Optimal pricing and advertising competition in two supply chains with deterministic demand

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Abstract: In this paper, the impact of advertisement on the competitiveness of supply chains is examined with a price and advertising sensitive linear deterministic demand. Within the two supply chains investigated, two retailers compete for supply to customers and two manufacturers compete for advertisement. The pricing and the advertising level decisions in the system are analysed by two two-stage Stackelberg games with different decision rights designated to the parties involved. The manufacturers first set the advertising levels. Then, the retailers choose the retail prices simultaneously and independently. The retail price equilibrium and the advertising level equilibrium are characterised in centralised supply chain and in decentralised supply chain. Also, this paper shows how the optimal retail prices and advertising levels change with system parameters. In the system, there is an optimal degree of demand substitution, which minimises the manufacturer’s advertising investment cost. Moreover, as retailer 1’s market potential increases, the average advertising level decreases in the industry.

Keywords: supply chain; pricing; competition; advertising level; deterministic demand; Stackelberg game; Nash equilibrium.


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1 Introduction

One of the most controversial issues in marketing is the role of advertising on the retail price competition. In the last two decades, a good deal of attention has been devoted to the study of advertising competition in a lot of different frameworks (Banerjee and Bandyopadhyay, 2003; Calloway and Mesak, 1999; Eckard and Woodrow, 1987; Erikson, 1995, 2003; Gabszewicz et al. 2004; Gal-Or et al., 2006; Lancaster, 1984; Mesak, 2003; Mesak and Calloway, 1995a,b; Naik and Tsai, 2000; Nguyen and Shi, 2006; Park and Hahn, 1991; Piga, 2000; Seldon et al., 1993). Among these, models of advertising competition and its effects on consumer behaviour and market performance can be classified into two broad groups. One looks at advertising as a channel that provides valuable information to consumers, enabling them to make rational choices by reducing informational product differentiation. The other group views advertising as a device that persuades consumers by means of intangible and/or psychic differentiators. It creates differentiation among products, which at times may not be real. This is especially true for most ‘feel’ products, such as beer, cigarettes, soft drinks, perfumes, etc. (Banerjee and Bandyopadhyay, 2003). In this paper, the latter point is an important assumption that advertisement can increase customer demand.

In most mature categories in the consumer goods industry, the role of affective persuasion through mass media advertising is an important aspect of the entire marketing strategy. The competition between Coke and Pepsi is a case in point. What’s more, the advertising competition in the broadcasting industry is another example (see Gabszewicz et al., 2004 for a detailed example). This paper assumes that there are two manufacturers (Figure 1) who come from two different countries selling their products (one product with different brands) in a third country. And their special retailers in the third country sell the products of the manufacturers. In order to maximise the profit each special retailer decides on the retail price considering the competing retailer’s decision. The manufacturers carry on advertisement in the third country to increase their demands. Each manufacturer chooses an advertising level for maximises his profit considering the competing manufacturer’s decision. The competition in motorbikes between Chinese manufacturer and Japanese manufacturer is a similar example in Vietnam in 20th century. Additionally, there are a lot of manufacturers who come from different countries (e.g. China, Korea and Japan) in American competing in electronic appliances. Hence, the purpose of this paper is to investigate the advertising competition and the following key questions are settled:

1 How do the supply chains interact in the advertising competition and how do they make their individual decisions in equilibrium?
2 How to get the optimal retail price and advertising level in decentralised supply chain, and how do system parameters affect the optimal retail price and advertising level?
3 As retailer 1’s market potential increases, how does the average industry advertising level change (increases or decreases)?

The investigation is performed under the following channel setting. Two upstream manufacturers produce a product at a constant marginal cost. They then sell the products through one retailer respectively. Each retailer incurs a wholesale price cost. Then the retailers sell the product to customer in an oligopoly market. Specially, customer demand is both price sensitive and non-factor price (e.g. advertisement) sensitive. The
manufacturers can increase their demand by carrying on advertisement, and their advertisement has a negative effect on the competing manufacturer’s demand. The model consists of two firms (belongs to a supply chain) as a Stackelberg (leader-follower) game: The manufacturer, acting as the leader, offers the retailer an advertising level. The retailer, acting as a follower, chooses how many units of the product to produce and the retail price. The retailer accepts the advertising level as long as he can earn a positive profit. Normalise that the retailer’s reservation profit to zero. The above supply chain model, though simple, is rich enough to capture the key trade-offs and interactions of the two firms in their decision making. Although the manufacturer has the freedom to choose the advertising level, the retailer will trade-off his revenue against his production cost in choosing the production quantity and the retail price.

Figure 1  Advertising competition in two supply chains

The remainder of this paper is organised as follows. Section 2 reviews related research in this area. Model and analysis in centralised supply chain are in Section 3. In Section 4, the analysis of the decentralised supply chain model is presented. Section 4.1 is the manufacturer-retailer Stackelberg game. Vertical Nash equilibrium in decentralised supply chain is in Section 4.2. Section 5 provides numerical study. Conclusion is in Section 6 and all mathematical proofs are in the appendix.

2 Literature review

This paper has (or has a combination of) three distinctive features:

1 advertising competition between manufacturers (or retailers)
2 Stackelberg game in supply chain
3 attributes (e.g. advertising).

Most of them investigated advertising competition by differential approach or game theoretic approach. Banerjee and Bandyopadhyay (2003) construct a multistage game-theoretic model of advertising and price competition in a differentiated products duopoly. Advertising simultaneously plays the dual role in reducing such inertia through awareness and enhancing perceived brand value (persuasion). They characterise the nature of equilibrium under symmetry and show that when a large proportion of consumers exhibit inertial tendencies. Mesak (2003) develops a market share attraction model which is estimated using a data set of six manufacturers related to the ready-to-eat cereal industry in the USA. Piga (2000) develops a differential duopolistic game where price is sticky and firms can invest in market-enlarging promotional activities which have a public good nature. Mesak and Calloway (1995) present a theoretical model supported by both numerical and empirical analyses to evaluate the advertising policies of pulsation versus uniform spending a static continuous Lanchester competitive model of two rival firms. Their model demonstrates that generalising monopolistic results might not be adequate. Also, Erikson (1995, 2003) and Park and Hahn (1991) investigated the advertising competition with game theoretic approach. The effects of advertising on the retail price competition are consistent with those stated in both the market power school and market competition school. The effects of advertising are twofold. It can increase and decrease retail price competition at the same time. The effects of advertising on brand penetration and promotional pricing have both negative and positive influences, respectively. The retail price competition is decreased as retailers are forced to meet the brand demand, and increased as retailers use advertised brand as loss leader for promoting the store’s overall sales (Chen, 2004). Shaffer and Zettelmeyer (2004) show that

1. manufacturers can be worse off from advertising that reduces the cross-price elasticity between their products
2. channel conflict need not arise, even when the sole purpose of advertising is to affect cross-price elasticity
3. depending on its bargaining power, a retailer can be better off when the manufacturers’ products are perceived to be less substitutable.

Another stream of research related to this paper is the Stackelberg game in supply chain. Wang et al. (2004) develop a Stackelberg game model in supply chain that a manufacturer produces the product and then sells it to market through a retailer under consignment. In their model, the retailer, acting as the leader, offer the manufacturer a take-it-or leave-it contract, which specifies the percentage allocation of sales revenue between himself and the manufacturer, acting as a follower, chooses how many units of the product to produce and the retail price. They fully characterise the decentralised decisions and then derive closed-form performance measures using multiplicative demand model. And then Cachon and Lariviere (2005) extend that model with competing retailers. Narayanan et al. (2005) model a manufacturer that contracts with two retailers, who then choose retail prices and stocking quantities endogenously in a Bayesian Nash equilibrium. If the manufacturer designs a contract that is accepted by both retailers, it sets the wholesale price as a compromise between two conflicting roles: reducing intrabrand retail price competition and inducing retailers to stock closer to first-best levels (that is, optimum for the supply chain as a whole). They state that the manufacturer eliminates retail competition by designing a contract accepted by only one retailer, and the assignment of consumers to retailers is inefficient. Choi (1991) examines
a channel structure with two competing manufacturers and one common retailer, which
sells both manufacturers’ products. The study includes a one-period problem with
deterministic, price-sensitive demand, and three non-cooperative games of different
power structures between the two manufacturers and the retailer, that is, two Stackelberg
games and one Nash game. This literature also includes Iyer (1998), Vilcassim et al.
(1999), Wang and Benaroch (2004), Yang and Zhou (2006), Xia and Gilber (2007) and
Lau et al. (2007).

This paper is also related to attributes (e.g. delivery time, design, distinctiveness or
advertising appeal). As has been explained in detail by Fisher (1997), the relative
importance of price and non-price factors depends on the product characteristics. Ray
(2005) develops an integrated operations-marketing model for a profit-maximising firm
dealing with an innovative product or service. He argues that attribute can increase
customer demand (Fisher, 1997; Ray, 2002). Li et al. (2002) argue that cooperative
advertising plays a significant role in marketing programmes in conventional supply
chains and makes up the majority of promotional budgets in many product lines for both
manufacturers and retailers. Li et al. (2002), Yue et al. (2006) and Nie and Xiong (2006)
develop the cooperative advertising model in a supply chain in deferent situations. They
also argue that advertising can increase customer demand. Ray and Jewkes (2004) model
an operating system consisting of a firm and its customers, where the mean demand rate
is a function of the guaranteed delivery time offered to the customers and of market
price, where price itself is determined by the length of the delivery time. They show that
it is imperative for managers to know whether customers are price or lead-time sensitive
based on the simultaneous dependence of price and demand on delivery time before
selecting a time-based competitive strategy. In addition, Sriram and Kalwani (2007)
study the optimal levels of advertising and promotion budgets in dynamic markets with
brand equity as a mediating variable. They find that advertising has a net positive impact
on demand both in the short-term and in the long-term.

This research differs in many ways from earlier research noted above:

1 This research is motivated by the quest to understand the impact of non-price
factor (e.g. advertisement) when products are substitutable. A significant
observation that underpins the model framework is that in the retail industry,
where advertising competition between manufacturers is perhaps more
dominant than price competition between the retailers for the same product.

2 In the model, the relationship between customer demand and advertisement is
considered. The manufacturer carries on advertisement, which can increase the
manufacturer or retailer’s demand. Meanwhile, the manufacturer’s
advertisement has a negative effect on the competing manufacturer’s demand.

3 In the model, the retailers (supply chains) have different intrinsic demand
potentials. In this setting, the average advertising level in the market increases
as the dominant firm’s intrinsic demand potential decreases.

3 Model and analysis in centralised supply chain

In this paper, the model consists of two supply chains. Each supply chain consists of one
manufacturer and one retailer (Figure 1). Denote the manufacturers by \( M_i (i = 1, 2) \) and
let \( R_i (i = 1, 2) \) be the set of retailers. The retailers are engaged in a pricing competition
by selling a homogeneous product at a constant marginal cost. Without losses of
generality, assume that the constant marginal costs for both manufacturers and retailers
are zero. The manufacturers provide the retailers with goods in a Stackelberg fashion.
Firstly, manufacturer \(i\) offers an advertising level, \(r_i\). Then retailer \(i\) decides on demand
quantity, \(q_i\), and manufacturer \(i\) produces the quantity, \(q_i\). Assume that the
manufacturer \(i\) is obliged to meeting retailer \(i\)’s order and has the capacity to do so (Table 1).

Table 1 Definition of variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) (\in{1,2})</td>
<td>supply chains, retailers or manufacturers</td>
</tr>
<tr>
<td>(\chi) (\in{c,d,N})</td>
<td>centralised supply chain, manufacturer-retailer Stackelberg game and Nash equilibrium in decentralised supply chain</td>
</tr>
<tr>
<td>(r_i) (\in(0,\infty))</td>
<td>the advertising level of manufacturer (i), a decision variable</td>
</tr>
<tr>
<td>(K_i(r_i)) (\in(0,\infty))</td>
<td>the advertising cost function of manufacturer (i)</td>
</tr>
<tr>
<td>(Q_i) (\in(0,\infty))</td>
<td>retailer (i)’s demand, a decision variable</td>
</tr>
<tr>
<td>(M_i) (\in(0,\infty))</td>
<td>retailer (i)’s market potential</td>
</tr>
<tr>
<td>(p_i) (\in(0,\infty))</td>
<td>retailer (i)’s retail price, a decision variable</td>
</tr>
<tr>
<td>(\omega_i) (\in(0,\infty))</td>
<td>retailer (i)’s wholesale price</td>
</tr>
<tr>
<td>(\lambda) (\in[0,1])</td>
<td>the degree of demand substitution</td>
</tr>
<tr>
<td>(\mu) (\in[0,1])</td>
<td>the degree of advertising substitution</td>
</tr>
<tr>
<td>(c_i) (\in(0,\infty))</td>
<td>supply chain (i)’s unit cost</td>
</tr>
<tr>
<td>(\alpha_i) (\in(0,\infty))</td>
<td>retailer (i)’s unit cost</td>
</tr>
<tr>
<td>(\Pi_i) (\in(0,\infty))</td>
<td>supply chain (i)’s profit in centralised supply chain</td>
</tr>
<tr>
<td>(\Pi_i^c) (\in(0,\infty))</td>
<td>supply chain (i)’s profit in the manufacturer-retailer Stackelberg game</td>
</tr>
<tr>
<td>(\Pi_i^v) (\in(0,\infty))</td>
<td>supply chain (i)’s profit in vertical Nash equilibrium</td>
</tr>
<tr>
<td>(\lambda_i^d(\lambda)) (\in[0,1])</td>
<td>the advertising level at the degree of demand substitution (\lambda) in manufacturer-retailer Stackelberg game</td>
</tr>
<tr>
<td>(\tau_i = [Q_i’/(Q_i’+Q_i)]c_i +[Q_i’/(Q_i’+Q_i)]\alpha_i)</td>
<td>the average advertising level in the industry in centralised supply chain, manufacturer-retailer Stackelberg game and Nash equilibrium in decentralised supply chain</td>
</tr>
<tr>
<td>(\Delta_i^c = (\Pi_i - \Pi_i^c)/\Pi_i^c \times 100)</td>
<td>the percentage profit loss of the supply chain in manufacturer-retailer Stackelberg game, compared with that of centralised supply chain</td>
</tr>
<tr>
<td>(\Delta_i^v = (\Pi_i - \Pi_i^v)/\Pi_i^v \times 100)</td>
<td>the percentage profit loss of the supply chain in Nash equilibrium, compared with that of centralised supply chain</td>
</tr>
</tbody>
</table>
3.1 Advertising investment

Some researchers state that the advertising can increase demand. This line of research includes Li et al. (2002), Ray (2005), Yue et al. (2006), Bass et al. (2007), Sriram and Kalwani (2007) and Nie and Xiong (2006). In this paper, manufacturer $i$ provides an advertising level, $r_i$, in the way of carrying on advertisement. There are a number of examples, which demonstrate how advertisement can influence demand (Ray, 2005). Though higher advertising level can increase demand, such an action is usually associated with substantial investment. Assume that manufacturer $i$’s advertising cost function to be (Nie and Xiong 2006; Ray, 2005; Tsay and Agrawal, 2000):

$$K_i(r_i) = k_i r_i^3, \quad i = 1, 2$$

where $k_i > 0$ represents the advertising cost to achieve the value of $r_i = 1$ and $r_i$ is the advertising level. Generally, $k_i > 0$ (e.g. $k_i > 1$), assuming it to be true for the rest of this paper. This condition implies that manufacturer $i$ increases the advertising level is not too cheap (Ray, 2005).

3.2 Demand function

Most of researchers who study supply chain competition assume the demand function to be linear and it’s

$$Q_i = M_i - p_i + \lambda p_j, \quad i, j = 1, 2, \quad j = 3 - i$$

This type of linear demand function is adopted frequently (e.g. Choi, 1991; Ingene and Parry, 1995; Trivedi, 1998) in supply chain literature. In this paper, the consumer demand for each product is assumed to be sensitive to both retail price and advertising level. Decreasing product retail price or increasing advertising level will trigger two phenomena. Firstly, a group of customers will decide to switch from the competitor’s product. Secondly, a group of customers who otherwise would not have bought either product will purchase at this lower price or higher advertising level. The opposite happens when price is increasing or advertising level is decreasing. Then the demand function is extended as to be (see Banker et al., 1998).

$$Q_i = M_i - p_i + \lambda p_j + r_j - \mu r_j, \quad i, j = 1, 2, \quad j = 3 - i, \lambda, \mu \in [0, 1]$$

where $Q_i$ is retailer $i$’s demand, $M_i$ is retailer $i$’s market potential (let $M = M_1 + M_2$ represents the total market potential), $p_i$ is retailer $i$’s retail price, $\lambda$ is the degree of demand substitution, $r_j$ is manufacturer $i$’s advertising level and $\mu$ is the degree of advertising substitution. This is similar to the demand function used in Banker et al. (1998), except their model is used to study two competing firms in his study. In addition, this is also similar to the demand function used in Tsay and Agrawal (2000), except their model is used to study a system with one manufacturer and two competing retailers in their study. All parameters are common knowledge to all parties (e.g. Lee and Whang, 2000) discuss how and why the information is shared using industry examples and relating them to academic research. Li (2002) develops a model of information sharing in a supply chain with horizontal competition.
3.3 Profit function and analysis in centralised supply chain

Centralised supply chain in this paper means that manufacturer 1 and retailer 1 are
controlled by supply chain 1 (supply chain 1 is a centralised supply chain), and
manufacturer 2 and retailer 2 are controlled by supply chain 2 (supply chain 2 is another
centralised supply chain). Hence, each centralised supply chain is controlled by one
decision-maker who chooses the retail price and the advertising level. Supply chain $i$’s
profit function can be described by the following equations

$$\Pi_i(p_i, r_i) = p_iQ_i - c_iQ_i - K_i(r_i)$$

where $p_iQ_i$ is the total revenue obtained by selling $Q_i$ units at a retail price $p_i$, $c_iQ_i$ is
the production cost incurred by selling $Q_i$ units and $K_i(r_i)$ is the advertising investment
cost. Supply chain $i$ determines $p_i^*$ and $r_i^*$ by solving

$$\frac{\partial \Pi_i(p_i, r_i)}{\partial p_i} = M_i - 2p_i + \lambda p_i + r_i - \mu r_i + c_i = 0$$

(5)

$$\frac{\partial \Pi_i(p_i, r_i)}{\partial r_i} = p_i - c_i - 2k_i r_i = 0$$

(6)

Since second-order condition, $\frac{\partial^2 \Pi_i}{\partial p_i^2} \times \frac{\partial^2 \Pi_i}{\partial r_i^2} - \left[\frac{\partial^2 \Pi_i}{\partial p_i \partial r_i}\right]^2 = 4k_i - 1 > 0$, for
$k_i > 1$, then supply chain $i$’s profit function is strictly concave in retail price and
advertising level. Solving for $p_i$ and $r_i$ simultaneously from the above equations, the
equilibrium retail price and advertising level for each supply chain are given by

$$p_i^* = \frac{X_iT_i + Y_iT_i}{X_iY_i - YY_i}$$

(7)

$$r_i^* = \frac{1}{2k_i} \times \frac{X_iT_i + Y_iT_i}{X_iY_i - YY_i} - \frac{c_i}{2k_i}$$

(8)

where

$$T_i = M_i + c_i - \frac{c_i}{2k_i} - \frac{\mu c_i}{2k_i}, \quad X_i = 2 - \frac{1}{2k_i}, \quad Y_i = \lambda - \frac{\mu}{2k_i}$$

(9)

Note that $T_i = T_j, X_i = X_j, Y_i = Y_j$ for $M_i = M_j, c_i = c_j, k_i = k_j$. Then, $p_i^* = p_j^*, r_i^* = r_j^*$. Furthermore, recall that $k_i > 1, T_i > 0, X_i > 0$ is observed. Hence, $r_i^* > 0$ is yielded for
$p_i^* > c_i$. Assume that $p_i^* > c_i, p_i > \alpha c_i + \omega_i$ and $\alpha_i > (1 - \alpha_i)c_i$ to be true for the rest of
this paper. From Equations (3), (7) and (8), the demand quantity for supply chain $i$ is
given by

$$Q_i^* = M_i - \left(1 - \frac{1}{2k_i}\right) \frac{X_iT_i + Y_iT_i}{X_iY_i - YY_i} + \left(\lambda - \frac{\mu}{2k_i}\right) \frac{X_iT_i + Y_iT_i}{X_iY_i - YY_i} + \frac{\mu c_i}{2k_i} - \frac{c_i}{2k_i}$$

(10)
Then, from Equations (7), (8) and (10), supply chain $i$’s profit is given by

$$\Pi_i' = \left(p_i' - c_i\right)Q_i' - 2k_i\left(r_i'\right)^2$$

(11)

## 4 Analysis in decentralised supply chain

In this section, the retail price equilibrium and the advertising level equilibrium are discussed under the Stackelberg competition model and Nash equilibrium model. In decentralised supply chain, retailer $i$ and manufacturer $i$’s profit functions can be described by the following equations

$$\Pi_{ri} = p_iQ_i - (\alpha_i c_i + e_i)Q_i$$

(12)

$$\Pi_{mi} = \omega_iQ_i - (1-\alpha_i)c_iQ_i - K_i(r_i)$$

(13)

### 4.1 Manufacturer-retailer Stackelberg game (MRS game)

In a Manufacturer retailer Stackelberg, the sequence of moves is as follows:

1. Keeping the wholesale price $\omega_i$ unchanged, manufacturer $i$ (the leader) decides to make advertising level $r_i$.
2. In response to $r_i$, then retailer $i$ (the follower) decides the retail price to maximise his profit function.

Retailer $i$, as the first mover, chooses the retail price to maximise retailer $i$’s profit function. In response to retailer $i$’s retail price, manufacturer $i$ then chooses the advertising level to maximise his profit function.

To pursue the Stackelberg equilibrium retail prices, retailer $i$ selects a best-reply retail price policy respectively, denoted by $p_i^d(i = 1, 2)$, which maximises retailer $i$’s profit function $\Pi_{ri}'$. Hence, retailer $i$’s best-reply price policy is given as

$$p_i^d = \arg \max_{r_i} \Pi_{ri}'$$

(14)

where $\Pi_{ri}'(p_i)$ is given in Equation (12).

Anticipating the best-reply retail price, manufacturer $i$ will choose the advertising level so that his profit function is maximised. That is, manufacturer $i$’s profit-maximisation problem can be expressed as:

$$\max_r \Pi_{mi}(r, p_i^d) = \omega_iQ_i - (1-\alpha_i)c_iQ_i - K_i(r_i)$$

(15)

### 4.1.1 Retailer $i$’s decision

For a given $r_i$ which is decided by manufacturer $i$, retailer $i$’s decision is to choose the retail price. Retailers 1 and 2 choose the retail prices simultaneously and the
first-order conditions characterising equilibrium prices are:

$$\frac{\partial \Pi_i^r(p_i)}{\partial p_i} = M_i + \omega_i + \alpha_i c_i - 2p_i + \lambda p_j + r_i - \mu r_j = 0 \quad (16)$$

Since $\frac{\partial^2 \Pi_i^r(p_i)}{\partial p_i^2} = -2 < 0$, retailer $i$’s profit function given advertising level is strictly concave in price. Solving for $p_1$ and $p_2$ simultaneously from the above two equations, the equilibrium price for retailer $i$ is given by

$$p_i^* = \frac{2S_i + \lambda S_j + (2 - \lambda \mu)r_i + (\lambda - 2\mu)r_j}{4 - \lambda^2} \quad (17)$$

where

$$S_i = M_i + \omega_i + \alpha_i c_i \quad (18)$$

The corresponding demand quantities at the equilibrium are

$$Q_i^d = M_i + \frac{(\lambda^2 - 2)S_i + \lambda S_j + R_i}{4 - \lambda^2} \quad (19)$$

where

$$R_i = (2 - \lambda \mu)r_i + (\lambda - 2\mu)r_j \quad (20)$$

From Equations (17) and (19), some results can be obtained. The following proposition describes how the retail price changes with system parameters.

Proposition 1: In MRS game, for given $r_i, r_j$, retailer $i$’s retail price is decreasing in $\mu$ and is increasing in $\lambda$ at $r_i = r_j$; also retailer $i$’s demand quantity is decreasing in $\mu$ and is increasing in $\lambda$ at $S_i = S_j$, $r_i = r_j$, $i = 1, 2$, $j = 3 - i$.

All proofs are in the appendix.

4.1.2 Manufacturer $i$’s decision

Knowing that retailer $i$ chooses $p_i^*(r_i, r_j)$ according to Equation (17) in response to the advertising level, manufacturer $i$ decides on $r_i$ to maximise his own profit. Substituting Equation (19) into Equation (13), someone can get manufacturer $i$’s profit

$$\Pi_i^m(r_i) = [\omega_i - (1 - \alpha_i)c_i]Q_i^d - k_i r_i^2 \quad (21)$$

Hence, manufacturer $i$’s optimal advertising level is given as

$$r_i^* = \arg \max_{r_i} \Pi_i^m(r_i) \quad (22)$$
where \( \Pi'_m(r_i) \) is given in Equation (21).

Knowing that retailer \( i \) chooses \( p^*_i(r_i, r_j) \), manufacturer \( i \) decides the optimal advertising level, differentiating Equation (21) with respect to \( r_i \) and equating it to zero.

\[
\frac{\partial \Pi'_m(r_i)}{\partial r_i} = \left[ \omega_i - (1 - \alpha_i) c_i \right] \left( 1 - \frac{\partial p^*_i}{\partial r_i} + \frac{\partial p^*_j}{\partial r_i} \right) - 2k_i r_i = 0
\]

Before giving the optimal advertising level, the lemma 1 is given.

Lemma 1:
\[
\frac{\partial p^*_d}{\partial r_i} = \frac{2 - \lambda \mu}{4 - \lambda^2}, \quad \frac{\partial p^*_d}{\partial r_j} = \frac{\lambda - 2 \mu}{4 - \lambda^2}, \quad i = 1, 2, \quad j = 3 - i
\]

By the Lemma 1, solving for \( r_i \) from the Equation (23), the equilibrium advertising level for manufacturer \( i \) is given by

\[
r_i^* = \frac{\omega_i - (1 - \alpha_i) c_i}{2k_i} \frac{2 - \lambda \mu}{4 - \lambda^2}
\]

Since \( \frac{\partial^2 \Pi'_m}{\partial r_i \partial r_j} = -2k_i < 0 \), manufacturer \( i \) ‘s profit function is strictly concave in the advertising level. Substituting Equation (24) into Equation (17), the optimal retail price for each retailer in MRS game is given by

\[
p^*_i = \frac{2S_i + \lambda S_j + \omega_i - (1 - \alpha_i) c_i}{2k_i} \left( \frac{2 - \lambda \mu}{4 - \lambda^2} \right)^2 + \frac{\omega_i - (1 - \alpha_i) c_i}{2k_i} \left( \frac{\lambda - 2 \mu}{4 - \lambda^2} \right) \left( \frac{2 - \lambda \mu}{4 - \lambda^2} \right)
\]

Recall that \( R_i = (2 - \lambda \mu) r_i + (\lambda - 2 \mu) r_j \). Then substituting Equation (24) into Equation (19), the optimal demand quantity for each retailer in MRS game is given by

\[
Q^*_i = M_i + \frac{\left( \lambda^2 - 2 \right) S_i + \lambda S_j}{4 - \lambda^2} + \frac{\omega_i - (1 - \alpha_i) c_i}{2k_i} \left( \frac{2 - \lambda \mu}{4 - \lambda^2} \right) + \frac{\omega_i - (1 - \alpha_i) c_i}{2k_i} \left( \frac{\lambda - 2 \mu}{4 - \lambda^2} \right) \left( \frac{2 - \lambda \mu}{4 - \lambda^2} \right)
\]

From Equations (24)–(26), supply chain \( i \) ‘s optimal profit in MRS game is given by

\[
\Pi_i^* = \left( p_i^* - c_i \right) Q_i^* - 2k_i \left( r_i^* \right)^2
\]

Recall that \( S_i = M_i + \omega_i + \alpha_i c_i, i = 1, 2 \), and then substituting \( S_i \) into Equation (25), the optimal retail price is given by

\[
p_i^* = \omega_i \theta_i + \omega_j \theta_j + c_j \xi_j + c_i \xi_j + \frac{2M_i + \lambda M_j}{4 - \lambda^2}
\]
where

\[ \theta_i = \frac{2}{4 - \lambda^i} + \frac{(2 - \lambda_2 \mu_{i})^2}{2k_i (4 - \lambda^i)^2}, \quad \theta_j = \frac{\lambda}{4 - \lambda} + \frac{(2 - \lambda \mu)(\lambda - 2 \mu)}{2k_j (4 - \lambda^j)} \]

\[ \xi_i = \frac{\alpha_i}{4 - \lambda^i} + \frac{(1 - \alpha_i)^2 (2 - \lambda_2 \mu_{i})^2}{2k_i (4 - \lambda^i)^2}, \quad \xi_j = \frac{\alpha_j}{4 - \lambda^j} - \frac{(1 - \alpha_j)^2 (2 - \lambda \mu)(\lambda - 2 \mu)}{2k_j (4 - \lambda^j)^2} \] (29)

Equation (28) shows linear relationship between retail price and wholesale prices and production costs. Obviously, \( \theta_i > \theta_j \) for \( k_i = k_j \). This implies that manufacturer’s wholesale price has more effect on retailer’s retail price than competing manufacturer’s wholesale price. Moreover, note that \( \xi_i < \xi_j \) for \( \alpha_i = \alpha_j \) and \( k_i = k_j \). This implies that supply chain’s unit production cost has less effect on retailer’s retail price than competing supply chain’s unit production cost. Hence, when supply chain’s unit production cost is higher than competing supply chain’s unit production cost, competing supply chain’s retail price is high and competing supply chain maybe earn more profit. Furthermore, from Equation (28), it is easy to know \( \partial p_i / \partial M_i = 1/(2 + \lambda) > 0 \). This implies that as retailer 1’s market potential increases, retailer 1’s retail price is increasing. Recall that \( \partial p_i / \partial M_i = 1/[2 + \lambda -(1 + \mu)/2k_i] > 0 \) according to proposition 1. Hence, \( \partial p_i / \partial M_i < \partial p_i / \partial M_j \). This indicates that as retailer 1’s market potential increases, the increasing speed of the optimal retail price in centralised supply chain is higher than in MRS game. According to Equation (24), the effect of \( \lambda \) and \( \mu \) on manufacturer i’s advertising level can be stated as the following result.

Proposition 2: In MRS game, when wholesale price is set exogenously, manufacturer i’s advertising level is decreasing in \( \mu \). Also, for \( \mu \in [0,0.8] \), manufacturer i’s advertising level is decreasing in \( \lambda \) for \( \mu \in [0, \lambda^*] \), where \( \lambda^* = (2 - 2\sqrt{1 - \mu^2}) / \mu \), and manufacturer i’s advertising level is increasing in \( \lambda \) for \( \lambda \in (\lambda^*, 1) \). Manufacturer i’s advertising level decreases in \( \lambda \) for \( \mu \in (0.8, 1] \). For example,

\[
\begin{align*}
\frac{\partial p_i}{\partial \lambda} &< 0, \quad \text{if } \lambda \in [0, \lambda^*], \quad \mu \in [0,0.8] \\
\frac{\partial p_i}{\partial \lambda} &> 0, \quad \text{if } \lambda \in [\lambda^*, 1], \quad \mu \in [0,0.8] \quad \text{for } i = 1, 2, \\
\frac{\partial p_i}{\partial \mu} &< 0, \quad \text{if } \lambda \in [0,1], \quad \mu \in (0.8, 1]
\end{align*}
\]

Proposition 2 provides useful managerial insights as to how the manufacturers should react to change in operating environment. The effect of advertising level of \( \mu \) on the optimal decision variable values is quite intuitive. As \( \mu \) becomes larger (e.g. two types of advertisement have the similar attributes), the probability of having a fewer advertising level increases. Hence, manufacturers would like to pay out fewer advertising investment cost in order to obtain more profit because consumer demand is more sensitive to price than advertisement. Similarly, when the degree of advertising
substitution is small comparatively speaking (e.g. $0 < \mu < 0.8$), as the degree of demand substitution increases (e.g. two kinds of products have the similar attributes), before the degree of demand substitution reaches $\lambda^*$, the probability of having a fewer advertising level increases (Figure 2 (a)–(d)). So the manufacturers would like to pay out fewer advertising investment cost to obtain more profit, and the reason is consumer demand is more sensitive to price changes than advertisement changes. However, when the degree of demand substitution is large comparatively speaking (e.g. $\lambda > \lambda^*$), the manufacturers would like to pay out more advertising investment because consumer demand is more sensitive to advertisement changes than price changes. If the degree of advertising substitution is large enough (e.g. $\mu \in (0.8,1)$), as degree of demand substitution increases the optimal advertising level decreases. This shows that consumer demand is more sensitive to price changes than advertisement changes. Hence, the manufacturers do not want to take more advertising investment.

Figure 2 Advertising level with different $\lambda$ and $\mu$ (a) $\mu \in [0,0.5]$, (b) $\mu \in [0.5,0.8]$, (c) $\mu = 0.5$, (d) $\mu \in [0.8,1]$ and (e) $\lambda = 0.1$
Figure 2  Advertising level with different $\lambda$ and $\mu$ (a) $\mu \in [0,0.5)$, (b) $\mu \in (0.5,0.8]$, (c) $\mu = 0.5$ (d) $\mu \in (0.8,1]$ and (e) $\lambda = 0.1$ (continued)
Corollary 1: For $\mu \in [0, 0.8]$, there is a degree of advertising substitution $\mu = \mu'$, where $\mu' = 0.5$, so as to $r_i'(\lambda = 0) = r_i'(\lambda = 1)$. Furthermore, for $\mu \in [0, 0.8]$, $\lambda' = (2 - 2\sqrt{1 - \mu^2})/\mu$ minimises $r_i'(\lambda)$ in $\lambda$.

Corollary 1 indicates that when the degree of advertising substitution equals 0.5, the advertising level is no difference between the degrees of 0 and 1 (Figure 2(c)). There is an optimal demand substitution, for example, $\lambda' = (2 - 2\sqrt{1 - \mu^2})/\mu$, which minimises the advertising level given by Equation (24). That is, when the degree of demand substitution equals $4 - 2\sqrt{3}$, manufacturer $i$’s advertising investment cost is minimal (Figure 2(c)).

From the Proposition 3 and Corollary 1, Figure 2(e) can be yielded. Figure 2(e) indicates that as the advertising level increases, the advertising level $r_i'(0) > r_i'(1)$ (e.g. $0 \leq \mu < 0.5$, see Figure 2(e)). When the advertising level is large enough (e.g. $0.5 < \mu \leq 1$), $r_i'(0) < r_i'(1)$. Especially, when $\mu = 0.5$, $r_i'(0) = r_i'(1)$. Comparing $r_i'(\lambda')$ at $\mu = 0$, $\mu = \mu'$ and $\mu = 1$, it is easy to know $r_i'(\lambda')|_{\mu=0} > r_i'(\lambda')|_{\mu=\mu'} > r_i'(\lambda')|_{\mu=1}$. This shows that as the degree of advertising substitution increases, consumer demand is more sensitive to price changes than advertisement changes. In summary, when the degree of advertising substitution is very high (e.g. $\mu = 1$), manufacturer $i$’s advertising level is $r_i'(\lambda') = (\omega - (1 - \alpha)c_i)/2k_i(2 + \lambda)$ according to Equation (24), so when $\mu = 1$ and $\lambda = 1$, manufacturer $i$’s advertising level is minimised.

In order to know how the average advertising level in the industry changes when retailer 1’s market potential increases. Let $\bar{r} = \eta_i r_i + \bar{r}$ denotes the weighted average equilibrium industry advertising level in MRS game, where $\eta_i = (Q_i^0)/Q_i^0 + Q_i^1$, $i = 1, 2$. The following results summarize the findings.

Lemma 2:

\[ \frac{\partial Q_i^0}{\partial M_i} > 0, \quad \frac{\partial Q_i^1}{\partial M_i} < 0 \]

Proposition 3: Suppose retailer 2 has an intrinsic demand advantage over retailer 1, for example, $M_1 < M_2$. Then as retailer 1’s market potential $(M_i)$ increases the average industry advertising level decreases, for example, $\partial \bar{r}/\partial M_i < 0$.

Lemma 2 indicates that if the industry is dominated by supply chain 2 that holds an advantage over its competitor in the intrinsic demand potential. Then as retailer 1’s market potential increases, supply chain 1’s selling quantity increases. At the same time, supply chain 2’s selling quantity decreases. Proposition 3 also provides useful managerial insights as to how the manufacturers should react to change in operating environment.

If the industry is dominated by supply chain 2 that holds an advantage over its competitor in the intrinsic demand potential, then as retailer 1’s market potential increases (e.g. as the difference in the intrinsic demand potentials decreases), the average
industry advertising level declines. The reason is that supply chain 1 and 2 have equal bargain power (e.g. $M_1 = M_2$).

4.2 Vertical Nash equilibrium in decentralised supply chain

In vertical Nash equilibrium, every firm (the manufacturers and the retailers) has equal bargain power. Hence, they make their decisions simultaneously. The first order conditions are given below.

\[
\frac{\partial \Pi_i}{\partial p_i} = M_i + \omega_i + \alpha_i c_i - 2p_i + \lambda p_j + r_i - \mu r_j = 0
\]

(30)

\[
\frac{\partial \Pi_{ij}}{\partial r_i} = \omega_i - (1 - \alpha_i) c_i - 2k_i r_i = 0
\]

(31)

From Equations (30) and (31), the optimal retail price and the optimal advertising level in vertical Nash equilibrium are given by

\[
p_i^\text{N} = \frac{1}{4 - \lambda^2} \left[ 2S_i + \lambda S_j + (2 - \lambda \mu) \frac{\omega_j}{2k_i} - \frac{1 - \alpha_j}{2k_i} c_j + (\lambda - 2\mu) \frac{\omega_j}{2k_i} - \frac{1 - \alpha_j}{2k_i} c_j \right]
\]

(32)

\[
r_i^\text{N} = \frac{\omega_j - (1 - \alpha_j)c_j}{2k_i}
\]

(33)

Then, from Equations (32) and (33), the demand quantities and the supply chain’s profits in vertical Nash equilibrium are given by

\[
Q_i^\text{N} = M_i - p_i^\text{N} + \lambda p_j^\text{N} + r_i^\text{N} - \mu r_j^\text{N}
\]

(34)

\[
\Pi_i^\text{N} = p_i^\text{N} Q_i^\text{N} - k_i \left( r_i^\text{N} \right)^2
\]

(35)

From the optimal retail prices, advertising levels and demand quantities in MRS game and Nash equilibrium in decentralised supply chain, some results can be concluded by proposition 4.

Proposition 4: If $\omega = \omega_i = \omega_j = c = c_i = c_j, k = k_i = k_j, \alpha = \alpha_i = \alpha_j, M = M_i = M_j$ (e.g. the two manufacturers are identical), then

\[
\begin{align*}
p_i^\text{N} & > p_d^\text{N} > p_i^\text{d} = p_d^\text{d} = p_j^\text{d} = p_j^\text{N} \\
r_i^\text{N} & > r_d^\text{N} > r_i^\text{d} = r_d^\text{d} = r_j^\text{d} = r_j^\text{N} \\
Q_i^\text{N} & = Q_i^\text{d} = Q_d^\text{N} > Q_i^\text{d} = Q_d^\text{d} = Q_j^\text{d} = Q_j^\text{N}
\end{align*}
\]

Proposition 4 compares the equilibrium prices and equilibrium advertising levels under MRS game and Nash equilibrium. When the two manufacturers are identical, it shows that the pricing equilibrium, advertising level equilibrium and demand quantity equilibrium are higher in Nash equilibrium than MRS game. Unfortunately, it is difficult in comparing the optimal profits between MRS game and Nash equilibrium. In Section 5, numerical example is given to illustrate the relationship.
5 Numerical study

To illustrate the results, first let us provide a numerical example assuming parameter values as in Table 2. Table 3 gives the numerical solution in centralised supply chain, MRS game and Nash equilibrium in decentralised supply chain. From Table 3, it can be seen that as the degree of demand substitution increases, the retail prices, demand quantities and average advertising levels in the industry are all increasing in the three models. This implies that supply chains would obtain more profit as the degree of demand substitution increases in an oligopoly (Table 4). Furthermore, supply chain 2 has a cost advantage over supply chain 1, then retailer 1’s retail price is higher than retailer 2’s retail price. However, retailer 1’s demand quantity is lower than retailer 2’s demand quantity. As a result, it is reasonable that supply chain 2’s profit is higher than supply chain 1’s profit as the degree of demand substitution increases (Table 4).

Table 2 System parameter values

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<tr>
<th>Parameters</th>
<th>$M_i$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\mu$</th>
<th>$k_1$</th>
<th>$c_1$</th>
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Table 3 Numerical solutions

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<td>860</td>
<td>911</td>
<td>968</td>
<td>1033</td>
<td>1109</td>
<td>1197</td>
<td>1301</td>
<td>1426</td>
<td>1578</td>
<td>1767</td>
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<td>$p_2^e$</td>
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<td>867</td>
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<td>311</td>
<td>368</td>
<td>434</td>
<td>509</td>
<td>598</td>
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<td>388</td>
<td>447</td>
<td>515</td>
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<td>681</td>
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<td>88.3</td>
<td>102.8</td>
<td>119.2</td>
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<td>160.5</td>
<td>186.6</td>
<td>217.9</td>
<td>255.9</td>
<td>303.3</td>
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| $p_1^d$    | 909  | 953  | 1001 | 1056 | 1119 | 1190 | 1271 | 1366 | 1476 | 1607 | 1765 |
| $p_2^d$    | 826  | 873  | 926  | 984  | 1049 | 1123 | 1207 | 1304 | 1417 | 1550 | 1709 |
| $Q_1^d$    | 110  | 153  | 202  | 257  | 319  | 390  | 472  | 566  | 677  | 808  | 965  |
| $Q_2^d$    | 186  | 234  | 286  | 345  | 410  | 484  | 568  | 664  | 776  | 909  | 1070 |
| $\tau^d$  | 21.8 | 21.7 | 21.6 | 21.6 | 21.7 | 21.9 | 22.2 | 22.7 | 23.3 | 24.2 | 25.3 |

| $p_1^N$    | 919  | 963  | 1012 | 1067 | 1130 | 1202 | 1284 | 1379 | 1490 | 1622 | 1780 |
| $p_2^N$    | 832  | 880  | 933  | 992  | 1058 | 1133 | 1217 | 1315 | 1428 | 1562 | 1722 |
| $Q_1^N$    | 119  | 163  | 212  | 268  | 331  | 402  | 484  | 579  | 690  | 821  | 980  |
| $Q_2^N$    | 193  | 241  | 294  | 353  | 419  | 493  | 578  | 675  | 789  | 922  | 1083 |
| $\tau^N$  | 19.1 | 20.2 | 20.9 | 21.5 | 22   | 22.4 | 22.8 | 23   | 23.3 | 23.5 | 23.7 |
Table 4 gives some new managerial insights into the degree of demand substitution. As the degree of demand substitution increases, the profits in centralised supply chain, MRS game and Nash equilibrium are all increasing. And supply chain 1’s profit loss $d_1$ and $N_1$ are all decreasing as the degree of demand substitution is increasing (e.g. $\lambda \leq 0.7$). But Supply chain 1’s profit loss $d_1$ and $N_1$ are all increasing as the degree of demand substitution is increasing (e.g. $\lambda \geq 0.8$). That is, for $\lambda \in [0.7, 0.9]$, there is a degree of demand substitution which minimises Supply chain 1’s profit loss $d_1$ or $N_1$. Furthermore, when two products are substitutable completely (e.g. as $\lambda \to 1$), Supply chain 2’s profit loss $d_2$ is minimised. However, supply chain 2’s profit loss $d_2$ is increasing (e.g. $\lambda \geq 0.8$) as the degree of demand substitution is increasing. That is, for $\lambda \in [0.6, 0.8]$, there is a degree of demand substitution, which minimises supply chain 2’s profit loss $d_2$.

Table 5 and Figure 3 provide some new managerial insights into the degree of advertising level. When the degree of advertising substitution $\mu \in [0, 0.8]$, as the degree of demand substitution increases, the advertising level increases for $\lambda \in (\lambda^*,1]$, where $\lambda^* = 4 - 2\sqrt{3} = 0.538$. But as the degree of advertising substitution and the degree of demand substitution increase simultaneously for $\mu, \lambda \in [0.538, 0.8]$, the advertising level is decreasing (Figure 3). This indicates that as the degree of advertising substitution and the degree of demand substitution increase simultaneously, the degree of advertising substitution has more effect on the advertising level than the demand substitution. It is implies that the advertising level lies much on the degree of the advertising substitution. That is, generally speaking, when the degree of the advertising substitution is high, then the advertising level is also high. Furthermore, from Equation (33), it is easy to know manufacture 1’s advertising level in Nash equilibrium is 50. However, manufacture 1’s advertising level in MRS game is not more than 40. This identifies the result in Proposition 4 ($r_1^N > r_1^\mu$).

<table>
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Table 5 Advertising level with $\mu$ and $\lambda$ in MRS game

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<td>14.9</td>
<td>13.3</td>
</tr>
</tbody>
</table>

Figure 3 Manufacturer 1’s advertising level in MRS game (see online version for colours)

6 Conclusion

In this paper, the primary objective is to highlight the importance of advertisement from manufacturers in the interactions between two competing manufacturers and their retailers, facing end consumers who are sensitive to both retail price and manufacturer’s advertisement. The role of the retailer and its bargaining power are explored by examining the supply chain over MRS game. Using the game-theoretic approach, the analysis found a number of insights into the economic behaviour of firms, which could serve as the basis for empirical study in the future.

In this research, a framework has been developed to study the interaction of pricing and advertising competition in oligopoly. The expressions for equilibrium retail prices and advertising levels are derived in centralised supply chain and in decentralised supply chain. Notice that a higher degree of advertising substitution or the degree of demand substitution reduces the retailers’ retail price levels under certain conditions. Moreover, surprisingly, when the degree of advertising substitution equals 0.5, there is no difference in advertising level between the demand substitution degrees of 0 and 1. In addition, there is an optimal degree of demand substitution, which minimises the advertising level under certain condition. The last but not the least, the optimal retail price, demand quantity and advertising level in MRS game is lower than in Nash equilibrium.

There are a number of possible extensions of this study, which can constitute future research endeavours in this area.
One possible extension is to remove the symmetry assumption of the model. Differences in production costs may support asymmetric equilibrium (e.g. Li, 2002). Presumably, a low-cost manufacturer will have a stronger incentive to advertise than a high-cost competing manufacturer. If the retailers know the demand information, and the manufacturers cannot observe the demand information. In this situation, it is interesting in studying the asymmetric equilibrium retail price and advertising level.

The assumption of linear demand is another limitation of this paper. Someone may further study it with demand uncertainty. In that case, the problem faced by the retailer will be a newsvendor problem with price-dependent demand (Wang et al., 2004).

What’s more, there are two competing supply chains in this paper. There is a possible extension in studying several competing supply chains because this is a better assumption (e.g. there are a lot of manufacturers who come from different countries (e.g. China, Korea and Japan) in US competing in electronic appliances).

Finally, there may be differences in product quality (Banker et al., 1998). This is especially interesting when the consumers observe only the quality offered by the ‘preferred’ seller (manufacturer or retailer). If the sellers can credibly communicate information about price but not quality, the advertised price will serve as a signal of quality. Price advertising then becomes complicated by the fact that consumers will use price advertising to draw inferences about the quality of the offer.

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References


Appendix

Mathematical proofs

Proof of Proposition 1

Differentiating Equation (17) with respect to $\mu$, the first-order condition is

$$ \frac{\partial \rho^d}{\partial \mu} = \frac{-\lambda r - 2r}{4 - \lambda^2} < 0 \quad (A1) $$

When $r_1 = r_2$, differentiating equation (17) with respect to $\lambda$, the first-order condition is

$$ \frac{\partial \rho^d}{\partial \lambda} = \frac{(S_1 - \mu r_1 + r_1)(4 - \lambda^2) + 2\lambda (2S_1 + \lambda S_2 + (2 - \lambda \mu + \lambda - 2\mu)r_1)}{(4 - \lambda^2)^2} \quad (A2) $$

Note that $S_1 - \mu r_1 + r_1 > 0$ and $2 - \lambda \mu + \lambda - 2\mu = (2 + \lambda)(1 - \mu) > 0$, hence $\partial \rho^d / \partial \lambda > 0$.

Similarly, $\partial Q^d / \partial \mu < 0$, $\partial Q^d / \partial \lambda > 0$, $i = 1, 2$.

Proof of Lemma 1

It is straightforward and therefore omitted.

Proof of Proposition 2

Differentiating equation (24) with respect to $\mu$, the first-order condition is

$$ \frac{\partial r^d}{\partial \mu} = \frac{-\lambda}{4 - \lambda^2} < 0 \quad (A3) $$

Similarly,

$$ \frac{\partial r^d}{\partial \lambda} = \frac{4\lambda - 4\mu - \mu \lambda^2}{(4 - \lambda^2)^2} \quad (A4) $$

Let $f(\lambda) = 4\lambda - 4\mu - \mu \lambda^2 = 0$, two solutions can be solved from the equation $f(\lambda)$. They are

$$ \lambda_1 = \frac{2 - 2\sqrt{1 - \mu^2}}{\mu} \quad \text{and} \quad \lambda_2 = \frac{2 + 2\sqrt{1 - \mu^2}}{\mu} \quad (A5) $$

Recall that $\lambda \in [0, 1]$, note that $\lambda_2 = (2 + 2\sqrt{1 - \mu^2}) / \mu > 1$, so $\lambda_2$ is not a feasible solution to the equation $f(\lambda)$. Let $\lambda^* = \lambda_1 = (2 - 2\sqrt{1 - \mu^2}) / \mu$. From $\lambda^* \leq 1$, $\mu \leq 0.8$ can be yielded. Hence, for $\mu \in [0, 0.8]$, if $\lambda \in (\lambda^*, 1)$, that is $f(\lambda) > 0$, then $\partial r^d / \partial \lambda > 0$; if
\(\lambda \in [0, \lambda']\), that is \(f(\lambda) \leq 0\), then \(\partial r_i^d / \partial \lambda < 0\). In addition, for \(\mu \in (0.8, 1]\), \(f(\lambda) < 0\). Hence for any \(\lambda \in [0, 1]\), \(\partial r_i^d / \partial \lambda < 0\) for \(\mu \in (0.8, 1]\).

**Proof of Corollary 1**

When \(\lambda = 0\),

\[
r_i^d (\lambda = 0) = \frac{\omega_i - (1 - \alpha_i) c_i 1}{2k_i}, \quad i = 1, 2
\]

When \(\lambda = 1\),

\[
r_i^d (\lambda = 1) = \frac{\omega_i - (1 - \alpha_i) c_i 2 - \mu}{2k_i}, \quad i = 1, 2
\]

Let \(r_i^d (\lambda = 0) = r_i^d (\lambda = 1)\), and solving the equation, \(\mu = 0.5\) can be yielded. The second-order condition of \(r_i^d\) in \(\lambda\) is

\[
\frac{\partial^2 r_i^d}{\partial \lambda^2} = \frac{(4 - 2\mu\lambda)(4 - \lambda)^2 + 4\lambda(4 - \lambda)^2(4\lambda - 4\mu - \mu\lambda^2)}{(4 - \lambda^2)^4} = \frac{(4 - 2\mu\lambda) + 4\lambda(4\lambda - 4\mu - \mu\lambda^2)}{(4 - \lambda^2)^3} = \frac{4(1 - 4.5\lambda\mu + 4\lambda^2 - \mu\lambda^2)}{(4 - \lambda^2)^3}
\]

(A6)

Now, it is needed to prove \(\partial^2 r_i^d / \partial \lambda^2 > 0\). Let \(h(\mu, \lambda) = 1 - 4.5\lambda\mu + 4\lambda^2 - \mu\lambda^2\), note that

\[
h(\mu, \lambda) = (4 - \mu) \left[ \lambda - \frac{(4.5\mu)^2}{8 - 2\mu} \right] + 1 - \frac{(4.5\mu)^2}{16 - 4\mu}
\]

(A7)

Now, it is only needed to prove \(1 - (4.5\mu)^2 / (16 - 4\mu) > 0\), that is \(81\mu^2 + 16\mu - 64 < 0\). Let \(g(\mu) = 81\mu^2 + 16\mu - 64\), note that \(g(\mu) = 81\mu^2 + 16\mu - 64\) increase in \(\mu\). Recall that \(\mu \in [0, 0.8]\), hence \(g(0.8) = 81\mu^2 + 16\mu - 64 < 0\), that is \(g(\mu) = 81\mu^2 + 16\mu - 64 < 0\). So the second-order condition \(\partial^2 r_i^d / \partial \lambda^2 > 0\).

**Proof of Lemma 2**

Recall that \(Q_i^d = M_i - p_i^d + \lambda p_i^d + r_i - \mu r_i\). From Equation (24), observe that \(r_i^d\) is choosed by manufacturer \(i\) does not depend on \(M_i\).

Hence,

\[
\frac{\partial Q_i^d}{\partial M_i} = 1 - \frac{\partial p_i^d}{\partial M_i} + \lambda \frac{\partial p_i^d}{\partial M_i} + \frac{\partial p_i^d}{\partial M_i} + \frac{\partial r_i}{\partial M_i} - \mu \frac{\partial r_i}{\partial M_i}
\]

(A8)
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Note that \( \frac{\partial p_t^u}{\partial \lambda} = (2 - \lambda)/(4 - \lambda^2) \) and \( \frac{\partial p_t^u}{\partial M_t} = \lambda - 2/4 - \lambda^2 \), substituting these two equations into equation (A11), and then the first-order condition can be given by

\[
\frac{\partial Q_t^u}{\partial M_t} = 1 - \frac{(2 - \lambda)(\lambda - 2)}{(4 - \lambda^2)}
\]

\[
+ \frac{\lambda}{(4 - \lambda^2)} \quad (A9)
\]

\[
= \frac{1}{2 + \lambda} > 0
\]

Similarly,

\[
\frac{\partial Q_t^e}{\partial M_t} = -\frac{1}{2 + \lambda} < 0.
\]

**Proof of Proposition 3**

Transforming \( r' = \eta_1 r_1 + \eta_2 r_2 \) into

\[
r' = r_1 - r_2 \frac{Q_t^e - Q_t^d}{Q_t^e + Q_t^d}
\]

(A10)

And then differentiating Equation (A10) with respect to \( M_t \), the first-order condition is

\[
\frac{\partial r'}{\partial M_t} = 2r_2 \left( Q_t^e \frac{\partial Q_t^e}{\partial M_t} - Q_t^d \frac{\partial Q_t^d}{\partial M_t} \right)
\]

(A11)

From Lemma 2, \( \frac{\partial Q_t^e}{\partial M_t} > 0 \) and \( \frac{\partial Q_t^d}{\partial M_t} < 0 \) can be yielded, hence \( \frac{\partial r'}{\partial M_t} < 0 \).

**Proof of Proposition 4**

If

\[
\omega = \omega_1, c = c_1, k = k_1, \alpha = \alpha_1, M = M_1 = M_2
\]

then

\[
\begin{align*}
p_N^t &= p_N^u, \quad p_t^e = p_t^d, \quad r_t^N = r_t^u, \quad r_t^e = r_t^d, \quad Q_N^t &= Q_N^u, \quad Q_t^e = Q_t^d
\end{align*}
\]

(A12)

Notice that \( S = S_t \), so

\[
p_N^t - p_t^e = \frac{1}{4 - \lambda^2} \left[ 2S + 2S + (2 - \lambda\mu) \frac{\omega(1 - \alpha)c}{2k} + (\lambda - 2\mu) \frac{\omega(1 - \alpha)c}{2k} \right]
\]

\[
= \frac{2S + 2S + \omega - (1 - \alpha)c \left( 2 - \lambda\mu \right)}{4 - \lambda^2} \left( \frac{2 - \lambda\mu}{2k} \right) + \frac{\omega - (1 - \alpha)c \left( 2 - \lambda\mu \right)}{2k} \left( 4 - \lambda^2 \right)
\]

\[
= \frac{(2 + \lambda)(1 - \mu)(2 + \lambda\mu - \lambda^2)}{4 - \lambda^2} \frac{\omega(1 - \alpha)c}{2k} > 0
\]

(A13)
\[
\frac{r^n - r^d}{2k} = \frac{\omega(1-\alpha)c}{2k} - \frac{\omega(1-\alpha)c}{2k} \frac{2 - \lambda \mu - \lambda^2 + \lambda^2}{4 - \lambda^2} > 0 \tag{A14}
\]

\[
Q^n - Q^d = (\lambda - 1)(p^n - p^d) + (1 - \mu)(r^n - r^d)
\]

\[
= \frac{\omega(1-\alpha)c}{2k} \left[ \frac{(2 + \lambda)(1 - \mu)(2 + \lambda \mu - \lambda^2)}{4 - \lambda^2} + 2 + \lambda \mu - \lambda^2 \right] \tag{A15}
\]

\[
= \frac{\omega(1-\alpha)c}{2k} \frac{(1 - \mu)(2 + \lambda \mu - \lambda^2)}{4 - \lambda^2(2 - \lambda)} > 0
\]