Coordination of a supply chain with consumer return under demand uncertainty

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Abstract

This paper investigates coordination of a supply chain consisting of one manufacturer and one retailer facing consumer return. We integrate consumer returns policy and manufacturer buyback policy within a modeling framework and explicitly model the positive effect of refund amount on demand and its negative effect on the probability that consumers keep the products. We design a buyback/markdown money contract to coordinate the supply chain under partial refund policy and find that the refund amount plays an important role in the decisions and profitability of the players. In the coordinated setting with given buyback price, the refund amount first increases the players' expected profits/quantity, and then decreases them. When the risk (variance) of the consumer's valuation increases, the manufacturer may raise the unit wholesale price to achieve a higher unit profit. The supply chain is better off using full refund policy if the risk is very small; otherwise, the supply chain prefers no returns policy. The results of this paper are robust to distribution form.

Keywords:
Supply chain coordination
Consumer return
Buyback contract
Robustness
Game theory

1. Introduction

With the increase of product variety, consumers feel much uncertain about whether specific items fit their needs or match their tastes. If the items do not fit, the consumers may return them. When a consumer purchases a product, firms typically offer money-back guarantees to ensure consumer satisfaction. Consumer returns policy is common in recent years. For example, in the United States, returns rate from consumer to manufacturer or retailer is about in the range of 6–15% (Gentry, 1999). Recent reports on the value of the products returned in the United States have shown that it exceeds $100 billion per year (Stock et al., 2002). In the European Union, most of mass retailers offer full refund within 30–90 days of purchase (Guide et al., 2006). Most companies identify the management of product returns and exchanges as the most challenging aspect of reverse logistics (Gecker and Vigoroso, 2006). This paper will focus on the effects of the uncertainty of the consumer's valuation.

In retail industries, a returned item is differently handled, depending on the status of the product and the relationship between retailers and manufacturers. If the item is not apparently damaged, it will go back to the shelf. However, if the manufacturer desires to keep a high standard, the item will not go back to the shelf until the manufacturer inspected the product. For example, welding equipment HP and Bosch follow this policy (Ferguson et al., 2005). If the item is damaged or some parts are missing, the product will follow different paths, depending on multiple factors such as shipping cost, retail price, and retailer's profit margin. Another important factor is the relative bargaining power of players. Big retailers often transfer part or even all of the cost associated with the returned item to the manufacturer. In some cases, the item is returned to the manufacturer, who incurs some handling costs associated with the return. In other cases, the retailer decides what to do with the returned product. We will consider two types of contracts, buyback contract and markdown money contract, to address coordination issue in the two settings.

Returns policy stimulates the market demand by signaling high quality or sharing the valuation uncertainty risk with consumers. However, they may incur excess inventories and handling costs for firms when items are returned. What are the implications of returns policy on supply chain management? How does the refund amount influence the coordination mechanism and profitability of players? This paper aims to address these issues.

The presence of product return adds one dimension to the relationship between manufacturers and retailers and underscores the importance of coordination. To improve the efficiency of a decentralized supply chain, the supply chain requires the collaboration of the players who independently maximize their own profits. Supply chain coordination may be achieved by modifying the structure of these relationships (Tsay, 1999). Contract is an effective tool to allocate the channel profit between...
the players (Lariviere, 1999; Wong et al., 2009). Another important function of contract is that it facilitates a long-term partnership and makes the terms more explicit (Cachon, 2003). How to coordinate the supply chain with consumer return?

In this paper, we consider a supply chain consisting of one manufacturer (seller) and one retailer to investigate the effect of consumer return on coordination of the supply chain and study the motivation to use full refund policy over no returns policy. The retailer faces stochastic demand and consumer return. Handling costs related to consumer return are incurred at both the retailer and manufacturer levels. We find that a buyback/markdown money contract can coordinate the supply chain when partial refund policy is taken into account. Although a higher salvage value of unsold units increases the channel profit, it may decrease the manufacturer’s profit, given the buyback price. The refund amount and the variance (risk) of the consumer's valuation influence the decisions and profitability of the players to a large degree. The risk influences the motivation of the supply chain to use full refund policy over no returns policy to a large degree.

The rest of this paper is organized as follows. In the next section, we review the related literature. Section 3 introduces the basic model. Section 4 studies coordination of the supply chain via buyback contract under partial refund policy and the motivation to use full refund policy over no returns policy. Section 5 considers coordination of the supply chain via markdown money contract. Section 6 summarizes the results and suggests directions for future research.

2. Literature review

This paper is closely related to consumer returns policy, valuation uncertainty and supply chain coordination management. Different functions of consumer returns policy have been highlighted in recent years. When there are substantial transaction costs, Moorthy and Srinivasan (1995) show that generous returns policy helps to signal high quality. Heiman et al. (2002) model money-back guarantees as an option. Generally, the guarantees can be used to implement risk-sharing between sellers and consumers. For simplicity, we assume that the players are risk neutral. In addition, there is a body of work on stochastic inventory models with product return. One common approach is to consider the situation where items are directly returned to reusable inventory (Fleischmann et al., 2002; Decroix and Zipkin, 2005). An alternative approach of incorporating product return is to consider an independent remanufacturing facility that processes returned items before they are resold (Dekker et al., 2004). In this paper, we first consider partial refund policy, and then compare the full refund policy with the no returns policy.

The operations management literature on consumer behavior explicitly studies consumer decision process. For example, Su (2007) and Aviv and Pazgal (2008) study dynamic pricing problems in which consumers make purchase decisions based on expected future price. Some papers analyze firm's inventory decisions when consumers anticipate and strategically wait for markdown, which occurs when there is excess inventory. Dana and Petruzzi (2001) examine scenarios in which the possibility of stock-outs may discourage consumers. These papers capture consumers' concern over price or availability risk. Consumer returns policy can be used to encourage purchasing. We will consider the two effects of refund amount, increasing demand (positive effect) and increasing returning probability (negative effect).

An important modeling ingredient in this paper is valuation uncertainty. Davis et al. (1995) formulate a model in which consumer ex post valuation is a Bernoulli random variable. Che (1996) allows consumer valuation to follow a general distribution and also incorporates risk aversion. However, these papers focus on how to prevent inappropriate return. Differing from theirs, we assume that the valuations of consumers are uncertain and focus on the effects of the distribution of consumer valuation on the coordination mechanism and the expected profits of the players.

When consumers face valuation uncertainty, firms may wish to offer advance purchase discounts to compensate them for bearing risk (Xie and Shugan, 2001). Other papers study the value of offering reservations to the consumers who face valuation uncertainty. Along the line, we study the role of consumer returns policy as a risk-sharing mechanism when the valuations of consumers are uncertain.

Our work is closely related to the literature on buyback contract within supply chain. The buyback policy may help to achieve Pareto improvement in a supply chain (Choi, 2006). Although we focus on consumer return, we will also consider the case where the retailer may return unsold units to the manufacturer. Pasternack (1985) shows that a supply chain may be coordinated when the supplier commit to buy back all unsold units at partial credit. However, Emmons and Gilbert (1998) incorporate retailer’s pricing decision and show that channel coordination using buyback contract may no longer be feasible, unless the retailer can commit the selling price prior to the selling season; otherwise, the retailer has an incentive to set price too high (Kandel, 1996). Donohue (2000) develops a model with forecast updating and shows how to coordinate the supply chain using buyback contract. Webster and Weng (2000) study a risk-free returns policy, in which the profits of both the retailer and manufacturer increase, compared to the situation in which no return is allowed. Padmanabhan and Png (1997) analyze the interaction between manufacturer returns policy and retail competition. Tsay (2001) suggests that markdown allowances may be more appropriate when the retailer is able to achieve a higher salvage value. Taylor (2002) incorporates a buyback contract with a target sales rebate contract to coordinate the supply chain when the demand is sensitive to the sales effort of retailer. Choi et al. (2008) carry out a mean-variance analysis of supply chains under a returns policy and illustrate how a returns policy can be applied for managing the supply chains to address the issues such as channel coordination and risk control. Wang and Zipkin (2009) study a two-stage supplier-retailer agent system, using a buyback contract. All the papers above consider either supply chain returns policy or consumer returns policy. Differing from theirs, we integrate the two returns policies into a single framework.

Although the importance of consumer behavior and supply chain returns policy is widely investigated, few researches integrate them. In view of this gap in the literature, there are three main contributions in this paper: First, we study the managerial implications of consumer returns policy. In particular, since buyback contract is a popular solution to the double marginalization problem, we integrate consumer return and manufacturer buyback in a supply chain setting. Second, we investigate the effects of consumer return on coordination mechanism and the expected profits of the players. Third, we consider the motivation of the supply chain to use full refund policy over no returns policy.

3. The basic model

Consider a supply chain consisting of one manufacturer and one retailer. The manufacturer produces a product with unit cost $c$, sells the products to end consumers through the retailer and
charges the retailer a unit wholesale price \( w (> c) \). We assume that the retail price \( p (> w) \) is exogenously given.

Similar to Che (1996), we assume that each consumer purchases at most a unit product and consumers did not fully know their preferences for the products until they obtained some experiences with the products. The consumer’s preference is parameterized by a random valuation \( v \) with an increasing cumulative distribution function \( G(\cdot) \) over the interval \([v, \bar{v}]\). We consider a returns policy in which the retailer offers a refund amount \( r \in [0, p] \) to consumers when the product is returned. \( r=0 \) means that no returns policy is provided and \( r=p \) represents a full refund policy, i.e., 100% money-back-guarantee is offered to ensure consumer satisfaction (Choi et al., 2004; Mostard and Teunter, 2006). We refer to the returns policy with \( 0 < r < p \) as partial refund policy.

Similar to Arcelus et al. (2008), we assume that the retailer faces a random demand \( X \) that can be decomposed into a stochastic, \( \varepsilon \), and non-stochastic, \( D(r) \), in additive, i.e., \( X = D(r) + \varepsilon \). \( D(r) \) is an increasing function of the refund amount \( r \) (referring to as the positive effect of refund) and the error \( \varepsilon \) is a random variable with probability density \( f(\cdot) \) and cumulative distribution \( F(\cdot) \) over \([-D(0), +\infty)\), where \( F(\cdot) \) is differentiable, invertible, and strictly increasing. For simplicity, we assume that the error \( \varepsilon \) is independent of the consumer’s valuation \( v \).

The retailer incurs a handling cost \( l_h \) per unit return of consumer. In the basic model, we consider the situation where there is a buyback contract between the manufacturer and the retailer. At the end of the selling season, the manufacturer buys back leftovers by paying the retailer a buyback price \( b (< w) \) per unit leftover. The leftovers include the products that were unsold as well as those that were sold but returned. The manufacturer incurs the inspection and disposition of the returned units by consumers at an average handling cost of \( l_h \) per unit. Let \( l = l_h + l_k \) denote the total handling cost per unit returned product. At the end of the selling season, the product that has not been sold has a unit salvage value \( s \). Without loss of generality, we assume \( s < c \) to rule out some trivial cases. Let \( s' (s < s') \) denote the salvage value per unit returned product. In other words, the manufacturer receives a unit salvage value \( s \) for unsold products, but the unit salvage value is reduced to \( s' \) for returned items due to physical damage.

Given the returns policy of the retailer, consumers will first attempt to purchase the product and then decide whether to return it after learning their own valuations. We assume that consumers return the products at a unit returns cost \( c_0 \). The consumer with \( v \) will return the product if and only if \( v < r - c_0 \). Thus, consumers return the products with probability \( G_1 = G(r - c_0) \) and will eventually keep the products with probability \( G_1 = 1 - G(r - c_0) \). To make a sense, we assume \( r < c_0 \geq y' \geq s' \); otherwise, consumers do not return the products. The higher the refund amount \( r \), the larger the returning probability \( G_1 \) will be. We refer to it as the negative effect of refund. Thus, the retailer will make a trade-off between the positive effect and the negative effect of refund. We will focus on how the results of this paper depend on the two effects.

The time sequence of this game is as follows:

(i) The manufacturer sets the buyback policy \((w, b)\).

(ii) The retailer determines order quantity \( Q \).

\[ S(Q) = E\min(D(r)+\varepsilon, Q) = \int_{-D(0)}^{Q-D(r)} (D(r)+\varepsilon)f(\varepsilon) d\varepsilon + \int_{Q-D(r)}^{+\infty} Qf(\varepsilon) d\varepsilon = Q - \int_{-D(0)}^{Q-D(r)} F(\varepsilon) d\varepsilon > 0, \]

where the second term in last line is the expected leftovers.

When the manufacturer considers a buyback policy, the expected profit of the retailer is

\[ E_{\pi_R}(Q) = pG_1S(Q) + bQ - S(Q) + (p - r + l_h)G_1S(Q) - wQ. \]

The second subscript of \( \pi_R(Q) \) represents model \( i \), \( i = 1, 2 \). \( i=1 \) denotes for the model with buyback contract and \( i=2 \) denotes for the model with markdown money contract. In Eq. (2), the first term denotes the revenue from the products that are sold and kept by consumers, the second term denotes the revenue of the unsold units bought back, the third term denotes the revenue of the units that are bought but returned by consumers, and the last term is the retailer’s procurement cost.

Further, we can rewrite Eq. (2) as

\[ E_{\pi_R}(Q) = [p - b (r + l_h - b)G_1S(Q) - (w - b)Q] = [p - b (r + l_h - b)G_1Q - \int_{-D(0)}^{Q-D(r)} F(\varepsilon) d\varepsilon] - (w - b)Q. \]

Eq. (3) and \( b < w \) imply that \( p - w > (r + l_h - b)G_1 \); otherwise, the retailer will make a negative expected profit. Furthermore, \( E_{\pi_R}(Q) \) is a concave function of \( Q \) because \( F(\cdot) \) is an increasing function of \( r \). \( p - w > (r + l_h - b)G_1 \) means that the (gross) profit margin of the retailer is higher than the expected returning cost per unit product. That is, the retailer makes a positive expected profit from selling a unit product.

The expected profit of the manufacturer is

\[ E_{\pi_M}(Q) = (w - c)Q + (s - b)(Q - S(Q)) + (s' - b - l_h)G_1S(Q) = (w + s - b - c)Q - \left[ s - b + G_1(b + l_M - s') \right] - \int_{-D(0)}^{Q-D(r)} F(\varepsilon) d\varepsilon. \]

From Eqs. (3) and (4), we know that the expected channel profit is

\[ E_{\pi_C}(Q) = [p - s - G_1(r + l - s')] \left[ Q - \int_{-D(0)}^{Q-D(r)} F(\varepsilon) d\varepsilon \right] - (c - s)Q. \]

Eq. (5) and \( s < c \) imply that \( s - c - G_1(r + l - s') > 0 \); otherwise, the expected channel profit is negative.

4. Coordination mechanism via buyback contract

4.1. Coordination mechanism

To provide a benchmark, we first consider the centralized system. From \( s < c \), \( p - c > G_1(r + l - s') \) and that \( F(\varepsilon) \) is an increasing function of \( \varepsilon \), it follows that \( E_{\pi_C}(Q) \) is a concave function of \( Q \). By solving the first-order condition \( dE_{\pi_C}(Q)/dQ = 0 \) for \( Q \), we have

\[ Q_C^* = D(r) + F^{-1}(p - c - G_1(r + l - s')). \]

From \( s < c \) and \( r > s' \), it follows that \( Q_C^* \) is an increasing function of \( c_0, s \) and \( s' \), and a decreasing function of \( c \) and \( l \).
Specifically, when the return cost \((c_0)\) of consumer increases, the probability that a product is returned decreases, which reduces the loss of the supply chain. Thus, the supply chain has an incentive to produce more. When the salvage values increase, the loss of leftovers decreases, which induces a higher quantity. We assume \(E[Z(TC_1 | Q^s_1)] > 0\) throughout this paper.

To coordinate the decentralized supply chain, the supplier must offer a mechanism that induces the retailer to order the quantity \(Q^s_1\).

Proposition 1 summarizes the coordination mechanism.

**Proposition 1.** The supply chain can be coordinated by the buyback contract \((w_t(b), b)\) with \(b_{M1} < b < b_{R1}\), where 
\[
b_{R1} = \left(\frac{p - G_1(l_b + r)}{1 - G_1}\right), \quad b_{M1} = \left(\frac{s - G_1(s' - l_M)}{(1 - G_1)}\right)\text{ and } w_t(b) = \left(\frac{(c - s)[p - G_1(r + l_b)] + b[p - c - G_1(r + l + s - c - s')] - b(s - c)}{p - s - G_1(r + l - s')}\right).
\]

We can obtain the coordination mechanism under full refund policy by inserting \(r = p\) into the expressions in Proposition 1, omitting it. From \(p - c - G_1(r + l - s') > 0\) and \(s < c\), it follows that \(w_t(b)\) is an increasing function of \(b\). Proposition 1 means that the supplier should offer an appropriate buyback price \((b_{M1} < b < b_{R1})\) to assure that both firms make positive profits. Specifically, if the buyback price is sufficiently high \((b > b_{R1})\), the retailer will make a negative profit due to a high unit wholesale price. If the buyback price is sufficiently low \((b < b_{M1})\), the supplier makes a negative profit because it must offer a low unit wholesale price to coordinate the supply chain. It is worthy to note that the coordination mechanism is independent of the deterministic part of demand \(D(r)\). That is, although the refund amount \(r\) has a positive effect on the output, the positive effect does not influence the coordination mechanism because the effects of the deterministic part of demand on the optimal quantity of the centralized system and that of the decentralized system are identical. Thus, the coordination mechanism is robust to the deterministic part of demand. However, the refund amount influences the equilibrium quantity and profits through the positive effect (see Eq. (6)).

From Proposition 1, we have
\[
b_{R1} - b_{M1} = \frac{p - s - G_1(r + l - s')}{1 - G_1},
\]
which is an increasing function of \(p\) and \(s'\); and a decreasing function of \(s\) and \(l\). That is, when the retail price or the salvage value of the returned units \((s')\) increases, the range of the buyback price becomes larger due to a higher channel profit. When the salvage value of the unsold unit \((s)\) increases, the supplier will offer a higher buyback price, which reduces the range of the buyback price. When the total handling cost associated with the returned product increases, the range of the buyback price becomes smaller due to a lower channel profit.

From Proposition 1, we derive Corollary 1.

**Corollary 1.** (i) If the refund amount is sufficiently high \((r > p - l - s + s')\), a higher refund amount will shrink the range of the buyback price; otherwise, it may expand the range;

(ii) If the refund amount is sufficiently high \((r > p - l - s + s')\), a higher unit returns cost \((c_0)\) of consumers will expand the range; otherwise, it will shrink the range.

Corollary 1 means that the effects of refund amount and returns cost of consumers on the coordinated range of the buyback price depend on the refund amount through the returning probability.

Note that consumers will not return any product if \(r \leq v + c_0\) although a returns policy is provided to them. Thus, from Proposition 1, we derive the following corollary.

**Corollary 2.** If \(r \leq v + c_0\), the supply chain can be coordinated by the buyback contract \((w_t(b), b)\) with \(s < b < p\), and \(w_t(b) = \frac{(c - s)p + b(p - c)}{p - s}\).

From \(s < c < p\) and \(b < p\), it follows that the buyback price is lower than the unit wholesale price, i.e., \(b < w_t(b)\). Thus, Corollary 2 means that the manufacturer shares the risk of demand uncertainty with the retailer. Corollary 2 implies that the optimal quantity increases as the unit wholesale price is an increasing function of \(c, b\) and \(p\), and a decreasing function of \(s\), if \(r \leq v + c_0\). When the salvage value \(s\) increases, the channel profit increases due to a lower loss of leftovers. Thus, the manufacturer should decrease the unit wholesale price to transfer a part of the benefit of higher salvage value to the retailer.

4.2. Sensitivity analysis

In Section 4.1, we have given a buyback contract to coordinate the supply chain. In this subsection, we illustrate the effects of some factors on the equilibrium outcome and expected profits of the players.

Table 1 illustrates the effects of some parameters, where we assume that the default values of parameters are used as \(b = 3.0, c = 2, c_0 = 0.2, D(r) = 5 + 0.5r, l_M = l_b = 0.2, p = 6, r = 4, s = 1.2, s' = 1, v \sim N(6, 1)\) and \(e \sim N(5, 1)\).

The above values of parameters satisfy the basic assumptions of this paper. Since the effects of \(l_M\) and \(l_b\) are intuitive (the quantity and expected profits decrease with them), we omit them.

From Table 1, given the buyback price, we derive the following observations:

(i) When the unit returns cost \((c_0)\) of consumer increases, the optimal quantity of the supply chain increases due to a lower returning probability, and the manufacturer will decrease the unit wholesale price to transfer a part of the benefit of higher quantity to the retailer; the profits of the players increase due to a higher order quantity and lower returning probability.

(ii) If the refund amount \(r\) is sufficiently small, when the refund amount increases, the manufacturer keeps the unit wholesale price because the consumers do not return the products (i.e., returning probability is zero); otherwise, the manufacturer raises the unit wholesale price to achieve a higher unit profit. When the refund amount increases, the optimal quantity and expected profits of the players first increase and then decrease. Specifically, if the refund amount is sufficiently small, when the refund amount increases, the positive effect of the refund amount (increasing the demand) outweighs its negative effect (increasing returning probability) such that the optimal quantity increases; otherwise, the optimal quantity decreases because the negative effect outweighs the positive effect. Similarly, the interaction between the positive effect and the negative effect accounts for the effect of the refund amount on the expected profits.

(iii) When the salvage value \(s\) or \(s'\) increases, the optimal quantity increases due to a lower loss for leftovers and the manufacturer decreases the unit wholesale price to transfer a part of the benefit of higher salvage values to the retailer, which in turn increases the retailer’s profit and the channel profit. However, they have different effects on the manufacturer’s profit. Specifically, when the unit salvage value \(s\) of unsold units increases, the manufacturer’s profit decreases because the negative effect of the lower unit wholesale price outweighs the positive effect of the higher order quantity and the lower loss of unsold units; however, when the unit salvage value \(s'\) of returned units increases, the
manufacturer's profit increases because the positive effects of the higher order quantity and the lower loss of returned units outweighs the negative effect of the lower unit wholesale price.

To investigate the effect of the distribution of the consumer's valuation, we consider two forms of distribution, normal and uniform. Since the effects of the mean of the consumer's valuation are intuitive, we omit them. To investigate the effect of the risk (variance) of the consumer's valuation, we keep the mean of \( \nu \) in Figs. 1 and 2 where the parameters have the same default values as Table 1. Since the effect of the variance on the optimal quantity is similar to its effect on the expected profits, we omit it.

Fig. 1 implies that, if the risk of the consumer's valuation is very small, the effect of the risk on the unit wholesale price is very small because consumers do not almost return products due to a high mean valuation. However, if the risk is very large, the manufacturer would like to raise the unit wholesale price to achieve a higher unit profit. This result may be counterintuitive as one expects that the manufacturer decrease the unit wholesale price to share the returning loss with the retailer. However, when the risk increases, the optimal quantity of the coordinated supply chain decreases due to a higher returning probability, and the manufacturer would like to raise the unit wholesale price to compensate the loss of the lower quantity.

Fig. 2 implies that the effect of the risk on the expected profit of the manufacturer is similar to that of the retailer. Specifically, if the risk is very small, the risk increases, their expected profits remain because the unit wholesale price and the order quantity remain; otherwise, their expected profits decrease due to a higher unit wholesale price and lower quantity, i.e., they share the risk cost. Thus, the manufacturer and the retailer have incentives to decrease the risk by clearly explaining the functions and defects of product.

It is worthy to note that these above results are robust to the distribution of the consumer's valuation. However, the effects of the risk on the unit wholesale price and expected profits/quantity under uniform distribution are larger (smaller) than that under normal distribution if the risk is sufficiently large (small).

4.3. Comparisons between full refund policy and no returns policy

In this subsection, we consider two extreme policies, full refund \( F \) and no return \( N \). We add superscripts \( F \) and \( N \) to differentiate them. It is easy extended to the comparisons between partial refund policy and no returns policy. We will examine when the supply chain is better off using full refund policy \( (r=p) \) over no returns policy.

When the retailer adopts no returns policy, the expected channel profit is

\[
E_{c}(\pi_{c}^{0}(Q)) = (p - s)(Q - \int_{-D(0)}^{0} F(v) dv) - (c - s)Q. \tag{8}
\]

Similar to Eq. (6), we have \( Q_{c}^{0} = D(0) + F^{-1}(p - c)/p - s \). By setting \( G_{1} = 0 \) and \( r = 0 \) in \( w_{1}(b) \), we can obtain the coordination mechanism under no returns policy.

For general distribution, it is difficult to compare the two policies. We first consider a special distribution from the analytic perspective, and then consider two distributions to investigate the effect of distribution form by using a numerical example. When the error \( \varepsilon \) is uniformly distributed over \([-D(0), A]\), we have \( F(\varepsilon) = (c + D(0))/A + D(0) \). For convenience, we define \( A_{1} = G(p - c_{0}) \), \( p + 1 - s') \), reflecting the expected returning cost per unit product. From Eq. (6), we know that the optimal quantity of the centralized system under full refund policy \( Q_{c1}^{*} = D(p) + F^{-1}((p - c - 1)/p - s - A_{1}) \).

Proposition 2 and 3 summarize the comparisons on the optimal quantity and profit of the coordinated supply chain under two returns policies.

**Proposition 2.** Assume that the error \( \varepsilon \) is uniformly distributed over \([-D(0), A]\). The optimal quantities of the centralized system under two return policies satisfy \( Q_{c1}^{*} > Q_{c2}^{*} \) if and only if

\[
s' = \int_{1}^{p} \frac{(D(p) - Q_{c2}^{*}(p + s) - (c + D(0))/A + D(0))}{G(p - c_{0})(A + D(p) - Q_{c2}^{*})} \, dp > 0. \tag{9}
\]

Proposition 2 means that when the salvage value \( s' \) per unit returned product is sufficiently high or the total handling cost \( l \)
Proposition 3. Assume that the error ε is uniformly distributed over \([-D(0), A]\). Full refund policy for the supply chain dominates no returns policy if and only if \(A_1 < A_1\), where \(A_1\) is the unique root of \(E_n[\pi^*_1(Q_{E1}^r)] = E_n[\pi^*_1(Q_{E1}^R)]\).

Proposition 3 implies that, when the expected returning cost per unit product is sufficiently small \((A_1 < A_1)\), the supply chain has an incentive to offer the full refund policy due to a higher quantity. Note that \(A_1\) is a decreasing function of \(c_0\) and \(s\), and an increasing function of \(l\); however, the threshold \(A_1\) is independent of them. Thus, a higher returns cost \((c_0)\) of consumers or salvage value \((s')\) will strengthen the motivation of the supply chain to use full refund policy while a higher handling cost \((l)\) will weaken the motivation.

Define \(\Delta \pi^*_1 = E_n[\pi^*_1(Q_{E1}^r)] - E_n[\pi^*_1(Q_{E1}^R)]\). Figs. 3 and 4 describe how the standard variance and mean of the consumer’s valuation influence the profit difference of the coordinated supply chain between full refund policy and no returns policy, where the parameters have the same default values as Table 1.

Figs. 3 and 4 imply that, when the risk (variance) is very small, the supply chain is better off using full refund policy; otherwise, the supply chain would like to use no returns policy. A higher risk weakens the incentive to use full refund policy because the returning probability increases. When the mean of the consumer’s valuation is not too large, the supply chain has a stronger incentive to use full refund policy under uniform distribution than that under normal distribution.

5. Coordination via markdown money contract

Under markdown money contract, the retailer salvages the leftover units including the unsold units and returned units. The supplier will carry out the inspection and final disposition of the returned units. The retailer charges the retailer a unit wholesale price \(w\) and pays the retailer a chargeback rate \(m\) for the leftover units at the end of the selling season (Tsay, 2001). We assume \(w \geq m + s\); otherwise, the retailer will order an infinite quantity. In reality, markdown money is prevalent in industries selling perishable goods such as fashion apparel because perishable goods will lose value in the eyes of the customer after the regular season.

Under a markdown money contract, the retailer’s expected profit is

\[
E_n[\pi_{k1}(Q)] = pG_1S(Q) + (m+s)(Q - S(Q)) + (p - r + m + s - hG_1S(Q) - wQ = [p - m - s - G_1(r + l - m - s')]Q - \int_{-D(0)}^{Q_{E1}^R} F(s) ds)
\]

\[ -(w - m - s)Q. \tag{9} \]

and the total channel profit is given by Eq. (5). Similar to Proposition 1, we show that markdown money contract can achieve supply chain coordination.

Proposition 4. The supply chain can be coordinated by the markdown money contract \((w_2(m), m)\) with \(0 < m < \tilde{m}_k\), where \(\tilde{m}_k = [p - s - G_1(l + r - s')]/(1 - G_1)\) and

\[
w_2(m) = \frac{clp - m - s - G_1(l + r - m - s') + m[p - G_1(l + r + s - s')]}{p - s - G_1(r + l - s')}.\]
Fig. 2. The profits versus the distribution of consumer’s valuation.

Fig. 3. $\Delta \pi_{C1}^{*}$ versus the distribution of consumer’s valuation with $E(v) = 6.0$. 
Proposition 4 shows that supply chain can be coordinated when the retailer salvages the leftovers. The effect of some factors on the coordinated range is similar to Corollary 1. We can show that the unit wholesale price is an increasing function of $m$. That is, the manufacturer shares the demand risk with the retailer and raises the unit wholesale price to offset the loss of higher markdown money for unsold products as markdown money increases. Differing from buyback policy, the manufacturer can obtain a positive profit for any positive markdown money.

6. Conclusions

In this paper, we study coordination of a two-echelon supply chain with consumer return and uncertain demand. We examine the situation where consumers face uncertainty in their valuations for products. This uncertainty is resolved only after purchased and used. Under the returns policy of the retailer, consumers decide whether to keep or return the products. We mainly investigate the effects of the consumer return on the coordination mechanism and expected profits; and study the robustness of the coordination mechanism.

We consider two types of contracts, buyback contract and markdown money contract. We design the mechanisms to coordinate the supply chain under partial refund policy. We focus on how to coordinate the supply chain via buyback contract and investigate the effects of some factors including salvage values, refund amount and the distribution of the consumer’s valuation. We find that, given the buyback price, the salvage value of unsold units and that of the returned units may have a contrary effect on the manufacturer’s profit although both increase the channel profit. The refund amount and variance of the consumer’s valuation play an important role in the decisions and profitability of the players. The variance of the consumer’s valuation influences the motivation of the supply chain to use full refund policy over no returns policy. The results of this paper are robust to distribution form.

The analysis of this paper can be extended in several directions. First, many retailers sell multiple products, and consumers who return one may buy another. One can develop a multi-product model to examine this situation. Second, we assume that the retail price is exogenously given. In reality, some retailers jointly determine retail price and order quantity. In the case with endogenous retail price, traditional buyback contract and markdown money contract cannot coordinate the supply chain. It is interesting to find a contract to coordinate the supply chain. In addition, we ignore inventory cost. The effects of the inventory cost on coordination mechanism and the expected profits of players may be interesting.

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Appendix A

Proof of Proposition 1. Note that $E_{p}[\pi_{p1}(Q)]$ is a concave function of $Q$. According to Eq. (3), by solving the first-order condition $dE_{p}[\pi_{p1}(Q)]/dQ = 0$ for $Q$, we obtain the optimal order quantity of the retailer

$$Q^*_R = D(r + F^{-1}\left(\frac{p - w - G_1(r + l_b - b)}{b - G_1(r + l_b - b)}\right)). \tag{A.1}$$

When the supply chain is coordinated, the optimal order quantity of the retailer should satisfy $Q^*_C = Q^*_C$. According to Eqs. (6) and (A.1), by solving $Q^*_C = Q^*_C$, for $b$, we have

$$w_1(b) = \frac{(c - s)p - G_1(r + l_b) + b[p - c - G_1(r + l + s - c - s')]}{p - s - G_1(r + l - s')}. \tag{A.2}$$

Inserting $w_1(b)$ into Eq. (3), we have

$$E_{p}[\pi_{p1}(Q^*_C)] = \frac{[p - b - G_1(l + r + b)][(p - c)Q^*_C - (p - s)\int_{-\infty}^{Q^*_C}F(c)dc - G_1(l + r + s')]}{p - s - G_1(l + r - s')}.$$ \tag{A.3}

Differentiating (A.2) with respect to $b$, we have

$$\frac{\partial E_{p}[\pi_{p1}(Q^*_C)]}{\partial b} = -\frac{G_1E_{p}[\pi_{p1}(Q^*_C)]}{(1 - G_1)} < 0. \tag{A.4}$$

That is, $E_{p}[\pi_{p1}(Q^*_C)]$ is a decreasing function of $b$. Note that $E_{p}[\pi_{p1}(Q^*_C)] = E_{p}[\pi_{p1}(Q^*_C)] - E_{p}[\pi_{p1}(Q^*_C)]$ is independent of $b$. Thus, $E_{p}[\pi_{p1}(Q^*_C)]$ is an increasing function of $b$.

Solving $E_{p}[\pi_{p1}(Q^*_C)] = 0$ for $b$, we have $b_R = [p - G_1(l + r + l) / (1 - G_1)]$. Solving $E_{p}[\pi_{p1}(Q^*_C)] = 0$ for $b$, we have $b_M = [s - G_1(s + l - l_m) / (1 - G_1)]$. From $p - s - G_1(r + l - s') > 0$ and $G_1 < 1$, it follows $b_R > b_M$. Furthermore, both firms make positive profits if $b_M < b < b_R$. \qed

Proof of Corollary 1. Part (i) Note that $G_1$ is an increasing function of $r$. From Eq. (7), we have

$$\frac{d(b_R - b_M)}{dr} = \frac{\frac{\partial b_R - \partial b_M}{\partial r} + \frac{\partial b_R - \partial b_M}{\partial r}}{\frac{\partial b_R - \partial b_M}{\partial r} + \frac{\partial b_R - \partial b_M}{\partial r}} = \frac{p - s - l - r - s - G_1}{(1 - G_1)^2} - \frac{1}{1 - G_1},$$

which is negative if $r \geq p - s - l - s'$. Otherwise, it may be positive. Part (ii) follows from $G_1$ is a decreasing function of $b_R$. \qed

Proof of Proposition 2. According to the definition of $F(c)$, we have $F^{-1}(p - c)/(p - s) = (\infty - D(0))(p - c)/(p - s)$. Therefore, the optimal quantity of the centralized system under no returns policy is $Q^*_C = \infty - D(0)/(p - c)/(p - s)$. Similarly, the optimal quantity of the centralized system under full refund policy is

$$Q^*_C = D(p) - D(0) + \frac{(\infty - D(0))(p - c - A_1)}{A_1 + D(p) - Q^*_C}. \tag{A.5}$$

We can show that $Q^*_C$ is a decreasing function of $A_1$ because of $s < c$. Thus, $Q^*_C > Q^*_C$ is equivalent to

$$A_1 < \frac{(D(p) - Q^*_C)(p - s) - D(0)(c - s) - \bar{A}(p - c)}{A_1 + D(p) - Q^*_C}. \tag{A.6}$$

Proof of Proposition 3. Inserting $r = p$ and $Q^*_C$ into Eq. (5), we can obtain the optimal expected channel profit under full refund policy

$$E_{p}[\pi_{p1}(Q^*_C)] = (p - s - A_1)S(Q^*_C) - (c - s)Q^*_C. \tag{A.7}$$

Differentiating $E_{p}[\pi_{p1}(Q^*_C)]$ with respect to $A_1$, we have

$$dE_{p}[\pi_{p1}(Q^*_C)]/dA_1 = -S(Q^*_C) < 0, \tag{A.8}$$

i.e., $E_{p}[\pi_{p1}(Q^*_C)]$ is a decreasing function of $A_1$. If $A_1 = 0$, we have $Q^*_C = D(p) - D(0)(p - c)/(p - s) > Q^*_C$ and $Q^*_C = D(p) - D(0)(p - c)/(p - s) > Q^*_C = D(0)$. Thus, it follows from $r > c$ that $E_{p}[\pi_{p1}(Q^*_C)] > E_{p}[\pi_{p1}(Q^*_C)]$ if $A_1 = 0$. If $A_1 = p + l - s'$, we have $E_{p}[\pi_{p1}(Q^*_C)] < 0 < E_{p}[\pi_{p1}(Q^*_C)]$. Note that $E_{p}[\pi_{p1}(Q^*_C)]$ is a continuous function of $A_1$. Thus, there exists a threshold $A_1$ where the expected channel profits are identical under two returns policies, i.e., $E_{p}[\pi_{p1}(Q^*_C)] = E_{p}[\pi_{p1}(Q^*_C)]$. Furthermore, full refund policy for the supply chain is better than no returns policy if and only if $A_1 < A_1$. \qed

Proof of Proposition 4. By solving the first-order condition of (9)

$$dE_{p}[\pi_{p2}(Q)]/dQ = 0$$

we have

$$Q^*_C = D(r + F^{-1}\left(\frac{p - m - s - G_1(r + l - s')}{p - s - G_1(r + l - s')}\right)). \tag{A.9}$$

By solving $Q^*_C = \frac{c(p - m - s - G_1(r + l - s') + m[p - G_1(r + l - s')]}{p - s - G_1(r + l - s')}$, we have

$$w_2(m) = \frac{c(p - m - s - G_1(r + l - s') + m[p - G_1(r + l - s')]}{p - s - G_1(r + l - s')}.$$ \tag{A.10}

Similar to Proposition 1, we can show that the expected profit of the retailer is a decreasing function of $m$ and both firms make positive profits if $0 < m < m_R$. \qed

References


