Coaxial waveguide mode reconstruction and analysis with THz digital holography

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Abstract: Terahertz (THz) digital holography is employed to investigate the properties of waveguides. By using a THz digital holographic imaging system, the propagation modes of a metallic coaxial waveguide are measured and the mode patterns are restored with the inverse Fresnel diffraction algorithm. The experimental results show that the THz propagation mode inside the waveguide is a combination of four modes $TE_{11}$, $TE_{12}$, $TM_{11}$, and $TM_{12}$, which are in good agreement with the simulation results. In this work, THz digital holography presents its strong potential as a platform for waveguide mode charactering. The experimental findings provide a valuable reference for the design of THz waveguides.

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References and links

1. Introduction

With the development of terahertz (THz) technology, integration and miniaturization of THz systems has become an urgent requirement, which strongly promotes the studies of THz waveguides. In recent years, many investigations about THz waveguides have been published [1–4]. THz propagation inside waveguides is determined by special modes of electromagnetic waves, so the measurement of these modes is a crucial step for studying wave-guiding performances, such as the transmission loss and group velocity dispersion. Generally, the mode profile can be detected by imaging the output THz field at the waveguide end [5, 6]. In 2009, J. A. Harrington et. al demonstrated that waveguide modes can be detected by using THz near-field microscopy [7]. In 2011, M. Roze et. al utilized THz near-field microscopy to measure the output mode profiles of two suspended porous core THz fibers [8]. However, as the reported techniques adopt a raster scan method to build THz images, it needs longer experimental time and the THz waveform may be distorted in the near-field detection [9]. In addition, most of the published results only contain one polarization component of the THz waves, so it is not easy to accurately analyze the waveguide modes. As a novel imaging technique, THz focal plane imaging [10], which was firstly proposed by X. C. Zhang et. al in 1996, has attracted more and more attentions in the latest years [11, 12]. Recently, the quick and precise acquirement of two-dimensional (2D) THz polarization images has been also reported [13]. Furthermore, it was shown that the influence of diffraction on THz waves can be effectively removed by applying the digital holography algorithm [14, 15].

The metallic coaxial waveguide, which can perfectly confine electro-magnetic waves between two metallic cylinders, is a typical design for radio frequencies. Compared with the unscreened waveguides, such as dielectric waveguides [3], wire waveguides [5], and parallel-plate waveguides [6], the metallic coaxial waveguide can effectively protect propagation signals from outside electro-magnetic interference. Additionally, the flexibility of the metallic coaxial waveguide is better than metallic hollow waveguides [1]. Therefore, it has obvious advantages for guiding electro-magnetic waves. Recently, this kind of waveguide has been introduced into the optical region [16]. O. Lamrous et. al theoretically discussed the effect of coaxial waveguide plasmonic modes for wave propagation in the visible optical range [17]. In 2004, D. Grischkowsky et. al firstly achieved the direct optoelectronic generation of THz pulses in a coaxial waveguide [18]. However, the measurement of the waveguide modes in the THz frequency range has not been reported.

In this paper, a significant application of THz digital holography for the determination of waveguide modes is proposed. The transmitted modes of a metallic coaxial waveguide are detected by a THz digital holographic imaging system and are reconstructed by a digital holography algorithm. The propagation properties of the waveguide modes are experimentally and theoretically discussed. This work is essential for the investigation and design of THz waveguides.
2. Experimental setup

In this experiment, a THz balanced electro-optic (EO) holographic imaging system [19] is used to measure the transmitted modes of the coaxial metallic waveguide, as shown in Fig. 1(a). Ultrafast 100 fs, 800 nm laser pulses with a repetition rate of 1 kHz and an average power of 750 mW are divided into the pump beam and the probe beam for generating and detecting THz waves. The pump beam illuminates a <110> ZnTe crystal with a 3 mm thickness to radiate a THz beam with an 8 mm diameter. The THz waves are coupled into the coaxial waveguide, which is held by two apertures to fix its direction. Another <110> ZnTe crystal with a 3 mm thickness is mounted close to the waveguide output end as the detection crystal. The probe beam firstly passes through a half wave plate (HWP) and a polarizer. Then, it is reflected by a 50/50 beam splitter (BS) to impinge on the detection crystal. The effect of the HWP and the polarizer is to adjust the probe polarization for detecting different THz polarization components [13]. The horizontal component of the THz field can be measured when the probe polarization is parallel to the <001> axis of the detection crystal, while the vertical component of the THz field is measured when the angle between the probe polarization and the <001> axis is 45° [13]. In the detection crystal, the probe polarization is modulated by the THz field via Pockels effect to carry the 2D THz information. The reflected probe beam is incident onto the imaging part of the system. This part is composed of a quarter wave plate (QWP), a Wollaston prism (PBS), two lenses (L1 and L2), and a CY-DB1300A CCD camera (Chong Qing Chuang Yu Optoelectronics Technology Company). It supplies the balanced EO detection for THz imaging whose detailed principle has been already discussed in Ref [19]. The CCD camera is synchronously controlled with a mechanical chopper to acquire the image of the probe beam with a 2 Hz frame ratio. The THz information is extracted by the dynamic subtraction technique [12]. Here, 50 frames are averaged to enhance
the signal to noise ratio of the system. By varying the optical path difference between the pump beam and the probe beam, the THz images at the different time-delay points are obtained. The time window is 20 ps and the time resolution is 0.13 ps. The total measurement time is about 2 hours and the average acquisition time of each image is about one minute. The total time consumption can be further reduced to about ten minutes by using a faster CCD camera.

The structure of the coaxial waveguide is shown in Fig. 1(c). It is a common commercial cable (Model: SYV-50-3-41), which is widely used in industry and research. The metal core is a solid copper cylinder with a radius of b = 0.45 mm. The outer conductor is a hollow copper wire mesh layer with the inner radius a = 1.50 mm. Because the arrangement of the wire mesh is quite dense, the layer is considered as a solid one. The medium between the inner and outer conductors is a dielectric layer. The outside of the waveguide is wrapped by a rubber layer for protection. The photograph of the waveguide cross-section is presented in Fig. 1(b). First of all, we measured the reference THz signal without any sample and obtained the averaged signal from all pixels. Figure 1(d) presents the two polarization components of the reference signal. The ratio between the THz horizontal and vertical fields is about 45:1, which indicates that the incoming THz waves have a very good linear polarization. Figure 1(e) shows the 2D distribution of the maximum of the temporal signal of the THz horizontal field, which exhibits a planar envelope. Because the size of the waveguide cross-section is less than the half of the THz spot diameter, the incident THz waves are considered as linear polarized plane waves.

3. Analysis of the mode structures

A coaxial waveguide with 30 mm length is selected as the sample and the transmitted THz fields on both the horizontal and vertical directions are measured. By performing Fourier transformation to the temporal THz signal on each pixel, 2D distributions of the THz waves at each frequency are obtained. The THz intensity images of the two polarization components at 0.50 THz are shown in Figs. 2(a) and 2(b), respectively. Using the THz holographic imaging system, it only spends about one minute to acquire an image. In conventional THz raster scan imaging, the time consumption is several hours for capturing an image with the same quality. Additionally, the THz holographic imaging system can more precisely reflect the THz field distribution after the introduction of polarization information [13]. Therefore, the superiority of the imaging technique is very significant for measuring waveguide modes.
However, the qualities of these two images are not ideal. The measured THz images are obviously influenced by diffraction, because there is always a certain propagation distance between the waveguide output end and the measurement plane in fact [20]. Therefore, it is not easy to observe and analyze the waveguide modes from the raw data. To solve this problem, we introduced a digital holography algorithm to reconstruct the distributions of the transmitted THz fields. In the experiment, the diffraction distance of the THz waves reaches several wavelengths, so the process can be seen as the Fresnel diffraction and the original THz field can be restored by [21]

\[
U(x, y) = \exp\left(-\frac{j k d_{\text{eff}}}{j d_{\text{eff}}} \right) \int U(x_0, y_0) \exp\left(-j \frac{k}{2 d_{\text{eff}}} \left[ (x_0 - x)^2 + (y_0 - y)^2 \right] \right) dx_0 dy_0, \tag{1}
\]

where \(U(x_0, y_0)\) and \(U(x, y)\) are the raw image and reconstructed field, \((x_0, y_0)\) and \((x, y)\) are the spatial coordinates on these two observation planes, respectively. \(k\) is the wave vector of the THz radiation in vacuum and \(\lambda\) is the wavelength. The parameter \(d_{\text{eff}}\) is the propagation distance of the THz waves between the waveguide output end and the measurement plane, as shown in the inset of Fig. 1(a). By varying the distance \(d_{\text{eff}}\) in the propagation kernel of Eq. (1), it is found that the clearest image can be acquired when \(d_{\text{eff}}\) is set as 1.0 mm. The first derivative of the reconstructed image is adopted to further check the recover distance and it is found that the edge of the waveguide is the sharpest for this distance. The reconstructed images for the horizontal and vertical polarization components are shown in Figs. 2(c) and 2(d), respectively.

Fig. 3. (a) and (b) present two polarization components of the electric field at 0.50 THz in polar coordinates. (c) and (d) are simulation results by weighted stacking four waveguide modes TE_{11}, TE_{12}, TM_{11}, and TM_{12}. (e), (f), (g) and (h) are measured polarization images for 0.35 THz and 0.70 THz in polar coordinates, respectively.
It is easy to analyze the waveguide modes in polar coordinates \((r, \phi)\) due to their axial symmetry. Therefore, the THz fields in Figs. 2(c) and 2(d) are converted into the polar coordinates with the transformation relationship,

\[
E_r = E_x \cos(\phi) + E_y \sin(\phi),
\]

\[
E_\phi = E_x \cos(\phi) - E_y \sin(\phi).
\]

where \(E_x\) and \(E_y\) are the THz fields in Cartesian coordinates, \(E_r\) and \(E_\phi\) are the THz fields in polar coordinates. Figures 3(a) and 3(b) show the intensities of \(E_r\) and \(E_\phi\) at 0.50 THz. The THz image of \(E_r\) exhibits a distribution of a pair of bilateral symmetric crescent moons around the inner axes. The \(E_\phi\) component is similar with that of \(E_r\) except for its upper and lower symmetry and its intensity being about half of \(E_r\). In the THz frequency range, the copper inner and outer conductors can be considered as perfect metals. By using the Helmholtz equation and special boundary conditions of the coaxial waveguide, the analytical expressions of \(E_r\) and \(E_\phi\) for the TE and TM modes \([22]\) can be expressed as

**TE modes:**

\[
E_r = \frac{m}{r} \sqrt{\frac{\pi}{2}} \frac{J_m\left(\frac{x_1 r}{a}\right)N'_m\left(x_1\right) - N_m\left(\frac{x_1 r}{b}\right)J'_m\left(x_1\right)}{\left[J_m^2\left(\frac{x_1}{c x_1}\right) - 1\right]^{\frac{m}{2}} \left[1 - \left(\frac{m}{x_1}\right)^2\right]\left[1 - \left(\frac{m}{x_1}\right)^2\right]} \cos(m \phi),
\]

\[
E_\phi = \frac{x_1}{b} \sqrt{\frac{\pi}{2}} \frac{J'_m\left(\frac{x_1 r}{b}\right)N'_m\left(x_1\right) - N_m\left(\frac{x_1 r}{b}\right)J'_m\left(x_1\right)}{\left[J_m^2\left(\frac{x_1}{c x_1}\right) - 1\right]^{\frac{m}{2}} \left[1 - \left(\frac{m}{x_1}\right)^2\right]\left[1 - \left(\frac{m}{x_1}\right)^2\right]} \sin(m \phi),
\]

**TM modes:**

\[
E_r = \frac{x_2}{b} \sqrt{\frac{\pi}{2}} \frac{J'_m\left(\frac{x_2 r}{b}\right)N_m\left(x_2\right) - N'_m\left(\frac{x_2 r}{b}\right)J_m\left(x_2\right)}{\left[J_m^2\left(\frac{x_2}{c x_2}\right) - 1\right]^{\frac{m}{2}} \left[1 - \left(\frac{m}{x_2}\right)^2\right]} \cos(m \phi),
\]

\[
E_\phi = \frac{m}{r} \sqrt{\frac{\pi}{2}} \frac{J_m\left(\frac{x_2 r}{b}\right)N'_m\left(x_2\right) - N_m\left(\frac{x_2 r}{b}\right)J'_m\left(x_2\right)}{\left[J_m^2\left(\frac{x_2}{c x_2}\right) - 1\right]^{\frac{m}{2}} \left[1 - \left(\frac{m}{x_2}\right)^2\right]} \sin(m \phi),
\]

where \(J_m\) and \(N_m\) are the \(m\)th-order Bessel and Neumann functions, \(J'_m\) and \(N'_m\) are the first-order derivatives of \(J_m\) and \(N_m\), and the parameter \(c = a / b\) is given by the geometry of the waveguide. \(x_1\) and \(x_2\) are the eigenvalues of the equations \(J'_m\left(cx\right)N'_m\left(x\right) - N'_m\left(cx\right)J'_m\left(x\right) = 0\) and \(J_m\left(cx\right)N_m\left(x\right) - N_m\left(cx\right)J_m\left(x\right) = 0\), which are related to the cut-off wave numbers of the waveguide modes. Utilizing Eqs. (4)-(7), the
distributions of the THz field for different modes can be calculated. However, it is found that the measured results do not match with any single waveguide mode. Because the chosen waveguide has a cross-section with a millimeter scale, it allows the existence of multiple modes. When four modes $\text{TE}_{11}$, $\text{TE}_{12}$, $\text{TM}_{11}$, and $\text{TM}_{12}$ are selected and weighted stacked as $\text{TE}_{11} + 0.625\text{TE}_{12} + 0.625\text{TM}_{11} + 0.625\text{TM}_{12}$, the simulation results are consistent with the measured ones very well, as shown in Figs. 3(c) and 3(d). The reason why the four modes are chosen can be explained as follows: the four modes have more obvious horizontal linear polarization than others, so they are easily excited when the incident light is horizontal linear polarized [17]. To more clearly observe the THz field vector in the waveguide, Figs. 3(c) and 3(d) are vector combined to build the electric field pattern, as shown in Fig. 4. In this figure, the red arrows show the direction of the THz field and the blue solid lines give the boundaries of the outer and inner conductors. It can be obviously seen that the whole electric field exhibits a certain unity of the horizontal polarization direction. It shows that the propagation mode is well compatible with a horizontal linearly incoming wave. In addition, the cut-off wavelengths of these four modes can be calculated by 

$$\lambda_{\text{TE}} = \frac{2\pi (a + b)}{(c + 1)x_1 \mu}$$

and

$$\lambda_{\text{TM}} = \frac{2\pi (a - b)}{(c - 1)x_2 \mu}$$

where $\mu = 1.50$ is the refractive index of the dielectric layer at THz frequencies, which is measured by standard THz time-domain spectroscopy. With these equations, the cut-off wavelengths of $\text{TE}_{11}$, $\text{TE}_{12}$, $\text{TM}_{11}$, and $\text{TM}_{12}$ are obtained as 8.9 mm, 2.8 mm, 3.0 mm, and 1.6 mm, respectively, which are larger than the longest wavelength of the incident THz waves. It means that these modes can appear in the coaxial waveguide and dominate the propagation of the THz waves. The intensities of $E_x$ and $E_y$ at 0.35 THz and 0.70 THz are also extracted, as shown in Figs. 3(e)-3(h). Their patterns are almost identical with the experimental results at 0.50 THz and the simulated ones, which indicates that the THz waves at other frequencies also propagate with the four modes in the waveguide. It should be still noted that the TEM mode cannot be effectively excited by a linearly polarized incident beam due to its radial polarization, so it does not appear in the measurement modes [22, 23].

![Fig. 4. Vector electric field pattern of the output mode in the coaxial waveguide converted from Figs. 3(c) and 3(d).](image-url)
4. Spatial-temporal analysis of the guided THz waves

Utilizing digital holography, the reconstructed THz fields in both time and frequency domains can be acquired. On the horizontal axis of the waveguide output plane (the dark dashed line in the inset of Fig. 5(a)), the $E_r$ component of the transmitted THz field on the space-time map is extracted and presented in Fig. 5(a). In this figure, the red and white dashed lines mark the boundaries of the outer and inner conductors. From the space-time map, it can be seen that the THz field is effectively localized in the dielectric layer and most of its energy is confined within the 1.5 ps pulse duration. Compared with the 1.3 ps pulse duration of the reference THz signal, the THz pulse is only slightly broadened after passing through the waveguide. An equivalent space-frequency map of the THz field is shown in Fig. 5(b). Its spectral range is from 0.20 THz to 1.30 THz. Although the propagation of the THz waves contains four waveguide modes, the interference between them is not obvious in this figure. To further uncover the propagation properties of the THz waves, a windowed Fourier transformation with an 1 ps long Gaussian window is performed to extract the temporal distribution of the Fourier component at a specified frequency. The processing result for 0.50 THz is shown in Fig. 5(c). The map in space and time clearly presents that the energy of all of the waveguide modes concentrate on an envelope with a 1.5 ps width. The four modes do not exhibit a division in the time domain, which demonstrates that their group velocities are almost the same. This is why no interference between the waveguide modes is apparent in Fig. 5(b). Comparing the pulse propagation time in the waveguide and in free space, the group velocity is estimated as 0.65 times the light speed in vacuum. By applying the same procedure, space-time maps for other frequencies are also obtained. The result for 0.70 THz is presented in Fig. 5(d) as an example. This figure shows the same phenomenon as in Fig. 5(c), which indicates that all frequency components are contained in the same THz pulse. This implies that the waveguide modes have small group velocity dispersion, which can explain that the broadening of the THz pulse after propagating through the waveguide is weak.

Fig. 5. (a) and (b) are measured space-time and space-frequency maps of the $r$ electric field component at the position of the dark dashed line in the inset. (c) and (d) are extracted space-time maps of the 0.50 THz and 0.70 THz components obtained by performing a windowed Fourier transformation. In these figures, the red and white dashed lines correspond to the boundaries of the outer and inner conductors.

To further check the conclusions drawn, the other coaxial waveguide with 40 mm length is chosen and measured by using the THz imaging system. The reconstructed images of the
transmitted THz waves are obtained by the digital holography algorithm and space-time maps for different frequency components are extracted by the windowed Fourier transformation. Figure 6(a) gives the transmitted modes in polar coordinates (inset) and the space-time distribution of the $E_z$ component at 0.50 THz from the waveguide. From these results, it can be found that the THz fields on the output plane of the waveguide present very similar distributions as those of Fig. 3 and the space-time map is also consistent with Fig. 5(c) except for a 17.6 ps time delay. To determine the dispersion property, the two waveguide sections with 30 mm and 40 mm lengths are used as the launch waveguide and the test waveguide, respectively. From the obtained space-time maps of the two waveguides, the two groups of THz temporal signals within the range from $r = -0.75$ mm to $r = -1.30$ mm are extracted as the reference data set and the sample data set, respectively. The average refractive index of the waveguide is calculated by $\mu_{ave} = \frac{\Phi(\omega)\nu}{\omega L} + 1$, where $\Phi(\omega)$ is the phase difference for each frequency component between the output THz signals from the launch waveguide and the test waveguide, $L$ is the length difference between the two waveguides, $\omega$ is the circular frequency, and $\nu$ is the light speed in vacuum. For comparison, the refractive index of the dielectric layer is also measured by standard THz time domain spectroscopy. Figure 6(b) shows the experimental results with the error bars. The red symbols are the average refractive index of the waveguide, while the blue symbols are the refractive index of the dielectric layer. It can be seen that the average refractive index of the waveguide almost keeps constant at 1.53 from 0.20 to 1.30 THz, which indicates that the dispersion of the waveguide is very low over this frequency range and also explains the weak broadening of the transmitted THz signal from the waveguide. In addition, the refractive index of the dielectric layer is about 1.50 in 0.20-1.30 THz. The average refractive index of the waveguide is slightly bigger than the refractive index of the dielectric layer, which reflects that the average group velocity $\nu / \mu_{ave}$ of the THz waves in the waveguide is a little lower than the light speed $\nu / \mu$ in the dielectric layer. These phenomena confirm the accuracy of above experimental findings.

5. Conclusion

In summary, an important application of THz digital holography for the measurement and reconstruction of THz waveguide modes is demonstrated. By applying this technique, the distribution of transmitted THz waves from waveguides can be clearly restored and the THz propagation properties can be accurately analyzed. It is found that the THz propagation inside a metallic coaxial waveguide mainly depends on the superposition of four modes TE$_{11}$, TE$_{12}$, TM$_{11}$, and TM$_{12}$. The spatial-temporal analyses of a specified spectral component show that these modes have similar group velocities and small group velocity dispersion. This work
provides a valuable research method and experimental basis for studies and designs of THz waveguides.

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