i - \tilde{p}_i + d\tilde{p}_j)^2 - \tilde{T} - c_w(\tilde{q}_1^* + \tilde{p}_i - d\tilde{p}_j - \bar{a}_i)^+ - c_w(\bar{a}_i - \tilde{p}_i + d\tilde{p}_j - \bar{a}_1)^+; \] \]

In the centralized decision-making case, we have shown that it is impossible that one retailer orders strictly more and the other orders strictly less. The following theorem shows this is still true for a decentralized supply chain.

**Theorem 7.** Assume $\sgn(\Delta a_1) = \sgn(\Delta a_2)$. It is impossible that one retailer orders strictly more and the other orders strictly less when the supply chain is coordinated.

**Proof.** Without loss of generality, suppose that retailer 1 orders strictly more and retailer 2 orders strictly less. In this case, when the decentralized supply chain is coordinated, the Nash equilibrium retail prices should be equal to the optimal retail prices of the centralized channel. From Theorem 5 we know that it is impossible that one retailer orders strictly more and the other orders strictly less. □

From Theorems 5, 6 and 7, we can derive the following.

**Corollary 2.** In both the centralized channel and the (coordinated) decentralized channel, both retailers order more than the planned quantities if $\sgn(\Delta a_1) = \sgn(\Delta a_2) > 0$; both retailers order less than the planned quantities if $\sgn(\Delta a_1) = \sgn(\Delta a_2) < 0$.

Note that Corollary 2 only says that both retailers simultaneously order more or less goods, not strictly more or less goods. As we already shown, when both $|\Delta a_1|$ and $|\Delta a_2|$ are very small, the retailers may keep the original order quantities.

According to Theorem 7 and Corollary 2, we only need to consider two cases: (a) market scales increase (i.e. $\sgn(\Delta a_1) = \sgn(\Delta a_2) > 0$), and (b) market scales decrease (i.e. $\sgn(\Delta a_1) = \sgn(\Delta a_2) < 0$). Note that two retailers simultaneously determine their retail prices. Furthermore, from Eq. (21), we can derive the following Theorem 8.

**Theorem 8.** The Nash equilibrium retail prices in the decentralized channel are

(1) if $\sgn(\Delta a_1) = \sgn(\Delta a_2) > 0$,
\[ p_i^N = \frac{2(1 - b)[\tilde{a}_i(1 - 2\tilde{b}) + c_i + c_{wu} + \tilde{w}] + d(1 - 2\tilde{b})[\tilde{a}_j(1 - 2\tilde{b}) + c_j + c_{wj} + \tilde{w}]}{4(1 - b)^2 - d^2(1 - 2b)^2}. \]

(2) If \( \text{sgn}(\Delta a_1) = \text{sgn}(\Delta a_2) < 0, \)
\[ p_i^N = \frac{2(1 - \tilde{b})[\tilde{a}_i(1 - 2\tilde{b}) + c_i - c_{si} + \tilde{w}] + d(1 - 2\tilde{b})[\tilde{a}_j(1 - 2\tilde{b}) + c_j - c_{sj} + \tilde{w}]}{4(1 - b)^2 - d^2(1 - 2b)^2}. \]

6.3. Coordination of supply chain

The supply chain is coordinated when the Nash equilibrium retail prices are equal to the optimal retail prices of the channel. Hence, we can solve \( \tilde{w}^* \) and \( \tilde{b}^* \) from \( p_i^N = \tilde{p}_i^* \) and \( p_j^N = \tilde{p}_j^* \), for \( k = 1, 2, \ldots, 7 \), respectively. From Theorems 6 and 8, we can derive the following.

**Theorem 9.** Assume \( \text{sgn}(\Delta a_1) = \text{sgn}(\Delta a_2) \) and the retailers bear deviation costs. For a supply chain with demand disruptions, we have the following linear quantity discount schedule to coordinate the decentralized supply chain:

1. If \( \Delta a_1 \geq \max\{c_{a1} - dc_{a2}, 0\} \) and \( \Delta a_2 \geq \max\{c_{a2} - dc_{a1}, 0\} \), then
\[ \tilde{w}^* = \frac{d(\tilde{q}_i^* + \tilde{q}_j^*)}{1 - d^2} + c_0, \quad \tilde{b}^* = \frac{d}{2(1 + d)}, \]
where
\[ \tilde{q}_i^* = q_i^* + \frac{1}{2}(\Delta a_1 - c_{wi} + dc_{wy}). \]

2. If \( \Delta a_1 \leq \min\{dc_{a2} - c_{a1}, 0\} \) and \( \Delta a_2 \leq \min\{dc_{a1} - c_{a2}, 0\} \), then
\[ \tilde{w}^* = \frac{d(\tilde{q}_i^* + \tilde{q}_j^*)}{1 - d^2} + c_0, \quad \tilde{b}^* = \frac{d}{2(1 + d)}, \]
where
\[ \tilde{q}_i^* = q_i^* + \frac{1}{2}(\Delta a_1 + c_{si} - dc_{sy}). \]

For the other cases, the analytic expression of the linear quantity discount schedule is very complex and thus omitted. Instead, some numerical examples are given in Section 8 for an illustration. Also note that we have omitted the range of the fixed fee \( \bar{T} \) in Theorem 9.

Again, we see that in the linear quantity discount schedule, the discount slope \( \tilde{b}^* \) keeps unchanged after demand disruptions for the above two cases. However, it is no longer true for other cases, as shown by the numerical examples in Section 8. So, the case of retailers bearing deviation costs is much more complicated than the case of manufacturer bearing deviation costs. This is mainly due to the fact that the deviation costs prevent one retailer or two retailers from changing order quantities rather than prevent the manufacturer from changing the total production quantity.

7. An all-unit quantity discount mechanism for supply chain coordination

Besides the linear quantity discount schedule, an all-unit quantity discount schedule is also often used to coordinate a supply chain (for example, Chen et al., 2001; Qi et al., 2004). In this paper, we briefly investigate whether and how an all-unit quantity discount schedule can coordinate a supply chain with demand disruptions. For simplicity, we assume that the two retailers are identical. Thus, the profit function of retailer \( i \) is given by
\[ \pi_i = (p_i - c - w)(a - p_i + dp_i), \]
where \( p_i > c + w \), i.e., the unit profit is positive. Then the total channel profit is
\[ \Pi(p_1, p_2) = \sum_{j=1}^{2} (p_i - c - c_0)(a - p_i + dp_i). \]

The optimal solution for Eq. (23) with respect to \( p_1 \) and \( p_2 \) are \( p^* = p_1^* = p_2^* = a/[2(1 - d)] + (c_0 + c)/2 \). Furthermore, the optimal order quantities are \( q^* = q_1^* = q_2^* = \frac{1}{2}[a - (1 - d)(c_0 + c)]. \)

For the supply chain with two identical retailers, it is sufficient for us to consider an all-unit quantity discount scheme with a breakpoint \( q^* \), and two corresponding wholesale prices \( w_0 > w_1 \). If the order quantity \( q \) is strictly less than \( q^* \), the unit wholesale price is \( w_0 \), and if \( q \geq q^* \), the unit wholesale price is \( w_1 \).
Similar to Qi et al. (2004) and Xiao et al. (2005), we can show that the all-unit quantity discount scheme with \( w = dq^*/(1 - d) + c_0 \) and a sufficient large \( w \) can coordinate the supply chain. Similar to Chen et al. (2001), we can show that an all-unit quantity discount scheme cannot be guaranteed to coordinate the supply chain with two non-identical retailers.

The optimal solution of Eq. (25) is

\[
\tilde{p}_i = \begin{cases} 
\tilde{p}_1^* + \tilde{p}_2^* & \text{if } \Delta a \geq (1 - d)c_u \\
\tilde{p}_1 - \tilde{p}_2 & \text{if } (1 - d)c_s < \Delta a < (1 - d)c_u \\
\tilde{p}_1 - \tilde{p}_2 & \text{if } -a < \Delta a \leq -(1 - d)c_s 
\end{cases}
\]

(25)

The proof is given in Appendix A. From Eq. (25), we can obtain the original order quantities \( \tilde{q}_1^*, \tilde{q}_2^* \) for the centralized supply chain. In any case, we always have \( \tilde{q}_1^* = \tilde{q}_2^* \), and their values are given in the following Theorem 11.

The decentralized supply chain will be coordinated if the Nash equilibrium retail prices satisfy \( \tilde{p}_i = (a + \Delta a + c + w)/(2 - d) = \tilde{p}^* \). The following theorem shows how an all-unit quantity discount schedule can coordinate a supply chain with demand disruptions.

Theorem 11. The decentralized supply chain with two identical competing retailers can be coordinated after demand disruptions by an all-unit quantity discount schedule with the breakpoint \( \tilde{q}^* \), a sufficient large wholesale price \( \tilde{w}^*_0 \) and

1. If \( \Delta a \geq (1 - d)c_u \), then \( \tilde{w}^*_0 = dq^*/(1 - d) + c_0 + c_u \), where \( q^* = \{a + \Delta a - (1 - d)(c_0 + c + c_u)\} \);
2. If \( - (1 - d)c_s < \Delta a < (1 - d)c_u \), then \( \tilde{w}^*_0 = (dq^*)/(1 - d) + (\Delta a)/(1 - d) + c_0 \), where \( q^* = dq^* \);
3. If \( -a < \Delta a \leq -(1 - d)c_s \), then \( \tilde{w}^*_0 = dq^*/(1 - d) + c_0 - c_s \), where \( q^* = \{a + \Delta a - (1 - d)(c_0 + c - c_s)\} \).

The wholesale price \( \tilde{w}^*_0 \) can be simply set to be sufficiently large so that the retailers would not like to order less than the breakpoint \( q^* \). Since the profit of the retailer is a decreasing function of its order quantity when the order quantity is larger than the breakpoint, it is optimal for the retailers to order \( q^* \) unit goods. Theorem 11 also shows that when the disruption is small, the retailers should keep the original order quantity; and Theorem 10 shows the retail prices should be changed as long as a disruption occurs. This is the same observation as the single-retailer model in Qi et al. (2004).

8. Decision-making for a supply chain without quantity discount

Up to this point, we have assumed that the supply chain is coordinated by a quantity discount policy. Sometimes, the manufacturer may not provide retailers such a policy and simply offers a constant unit wholesale price \( \tilde{w} \) for all products. Each retailer then determines an optimal order quantity to maximize his own profit. Still, it is a Stackelberg game where the manufacturer is the leader and the retailers are followers. However, the supply chain is not coordinated. We will now discuss how the previous approach can be applied to analyze the impact of demand disruption to such an uncoordinated supply chain.

For simplicity, we only consider the case where the manufacturer bears the production deviation cost after two demand disruptions. Thus, the profit of retailer \( i \) is

\[
\pi_i = (\tilde{p} - c_i - \tilde{w})(\tilde{a}_1 - \tilde{p}_1 + d\tilde{p}_2),
\]

(26)

the profit of the manufacturer is

\[
\pi_M = (\tilde{w} - c_0)[\tilde{a}_1 + \tilde{a}_2 - (1 - d)(\tilde{p}_1 + \tilde{p}_2)]
- c_u[\tilde{a}_1 + \tilde{a}_2 - (1 - d)(\tilde{p}_1 + \tilde{p}_2) - \tilde{q}_1^* - \tilde{q}_2^*] +
- c_s[(1 - d)(\tilde{p}_1 + \tilde{p}_2) + \tilde{q}_1^* + \tilde{q}_2^* - \tilde{a}_1 - \tilde{a}_2] +
\]

(27)
where $\hat{q}_i^\ast = [a_i(6 - d) - a_i(2 - 3d) - (2 - d^2)(2c_0 + 3c_i - c_0)]/[4(4 - d^2)]$ is the optimal order quantity of retailer $i$ in a normal environment, and $\hat{a}_i = a_i + \Delta a_i$ models the disruption. The case of $\Delta a_1 = \Delta a_2 = 0$ represents the normal condition.

We analyze the problem by backward induction. Firstly, solving the first-order conditions of Eq. (26), we obtain the reaction function of retailer $i$

$$\hat{p}_i(\hat{w}) = \frac{2(\hat{a}_i + c_j + \hat{w}) + d(\hat{a}_j + c_j + \hat{w})}{4 - d^2}.$$  

(28)

which is an increasing function of wholesale price, market scales and unit costs of retailers. Inserting Eq. (28) into Eq. (27), similar to Theorem 1, we obtain the following.

**Theorem 12.** For the Stackelberg game, the equilibrium wholesale price of manufacturer is

1. if $\Delta a_1 + \Delta a_2 \geq 2(1 - d)c_u$, then

$$\hat{w}^\ast = \frac{\hat{a}_1 + \hat{a}_2}{4(1 - d)} + \frac{1}{2}(c_0 + c_u) - \frac{1}{4}(c_1 + c_2)$$

2. if $-2(1 - d)c_u < \Delta a_1 + \Delta a_2 < 2(1 - d)c_u$, then

$$\hat{w}^\ast = \frac{\hat{a}_1 + \hat{a}_2 + \Delta a_1 + \Delta a_2}{4(1 - d)} + \frac{1}{2}c_0 - \frac{1}{4}(c_1 + c_2),$$

3. if $\Delta a_1 + \Delta a_2 \leq -2(1 - d)c_u$, $\Delta a_1 > -a_1$, and $\Delta a_2 > -a_2$, then

$$\hat{w}^\ast = \frac{\hat{a}_1 + \hat{a}_2}{4(1 - d)} + \frac{1}{4}(c_0 - c_s) - \frac{3}{4}(c_1 + c_2).$$

The equilibrium retail price of retailer $i$ is the $\hat{p}_i^\ast(\hat{w}^\ast)$ given by Eq. (28).

Theorem 12 implies that the equilibrium wholesale price depends on the overall effect of the disruption $\Delta a_1 + \Delta a_2$ for all cases and is an increasing function of $\Delta a_1 + \Delta a_2$, which differs from the result of Theorem 4 for the case of a coordinated supply chain. Compare Theorem 12 with Theorem 4, we find that the two cases, with and without quantity discount policies, have the same thresholds of $\Delta a_1 + \Delta a_2$ for the manufacturer to make decisions: the manufacturer will determine a wholesale price to keep the original total production quantity if the total changed amount of demands $|\Delta a_1 + \Delta a_2|$ is sufficiently small. However, the members of an uncoordinated supply chain have different pricing and ordering strategies comparing with a coordinated one, which implies that the demand disruption affects the ‘coordination mechanism.’

9. Numerical examples

In the above sections, we investigate the coordination of the supply chain with demand disruptions theoretically. To well understand the effects of demand disruptions under different situations on the coordination of supply chain, we will give some numerical examples in this section.

**Example 1.** Manufacturer bears the deviation cost.

Consider the linear quantity discount schedule. Recall that the discount slope is always $d/[2(1 + d)]$ when the manufacturer bears the production deviation cost. Hence, we only need to consider the maximum variable wholesale price $\hat{w}^\ast$ for the linear quantity discount schedule. Consider the following problem with parameters:

$$a_1 = a_2 = 10, \quad c_0 = 2, \quad c_1 = c_2 = 1, \quad c_u = c_s = 0.5, \quad d = 0.5.$$

Suppose retailer 1 experiences a demand disruption. Based on Theorem 2, we can illustrate the linear quantity discount schedule for different demand disruptions, which are depicted in Figs. 1 and 2. Note that in the figures the demand disruption is indicated by ‘$\Delta a^\ast$’.

From Fig. 1, we see that the curve of the maximum variable wholesale price $\hat{w}^\ast$ during the interval $\Delta a \in [-0.5, 0.5]$ is the steepest in order to prevent the retailers from deviating the planned quantities.

Fig. 1. The maximum variable wholesale price verses the demand disruption.
Meanwhile, Fig. 2 shows that the optimal quantity of the channel for any \( \Delta \alpha \in [-0.5, 0.5] \) is unchanged. But for any \( \Delta \alpha \notin [-0.5, 0.5] \), the channel will change the total quantity in despite of the deviation cost. In other words, it is optimal for the channel to change the total quantity when the change of the market scale is very large.

Example 2. Retailers bear the deviation costs.

In Section 6, we study the case where the retailers bear the deviation costs. We have given the coordination mechanism for the supply chain for two symmetrical cases in Theorem 9, but left other cases open. We now use some numerical examples to show how the coordination scheme works.

Consider the following example with parameters as

\[
a_1 = a_2 = 10, \quad c_0 = 2, \quad c_1 = c_2 = 1, \\
c_{u1} = c_{u2} = c_{s1} = c_{s2} = 0.5, \quad d = 0.5.
\]

In Table 1, we present the coordination schemes for the example for some specific demand disruptions \( \Delta a_1 \) and \( \Delta a_2 \). In the table, each entry is represented by a vector \((\bar{b}^*, \tilde{\nu}^*)\) that defines a linear quantity discount schedule.

Note that in the table the blank spaces represent the cases where \( \text{sgn}(\Delta a_1) \neq \text{sgn}(\Delta a_2) \), which is inconsistent with our assumption. It is interesting that the discount slope \( \bar{b}^* = 1.5 \) when Program (A.1) or (A.2) has at least one binding constraint, and for other cases \( \bar{b} = 1/6 \), the original value without disruptions. This illustrates the managerial complexity for the case of retailers bearing deviation costs.

Example 3. Decision-making for a supply chain without quantity discount.

In the Section 8, we discuss the members’ decisions when the manufacturer plays a Stackelberg game with the retailers. We assume that the parameters have the same values as Example 1. Based on Theorem 12, we can illustrate the effect of demand disruptions on the wholesale price, which is depicted in Fig. 3.

Compare Fig. 3 with Fig. 1, we find some similar characteristics. But the curve in Fig. 3 is steeper than that in Fig. 1, which means that the effect of demand disruptions on the wholesale price for the uncoordinated supply chain is larger than that for the coordinated supply chain. This also implies the importance of coordination.

10. Conclusions

In this paper, we have considered the coordination of a supply chain with one manufacturer and
two competing retailers. We mainly focus on the coordination of the supply chain when the demands are disrupted and analyze the effects of the changed amount of market scales on the coordination mechanism and the optimal decision-making. We also study the decision-making of members when the manufacturer does not use a quantity discount schedule.

We first study the coordination of the supply chain with one demand disruption, and then extend to the case with two demand disruptions. In the first two models, we assume that the manufacturer bears the production deviation cost and the supply chain is to be coordinated by a linear quantity discount schedule. The results are further extended to the case where the retailers bear the deviation cost. We also briefly discussed the case of using an all-unit quantity discount coordination scheme.

We believe that there are still many interesting problems to study in this field. A direct extension of the paper will be the case in which both manufacturer and retailers share the deviation cost. One can also investigate how a supply chain with two competing non-identical retailers is coordinated by an all-unit quantity discount schedule. The more general case with more than two competing retailers is still another important problem to be studied.

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Appendix A. Proof of Theorem 6

Firstly, we consider the case in which two retailers order more products simultaneously, i.e., \(\Delta a_1 \geq \tilde{p}_1 + q_1^* - a_1 - d\tilde{p}_2 \) and \(\Delta a_2 \geq \tilde{p}_2 + q_2^* - a_2 - d\tilde{p}_1\). In this case, Eq. (19) is equivalent to the following:

\[
\hat{H}_1 = \sum_{j=1}^{2} \left[ (p_i - c_0 - c_i)(\tilde{a}_i - \tilde{p}_i + d\tilde{p}_j) - c_u(\tilde{a}_i - \tilde{p}_i + d\tilde{p}_j - q_i^*) \right].
\]  

(A.1)

The Kuhn-Tucker condition of (A.1) is that at the optimal retail price profile \((\tilde{p}_1^*, \tilde{p}_2^*)\), \(\exists \lambda_1, \lambda_2 \geq 0\) such that

\[
\begin{align*}
\hat{\theta}H_1 &= \hat{\theta}p_i - \lambda_i + d\lambda_j = 0, \\
\hat{\theta}a_i - \hat{\theta}p_i + d\hat{\theta}p_j - q_i^* = 0, \\
\lambda_i[\tilde{a}_i - \tilde{p}_i + d\tilde{p}_j - q_i^*] = 0,
\end{align*}
\]

where \(i = 1, 2, j \neq i\) and \(\lambda_i\) is Lagrangian multipliers.

Recall that \(\text{sgn}(\Delta a_1) = \text{sgn}(\Delta a_2)\).

If \(\Delta a_1 \geq \max\{c_{u1} - d\tilde{c}_{u1}, 0\}\) and \(\Delta a_2 \geq \max\{c_{u2} - d\tilde{c}_{u2}, 0\}\), then the optimal solution of (A.1) is

\[
\tilde{p}_1^* = p_1^* + \frac{1}{2}c_{u1} + \frac{(2 - d^2)\Delta a_1 + d\Delta a_2}{2(1 - d^2)}, \quad \tilde{p}_2^* = p_2^* + \frac{1}{2}c_{u2} + \frac{d\Delta a_1 + \Delta a_2}{2(1 - d^2)}.
\]

If \(0 \leq \Delta a_1 < c_{u1} - d\tilde{c}_{u1}\), \(\Delta a_2 \geq c_{u2}(1 - d^2)\) and \(\Delta a_2 \geq 0\), then the optimal solution of (A.1) is

\[
\tilde{p}_1^* = p_1^* + \frac{1}{2}c_{u1} - \frac{d\Delta a_1 + d\Delta a_2}{2(1 - d^2)}, \quad \tilde{p}_2^* = p_2^* + \frac{1}{2}c_{u2} + \frac{(2 - d^2)\Delta a_2 + d\Delta a_1}{2(1 - d^2)}.
\]

If \(d\Delta a_1 + d\Delta a_2 < (1 - d^2)c_{u1}\) and \(d\Delta a_2 + d\Delta a_1 < (1 - d^2)c_{u1}\), the optimal solution of (A.1) is

\[
\tilde{p}_1^* = p_1^* + \frac{1}{2}c_{u1} + \frac{(2 - d^2)\Delta a_2 + d\Delta a_1}{2(1 - d^2)}, \quad \tilde{p}_2^* = p_2^* + \frac{1}{2}c_{u2} + \frac{(2 - d^2)\Delta a_1 + d\Delta a_2}{2(1 - d^2)}.
\]

The Kuhn-Tucker condition for (A.2) is that at the optimal retail price profile \((\tilde{p}_1^*, \tilde{p}_2^*)\), \(\exists \lambda_1, \lambda_2 \geq 0\) such that

\[
\begin{align*}
\hat{\theta}H_2 &= \hat{\theta}(\tilde{a}_i - \tilde{p}_i + d\tilde{p}_j - q_i^*) \quad \text{for } i, j = 1, 2, \lambda_i \neq \lambda_j \neq 0, \\
q_i^* - \tilde{a}_i + \tilde{p}_i - d\tilde{p}_j &\geq 0, \\
\lambda_i[q_i^* - \tilde{a}_i + \tilde{p}_i - d\tilde{p}_j] &= 0,
\end{align*}
\]

where \(i = 1, 2, j \neq i\) and \(\lambda_i\) are Lagrangian multipliers.
The optimal solution of (A.2) is the following:
If $\Delta a_1 \leq \min\{dc_{s2} - c_{s1}, 0\}$ and $\Delta a_2 \leq \min\{dc_{s1} - c_{s2}, 0\}$, then
$$p^*_1 = p^*_2 = \frac{1}{2}c_{s1} + \frac{\Delta a_1 + d\Delta a_1}{2(1 - d^2)},$$
$$p^*_{s1} = p^*_{s2} = \frac{1}{2}d\Delta a_2 + \frac{\Delta a_2}{2(1 - d^2)}.$$

If $dc_{s2} - c_{s1} < \Delta a_1 \leq 0$, $\Delta a_2 \leq 0$ and $d\Delta a_2 + dc_{s2} \leq 0$, then
$$p^*_{s1} = p^*_1 = \frac{1}{2}d\Delta a_2 + \frac{\Delta a_2}{2(1 - d^2)},$$
$$p^*_{s1} = p^*_2 = \frac{1}{2}c_{s2} + \frac{d\Delta a_1 + dc_{s2}}{2(1 - d^2)}.$$

If $\Delta a_1 + d\Delta a_2 < -(1 - d^2)c_{s1}$, $\Delta a_1 \leq 0$ and $dc_{s1} - c_{s2} < \Delta a_2 \leq 0$, then
$$p^*_{s1} = p^*_2 = \frac{1}{2}c_{s1} + \frac{d\Delta a_2 + dc_{s2}}{2(1 - d^2)},$$
$$p^*_{s2} = p^*_1 = \frac{1}{2}d\Delta a_1 + \frac{\Delta a_1}{1 - d^2}.$$

Note that we can merge the two cases for $\tilde{p}^*_i$ and $\tilde{p}^*_{s1}$ into a single case, i.e., the case 4 in the theorem $-(1 - d^2)c_{s1} < \Delta a_1 + d\Delta a_2 < (1 - d^2)c_{s1}$, $i = 1, 2, j \neq i$. □

Proof of Theorem 10.
We can differentiate Eq. (24) into two cases: (a) $\tilde{a} \geq \frac{1}{2}(1 - d)(\tilde{p}_1 + \tilde{p}_2) + \tilde{q}^*$, and (b) $\tilde{a} \leq \frac{1}{2}(1 - d)(\tilde{p}_1 + \tilde{p}_2) + \tilde{q}^*$. For case (a), Eq. (24) becomes
$$\tilde{H}_1 = \sum_{j=1}^{2}(\tilde{p}_j - c_0 - c)(\tilde{a} - \tilde{p}_i + d\tilde{p}_y)$$
$$- c_u[2\tilde{a} - (1 - d)(\tilde{p}_1 + \tilde{p}_2) - 2\tilde{q}^*]. \quad (A.3)$$
For case (b), Eq. (24) becomes
$$\tilde{H}_2 = \sum_{j=1}^{2}(\tilde{p}_j - c_0 - c)(\tilde{a} - \tilde{p}_i + d\tilde{p}_y)$$
$$+ c_u[2\tilde{a} - (1 - d)(\tilde{p}_1 + \tilde{p}_2) - 2\tilde{q}^*]. \quad (A.4)$$

The optimal solution of (A.3) is
$$\tilde{p}^* = \tilde{p}^*_1 = p^*_2 = p^* + \frac{\Delta a}{2(1 - d)} + \frac{1}{2}c_u,$$
for $\Delta a \geq (1 - d)c_u$.

Thus, the theorem follows. □

References