Two-echelon supply chain coordination through the unified number of annual orders

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ABSTRACT

This paper studies the problem of how to coordinate a one-supplier, multi-retailer supply chain by requiring retailers to adopt the unified number of annual orders proposed by the supplier. The paper presents two coordination models: one considering a small number of retailers; the other considering a large number of retailers. In the proposed models, the supplier views all retailers or some selected retailers (if more beneficial) in the supply chain as one group, then offers a price discount to entice the retailers in this group placing their orders only with the specific number of annual orders. A heuristic is developed for each model to let the supplier know which retailers will be selected to implement the common number of annual orders, and how to determine the corresponding optimal number of annual replenishments and price discount. By using a numerical study, we evaluate the benefit of the proposed coordination strategy.

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1. Introduction

In reality, it is very common that one supplier provides a single product to many retailers who belong to different entities. Under such a situation, each retailer will have to make her order decision independently to minimize her own inventory-related cost. Thus, the number of annual orders from each retailer will have to be different. Hence, orders that the supplier receives from different retailers will be required to deliver at different moments. Consequently, meeting all the retailers' orders will have the supplier's order processing and delivery costs very high. From the supplier's perspective, he would definitely economize on the order processing and delivery costs if he could consolidate orders from all or some of the retailers. For doing so, it will be necessary for the supplier to have all or some of the retailers place their orders with the unified number of annual orders (hereafter, UNAOs).

From the retailer's side, however, accepting such a UNAOs means that her corresponding inventory-related cost deviates from the original minimum cost. Then, to entice retailers to accept the UNAO, the supplier must design a suitable incentive mechanism that can compensate retailers for any increase in inventory-related cost and provide additional savings in cost as well. Therefore, the supplier must consider trade-off between payment for compensation and savings in cost when designing the UNAO strategy.

This paper considers a two-echelon supply chain in which a supplier sells a single product through multiple retailers who face a constant annual demand. It assumes that the supplier follows the "lot-for-lot" policy, and that there is a fixed cost for the supplier to process orders from any group of the retailers and an additional cost for each retailer included in the group. The objective is to let the supplier know how to maximize his savings in order processing and delivery costs by implementing the UNAO strategy. The paper develops two models: one with a small number of retailers and the other with a large number of retailers. The corresponding heuristic...
algorithms are provided to help the supplier make sure for which retailers the UNAO policy should be implemented, and how to design it.

1.1. Literature review

In the recent two decades, with increasing emphasis on the significance of effective supply chain management, many researchers have paid much attention to coordination issues between manufacturers (suppliers) and retailers (buyers) in a multi-echelon supply chain. Various coordination mechanisms that can improve supply chain efficiency have been proposed. Among these mechanisms, supply chain contract and quantity discount are two most representative and widely studied ones.

1.1.1. Supply chain contract

On the topic of supply chain contract, there is now extensive literature. Among the existing literature, various contracts have been proposed, such as the backup agreements (Eppen and Iyer, 1997), the return policies (Emmons and Gilbert, 1998), the quantity flexibility contracts (Tsay, 1999), the incentive mechanisms (Lee and Whang, 1999), the revenue sharing contracts (Cachon and Lariviere, 2005; Giannoccaro and Pontrandolfo, 2004), etc. Most of these existing contracts are designed under the single-period newsvendor framework. Milner and Rosenblatt (2002) developed a two-period contract model, in which a retailer is required to make an affirmative order commitment for the first period and a combination of affirmative and flexible order commitments for the second period at the beginning of the planning period. Tsay and Lovejoy (1999) studied contract models that allow commitments to be changed in each period by a given percentage of the commitments in the preceding period without penalty. Xu (2005) extended the two-period contracts to a class of multi-period dynamic supply contracts, in which a supplier allows a retailer to cancel a portion of an outstanding order with penalty in each period during a planning horizon. Because the discussed issue in this paper is not involved with contract mechanism, we merely present a short review about the literature on this direction. Interested readers can read a recent review paper on supply chain contracts for details (Cachon, 2004).

1.1.2. Quantity discount mechanism

The previous studies on quantity discount mechanism can be classified by two categories. One category has been done from the viewpoints of marketing and the other from the perspective of operations management. The former has concentrated on sales profit maximization under the assumption that operational costs are independent of the pricing decision (e.g., Jeuland and Shugan, 1983; Raju and Zhang, 2005; Lau et al., 2008, etc.). In contrast, the latter has focus on improving channel efficiency by managing operations activities under the assumption that annual demand is exogenous (e.g., Zhou and Yang, 2008). This paper is more closely related to the latter. So the studies that fall into this category are briefly reviewed in the following.

Various quantity discount models have been developed to coordinate supply chain system from the viewpoints of operations management. Many of them have analyzed quantity discount policies in the setting of a single-supplier and a retailer. For instance, Monahan (1984) presented a quantity-discount model to coordinate order decisions of a single-supplier–single-retailer supply chain. Assuming a lot-for-lot replenishment policy for the supplier, he showed that the supplier could reduce his own operation-related cost by offering a quantity discount to entice the retailer increasing her order quantity by a factor, without any additional cost to the retailer. By relaxing the lot-for-lot assumption, Lee and Rosenblatt (1986) incorporated the supplier’s inventory holding cost into Monahan’s model (1984), while Banerjee (1986) further considered the situation where the replenishment rate in the supplier’s side is limited. Dolan (1987) provided a review on some earlier quantity discount models. Later, Parlar and Wang (1994) presented a quantity discount model for a two-echelon channel consisting of a supplier and homogenous buyers whose annual demand is a linear function of wholesale price. They specified the conditions under which the supplier would offer quantity discount. Weng (1995) further studied the problem considered by Parlar and Wang (1994) and showed that quantity discount alone are not sufficient to guarantee joint profit maximization. By simultaneously using quantity discount and a franchise fee, he proposed a perfect coordination model that maximizes the system profit. Munson and Rosenblatt (2001) considered a supplier–manufacturer–retailer channel and analyzed the benefits of using quantity discount on both ends of the channel to decrease costs. They showed that incorporating quantity discounts into both ends of the channel could significantly decrease costs compared to concentrating only on the lower end. Chen and Chen (2005) incorporated joint replenishment of multiple products into coordination issues for a single-supplier–single-retailer channel, whereas Li and Liu (2006) presented a quantity discount model to coordinate a single-supplier–single-retailer system with probabilistic customer demand under a continuous review framework. Noting the situation that the retailer may face amplified overstocking risks related to increased order quantities incurred by the supplier’s discount policy, Shin and Benton (2007) proposed a retailer’s risk adjustment model for implementing coordination policy under a normally distributed demand. Through quantity discount mechanism, Zhou et al. (2008) discussed coordination issues of a two-echelon supply chain with an inventory-level-dependent demand rate.

As pointed out earlier, all the quantity discount models mentioned above were developed in the setting of one supplier and one retailer or multiple homogenous retailers. In reality, however, a supplier usually faces multiple heterogeneous retailers. When individual incentive schemes are allowable, a natural way that optimizes system performance for the supplier is to offer a separate discount policy to incentive each retailer to select the order policy that maximizes the profit of individual
subsystem composed of the supplier and each retailer. If taking fairness of trade in practice into account, suppliers should offer heterogeneous retailers a common discount pricing policy to coordinate the whole channel. Then, a natural question to ask is how to design a unified discount scheme for the supplier. Lal and Staelin (1984) first answered this question and developed a unitary quantity discount model for multiple heterogeneous retailers with a constant annual demand, assuming that the supplier adopts a lot-for-lot replenishment policy and a continuous price function that decreases exponentially with order size. Considering convenience of implementation in practice, Kim and Hwang (1988) proposed a single incremental discount policy under similar channel conditions. Under the assumption that both a supplier and any of retailers follow a power-of-two replenishment policy, Chen et al. (2001) demonstrated that a common quantity discount policy could achieve optimal system profit. By assuming that each retailer faces a price-sensitive demand, Yang and Zhou (2006) built both a single price-break all-unit discount model and a continuous discount-pricing model for a single-supplier multi-retailer channel. Wang (2002) developed an optimal general discrete quantity discount model under an approximate formulation for the supplier’s inventory-related cost. Wang and Wang (2005) extended this work to cover the situation with a price-sensitive demand. To avoid the approximation of the supplier’s inventory-related cost, Wang (2001) developed a hybrid discount policy of time coordination and quantity discounts when the replenishments of the supplier and retailers follow a power-of-two policy. Klasterin et al. (2002) presented another two-echelon coordination model, in which retailers replenish stock of a single product from a manufacturer who in turn supplied by the OEM at a reorder cycle. In their model, the manufacturer offers the retailers a price discount for their orders placed at the time that coincides with the beginning of the manufacturer’s cycle but a normal price for other orders in the manufacturer’s cycle. However, these existing unitary discount models assumed implicitly that the supplier processes orders from each retailer separately. Obviously, if the supplier can process orders from more than one retailer simultaneously, then he might be able to gain savings in the fixed order processing cost, independent of the number of retailers who place orders. Observing this point and following the “lot-for-lot” policy, Viswanathan and Piplani (VP, 2001) presented a Common Replenishment Epochs (CRE) mechanism to coordinate order decisions of a two-echelon channel consisting of one supplier and multiple buyers. Under this CRE policy, the supplier offers a price discount to induce all buyers to place orders only at times specified by the supplier (e.g., the first day of every week). They verified that implementing the CRE policy benefits the supplier in most cases. Mishra (2004) generalized their model to allow for a selective discount policy that, if beneficial, excludes some buyers to minimize the supplier’s total cost. He showed that offering discount to selective buyers could reduce the supplier’s cost in many scenarios. Since this CRE strategy only requires buyers to place their orders at specified times, the buyers who accept the CRE may still choose the number of annual orders different from each other when considering individual inventory cost. For instance, given the time specified by the supplier is Monday, one participating buyer may choose to order once every week, another may place orders once every two weeks, and so on. Therefore, a natural question we should ask is whether it is better for the supplier to further require retailers placing their orders with the united number of annual orders specified by the supplier. The models this paper develops subsequently will positively answer this question.

The main contribution of this paper is the following. First, it uses a UNAO discount policy to coordinate a two-echelon supply chain consisted of one supplier and multiple retailers. Unlike the CRE mechanism proposed by Viswanathan and Piplani (2001), the UNAO policy requires that the supplier offer the retailers a price discount to entice them to place their orders both at specified times and with the united number of annual orders specified by the supplier. Numerical experiments have revealed that this UNAO discount strategy dominates the existing CRE strategies in many situations, especially in situations with lower fixed processing cost for the supplier. Then, it further considers the corresponding coordination problem of a two-echelon channel where a supplier provides a product to a very large number of homogeneous retailers. It is also shown that under such a situation, implementing the UNAO discount strategy is still profitable for both the supplier and system.

The remainder of the paper is organized as follows. Section 2 introduces notations and assumptions used throughout the whole paper. Section 3 presents a coordination model where a supplier who sells a product through a small number of retailers offers a price discount to induce the retailers choosing the UNAO specified by the supplier. In Section 4, the model is extended to cover the situation with a very large number of retailers by approximating all retailers’ annual demands as a certain continuous distribution, which can be interpreted as the empirical frequency distribution of past-observed annual demands. Conclusions are in Section 5.

2. Assumptions and notations

2.1. Assumptions

(1) Consider a supply chain in which a supplier sells a single product through multiple heterogeneous retailers who have deterministic annual demands.

(2) The number of annual orders for each retailer is determined according to standard EOQ formula before coordination.

(3) Replenishment orders from all retailers have to be met immediately.

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1 In some countries, offering discriminated discount policies to different retailers violates fair trade laws, such as the US Federal Robinson-Patman Act, etc. (Stein and EL-Ansary, 1992).
The supplier follows a lot-for-lot policy. It is a common assumption when dealing with multiple retailers. The major benefit of following this policy is savings in handling cost because products do not have to be moved into and out of storage. Under such a situation, the supplier actually plays a role of passing products to retailers. In reality, some distribution centers implementing cross-docking usually follow this policy, such as Wal-Mart.

The retailers are willing to accept the UNAO discount scheme set by the supplier only if the price discount they get from the supplier results in at least 100% savings in their annual ordering and holding costs. Where $S > 0$ is the retailer’s psychological expectation in cost savings if accepting the UNAO discount scheme.

The supplier can consolidate orders from all retailers who accept the UNAO strategy and process their orders at the same point in time.

2.2. Notations

- $m$: the number of retailers considered in the supply chain
- $D_i$: the annual demand for retailer $i$, $i = 1,\ldots,m$
- $K_i$: the ordering cost for retailer $i$
- $h_i$: the holding cost rate for retailer $i$
- $P$: the purchasing price of the product
- $A_i$: the fixed order processing cost incurred by the supplier each time an order or a set of orders of retailers arrive
- $A_i$: the additional order processing and delivery cost incurred by the supplier for retailer $i$’s order
- $f_i$: the number of annual orders for retailer $i$ before implementation of the UNAO discount scheme
- $f_o$: the UNAOs specified by the supplier
- $d$: a price discount on the purchasing price $(0 < d < 1)$ enjoyed by the retailer who places his/her orders with the number of annual orders equal to $f_o$

3. Coordination model with a small number of retailers

This section considers the situation in which a supplier sells a product through a relatively small number of retailers, whose cost and market parameters are all known to the supplier.

3.1. Before implementation of the UNAO discount strategy

For retailer $i$, by the classical EOQ model, the optimal number of annual orders is given by

$$f_i = \sqrt{(2K_iP h_i)/(dP h_i)}$$  
(1)

Correspondingly, the total cost for retailer $i$ is

$$TC_i = PD_i + \sqrt{(2K_iP h_i)}$$  
(2)

The supplier incurs an order and delivery cost of $A_i + A_i f_i$ for retailer $i$ due to processing each individual retailer’s order separately. Therefore, total order processing costs of the supplier are given by

$$TC_s = \sum_{i=1}^{m} (A_i + A_i f_i)$$  
(3)

3.2. After implementation of the UNAO discount strategy

Now suppose the supplier decides to implement a UNAO discount strategy so as to reduce his own order processing cost. It is obvious that such a UNAO strategy would require retailers to change their original numbers of annual orders that yield the minimum inventory-related costs. Recall that assumption (5) we have already shown in the previous section, to make the UNAO strategy accepted by retailers, the supplier must compensate retailers for any increase in the inventory-related cost but also offer minimum savings of 100%. Therefore, the supplier has to determine which retailers should be included in this UNAO scheme and then make sure how to design the UNAO through trade-off between payment for compensation and savings in cost. The further details are given below.

Suppose a UNAO discount strategy includes all retailers. Then, under this strategy retailer $i$ will be asked to place her orders with a specific number of annual orders $f_o$. Thus, the sum of order and holding cost for retailer $i$ is given by $K_i f_o h_i P D_i/(2 f_o)$. The total cost that retailer $i$ pays is

$$TC_i^C = K_i f_o h_i P D_i/(2 f_o) + (1 - d)P D_i$$  
(4)

According to assumption (5), for any given UNAO strategy $(f_o,d)$, retailer $i$ is willing to accept it only if the following condition is satisfied.

$$TC_i - TC_i^C \geq S \sqrt{(2K_iP h_i)}$$  
(5)

which gives

$$d \geq (K_i f_o h_i P D_i/(2 f_o) - (1 - S) \sqrt{(2K_iP h_i)})/(P D_i)$$  
$\equiv g_i(f_o)$, $i = 1,2,\ldots,m$  
(6)

(6) implies that if the supplier wants retailer $i$ to accept the UNAO strategy, he should set the discount $d$ at least as $g_i(f_o)$ while determining this strategy. The gray area in Fig. 1(a) intuitively represents the set of all UNAO discount policies that retailer $i$ is willing to accept.

The total cost for the supplier under the UNAO strategy will be the sum of the payment for compensation and the order and delivery cost that is incurred by jointly processing orders from all retailers who are willing to adopt the UNAO, which is

$$A_i f_o + \sum_{i=1}^{m} (A_i f_o + d P D_i)$$

Therefore, after implementing the UNAO the supplier obtains the gains as follows:

$$G_s = \sum_{i=1}^{m} (A_i + A_i f_i) - \left\{ A_i f_o + \sum_{i=1}^{m} (A_i f_o + d P D_i) \right\}$$  
(7)
Hence, there is no root to equation $D$ beneficial to the supplier only if the point ($d$, $f_0$) in the gray area below the line $d = g_i(f_0)$ is located in the gray area in Fig. 2 graphically depicts the set of all UNAO policies that the supplier is willing to offer.

Clearly, the supplier is willing to offer the UNAO to retailers only if $G_i > 0$, i.e.,

$$d < \left\{ \sum_{i=1}^{m}(A_i + A_i)f_i - f_0 + \frac{m}{2}(A_0 + \sum_{i=1}^{m}A_i) \right\} / (PD) \equiv g_i(f_0) \quad (8)$$

where $D = \sum_{i=1}^{m}D_i$.

Represented graphically in a $<f_0, d>$ plane (see, Fig. 1(b)), (8) means that offering the UNAO strategy is beneficial to the supplier only if the point $(f_0, d)$ is located in the gray area below the line $d = g_i(f_0)$ in the $(f_0, d)$ plane.

Thus, a UNAO strategy that is able to be accepted by both the supplier and retailer $i$ ($i = 1, 2, \ldots, m$) should satisfy conditions (6) and (8) simultaneously. The gray area in Fig. 2 graphically depicts the set of all UNAO policies that are accepted by both parties. Whether this gray area is null depends on if there is intersection points between line $d = g_i(f_0)$ and curve $d = g_i(f_0)$ in Fig. 2. This is equivalent to judging if there is any root to equation $g_i(f_0) - g_i(f_0) = 0$. To simplify notations, define

$$U_i = D_i \sum_{i=1}^{m}(A_i + A_i)f_i / D + (1 - S)\sqrt{2K\phi D_i}$$

$$V_i = K_i + D_i \left( A_0 + \sum_{i=1}^{m}A_i \right) / D$$

Then, known from (6) and (8), the equation $g_i(f_0) - g_i(f_0) = 0$ can be rewritten as the following quadratic equation with respect to $f_0$.

$$V_i f_0^2 - U_i f_0 + h_iPD_i / 2 = 0, \quad i = 1, 2, \ldots, m$$

Denote $\delta_i$ be the discriminant of the above quadratic equation. One then has $\delta_i = U_i^2 - 2h_iPD_iV_i, \quad i = 1, 2, \ldots, m$.

Hence, there is no root to equation $g_i(f_0) - g_i(f_0) = 0$ if $\delta_i \leq 0$, otherwise there are two roots: $(U_i - \sqrt{\delta_i})/(2V_i)$ and $(U_i + \sqrt{\delta_i})/(2V_i)$.

Very obviously, any UNAO strategy that could be ultimately implemented should be beneficial to each of ($m+1$) participators including the supplier. That is, a feasible UNAO strategy must be the one that meets conditions (6) and (8) simultaneously. The following lemmas show how to identify if there is such a feasible UNAO strategy for all retailers. The proof of lemmas can be seen in Appendix A.

**Lemma 1.** If there is some retailer (say, retailer $j$) whose cost parameters satisfy $\delta_j \leq 0$, there does not exist any feasible UNAO strategy.

**Lemma 2.** If cost parameters of each retailer $i$ ($i = 1, 2, \ldots, m$) satisfy $\delta_i > 0$, then

1. there does not exist any feasible UNAO strategy when $\alpha > \beta$;
2. the UNAOs for a feasible UNAO strategy will drop in the interval $[\alpha, \beta]$ when $\alpha \leq \beta$.

where $\alpha = \max\{|U_i - \sqrt{\delta_i}|/(2V_i), \quad i = 1, 2, \ldots, m\}$; $\beta = \min\{|U_i + \sqrt{\delta_i}|/(2V_i), \quad i = 1, 2, \ldots, m\}$.

Known from Lemma 2, if $\delta_i > 0$ for each $i$ from 1 to $m$ and $\alpha \leq \beta$ as well, the supplier will be able to find the feasible UNAO strategy that can be accepted by all retailers. In this situation, the problem of determining the UNAO strategy for the supplier can then be formulated as follows:

$$\begin{align*}
\text{Max} & \quad G_i = \sum_{i=1}^{m}(A_i + A_i)f_i
- \left\{ A_i f_0 + \frac{m}{2}(A_0 + \sum_{i=1}^{m}A_i) \right\} \\
\text{s.t.} & \quad d \geq (K_i f_0 + h_iPD_i)/(2f_0) - (1 - S)\sqrt{2K\phi D_i}/(PD_i) \\
& \quad \equiv g_i(f_0), \quad i = 1, 2, \ldots, m \\
& \quad \alpha \leq \beta
\end{align*}$$

(9)

For a given $f_0$ that belongs to the interval $[\alpha, \beta]$, the discount offered by the supplier that maximizes the
supplier’s gains will be
\[ d'(f_o) = \text{Max}(g(f_o), \ i = 1, 2, \ldots, m) \]  
(10)
Additionally, if considering cases in reality, the UNAOs \( f_o \) should be chosen from a set of UNAO, probably 12 times per year, 24 times per year etc. Thus, the supplier’s problem can be rewritten as
\[ \text{Max } G_o = \sum_{i=1}^{m} (A_i + A_d)f_i - \left\{ A_if_o + \sum_{i=1}^{m} (Af_o + d'(f_o)PD_i) \right\} \]
\[ \text{s.t. } f_o \in Y \cap [x, \beta] \]
(11)
where \( Y \) is the set of UNAO being considered.

The optimal \( f_o \) can be determined by evaluating the objective function (11) through an exhaustive search for all \( f_o \in Y \cap [x, \beta] \). When the cardinality of \( Y \) is limited and small as in subsequent numerical study, the computation to search for the optimal \( f_o \) can be neglected.

3.3. Finding optimal UNAO discount strategy

One should observe that in the above analysis, we assume all \( m \) retailers to be asked to participate in this UNAO coordination scheme. However, it is not the best choice for the supplier. It may be better to allow some retailers, whose original numbers of annual orders are very low, to order as earlier, because their original order policies will not make the supplier’s order processing cost too high. Based on the analysis above, we show the following heuristic approach to determine the optimal UNAO strategy for the supplier.

**Algorithm:**
Step 1: Rank \( m \) retailers based on the order of their original numbers of annual replenishments from high to low.
Step 2: Compute \( \delta_i \) for each \( i \) from \( m \) to 1. If there is some \( i \) such that \( \delta_i < 0 \), go to step 6; otherwise go to the next step.
Step 3: Calculate \( z \) and \( \beta \). If \( z > \beta \), go to step 6; otherwise go to the next step.
Step 4: For each \( y \in Y \cap [x, \beta] \), set \( f_o = y \). Then determine discount \( d'(f_o) \) from (10) and the objective function \( G_o(m) \) from (11). Select the pair \( (f_o, d) \) that maximizes \( G_o(m) \).
Step 5: If \( m = 2 \), the pair \( (f_o, d) \) corresponding to \( \text{Max}(G_o(j), j = 2, \ldots, m) \) is the optimal UNAO strategy for the supplier, stop; otherwise go to step 6.
Step 6: Set \( m = m - 1 \), go to step 2.

3.4. Numerical study

This section will present a numerical example to demonstrate the benefits of the UNAO strategy. In this example, a supplier provides a single product to 15 retailers. For comparative purpose, we still use the same data for retailers as used in Mishra (2004), which is shown in Table 1. The purchasing price of the product for all 15 retailers is set at \( P = 1 \), holding cost rate for each retailer at 15% and minimum retailer cost savings required to 10% (i.e., \( S = 0.1 \)). Before coordination, the optimal number of annual orders for the retailers ranges from 4 to 55 times per year. Order processing costs for the supplier are \( A_i = 100 \) and \( A_i = 200 \) for retailer \( i \) (1, 2, ..., 15). \( f_o \) is restricted to the set \( Y \) (in times per year) = \{1, 2, ..., 60\}.

Following the solution procedure proposed above, one can easily obtain that it is the best for the supplier to select 6 retailers labeled from 1 to 6 to implement the UNAO. The optimal UNAO strategy is given by \( f_o^* = 19 \) and \( d' = 0.12\% \). The supplier’s corresponding total order processing cost for all retailers is $61,033. The system total cost is $17,793,010.3. Known from (2) and (3), however, they are, respectively, $80,901.7 and $17,820,732.9 before implementing the UNAO. Thus, implementing the UNAO brings the supplier 24.56% cost savings and the whole system 0.16% cost savings.

To compare with the existing CRE strategies, a numerical study is carried out for various values of \( A_i \) and 4. Table 2 shows the results for \( A_i = 10, 100, 200, 500, 1000 \) and \( A_i = 10, 100, 200, 500, 1000 \), including optimal UNAO strategy and corresponding percentage cost savings of the supplier and the system.

From Table 2, it can be observed that implementing the UNAO strategy always benefits the supplier and the whole system, and that the higher the supplier’s fixed processing cost \( A_i \) incurred by retailers’ orders, the more the percentage of the supplier’s and system cost savings yielded by the implementation of the UNAO. Compared to VP’s CRE strategy, then, from the supplier’s perspective, the effectiveness of implementing the UNAO strategy in reduction of costs will be better in 23 of the 25 scenarios in Table 2. Another observation from Table 2 is that as the values of \( A_i \) is not very high (say, \( A_i \leq 200 \) in this example), the UNAO strategy proposed in this paper is superior to Mishra’s CRE strategy but it reverses in the opposite case. It implies that the supplier would prefer the UNAO strategy to the CRE strategy for lower fixed processing cost rather than for higher one, which seems contrary to common expectation. As a matter of fact, it can be explained as the following. Because under the CRE strategy the supplier only requires the selected retailers

| Table 1 |
| Data for retailers, labeled by the order of number of annual orders from big to small. |

<table>
<thead>
<tr>
<th>Retailers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_i ) (in $)</td>
<td>50</td>
<td>50</td>
<td>150</td>
<td>50</td>
<td>150</td>
<td>100</td>
<td>500</td>
<td>500</td>
<td>1500</td>
<td>500</td>
<td>1500</td>
<td>1000</td>
<td>5000</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>( D_i ) (in millions)</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>2</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>( f_i ) (in times per year)</td>
<td>55</td>
<td>39</td>
<td>32</td>
<td>27</td>
<td>22</td>
<td>19</td>
<td>17</td>
<td>12</td>
<td>10</td>
<td>9</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Price = 1 and holding cost rate = 0.15.
As AI UNAOs so as to lower his own order processing cost as discount) to induce the retailers to place their orders at a heterogeneous retailers should set incentives (e.g., a price supplier who supplies a product to a large number of 4.1. Problem statement

UNAO strategy.

the compensation he pays, but it just reverse under the supplier's cost savings increases much faster than does probably leads to that under the CRE strategy the retailers to accept the specified UNAO or CRE. This to gain more cost savings, the supplier would hope more selected retailers is not too big. Under the UNAO strategy, of annual orders to a lower specified number. Conse-

Moreover, as the supplier's fixed processing cost increases, incurred by the UNAO strategy may be relatively great. the selected retailers and the supplier's cost savings quently, both the compensation that the supplier offers would merely deviate from their original ones a little.

order cycles of the retailers who accept the specified CRE still choose different numbers of annual orders. Thus, the placing their orders at specified times, these retailers can

hence, the supplier's cost savings incurred by the CRE as well as the compensation that the supplier offers the selected retailers is not too big. Under the UNAO strategy, however, all the selected retailers (usually, the most-frequent-ordering retailers) must cut down their number of annual orders to a lower specified number. Consequentely, both the compensation that the supplier offers the selected retailers and the supplier's cost savings incurred by the UNAO strategy may be relatively great. Moreover, as the supplier's fixed processing cost increases, to gain more cost savings, the supplier would hope more retailers to accept the specified UNAO or CRE. This probably leads to that under the CRE strategy the supplier's cost savings increases much faster than does the compensation he pays, but it just reverse under the UNAO strategy.

4. Coordination model with a large number of retailers

4.1. Problem statement

In this section we consider the problem of how a supplier who supplies a product to a large number of heterogeneous retailers should set incentives (e.g., a price discount) to induce the retailers to place their orders at a UNAOs so as to lower his own order processing cost as well as the whole system cost. Generally, because $m$ is very large, it is impractical or very difficult for the supplier to obtain precisely the cost and market parameters of each retailer. Therefore, the analysis we present subsequently will be based on two assumptions below. (1) The supplier views the retailers' annual demands $D_i$’s as following a certain continuous distribution $\phi(D)$, which can be interpreted as the empirical frequency distribution of past observed $D_i$’s and (2) The cost parameters of each of the retailers are assumed to correspond to some “standard” values of which the supplier will be aware, i.e., set $A_i = A, K_i = K$ and $h_i = h$ for all $i$. Although this assumption might be restrictive, it is useful to simplify the analysis as an initial step to study.

Now suppose that to induce his retailers to accept the UNAO, the supplier wants to implement the following discount scheme defined by parameters $(f_o, d)$:

\[
\begin{align*}
\text{Unit purchasing price is } P & \text{ if the number of annual orders } f \neq f_o \\
\text{Unit purchasing price is } (1 - d)P & \text{ if the number of annual orders } f = f_o
\end{align*}
\]

Next the main target is to present a procedure for the supplier to design the optimal scheme $(f^*_o, d^*)$ and the advantages of implementing the UNAO strategy through numerical illustrations.

4.2. Retailers' reactions to the UNAO

Before the UNAO strategy stated in (12) is implemented, the order batch size of a retailer whose annual

<table>
<thead>
<tr>
<th>Serial</th>
<th>$A_i$</th>
<th>$A_f$</th>
<th>VP's CRE strategy</th>
<th>Mishra’s CRE strategy</th>
<th>The UNAO strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Supplier's savings (%)</td>
<td>System savings (%)</td>
<td>Supplier's savings (%)</td>
</tr>
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<td>1000</td>
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<td>0.08</td>
<td>26</td>
</tr>
<tr>
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<td>0.10</td>
<td>25</td>
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<tr>
<td>7</td>
<td>10</td>
<td>200</td>
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<td>0.12</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
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<td>500</td>
<td>29.99</td>
<td>0.34</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>1000</td>
<td>25.58</td>
<td>0.77</td>
<td>9</td>
</tr>
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<td>0.21</td>
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<td>24</td>
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<td>200</td>
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<td>0.25</td>
<td>18</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td>500</td>
<td>48.86</td>
<td>0.47</td>
<td>12</td>
</tr>
<tr>
<td>14</td>
<td>20</td>
<td>10</td>
<td>30.21</td>
<td>0.33</td>
<td>9</td>
</tr>
<tr>
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<td>0.73</td>
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<tr>
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<td>63.27</td>
<td>0.59</td>
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<td>1000</td>
<td>57.72</td>
<td>0.63</td>
<td>18</td>
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<td>10</td>
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<td>0.69</td>
<td>13</td>
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<td>20</td>
<td>100</td>
<td>31.73</td>
<td>0.88</td>
<td>9</td>
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<tr>
<td>20</td>
<td>20</td>
<td>1000</td>
<td>45.41</td>
<td>1.25</td>
<td>24</td>
</tr>
<tr>
<td>21</td>
<td>1000</td>
<td>10</td>
<td>74.72</td>
<td>1.28</td>
<td>13</td>
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<td>50</td>
<td>71.54</td>
<td>1.32</td>
<td>13</td>
</tr>
<tr>
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<td>1000</td>
<td>100</td>
<td>68.57</td>
<td>1.38</td>
<td>13</td>
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<tr>
<td>24</td>
<td>1000</td>
<td>500</td>
<td>62.03</td>
<td>1.53</td>
<td>12</td>
</tr>
<tr>
<td>25</td>
<td>1000</td>
<td>5000</td>
<td>58.04</td>
<td>2.04</td>
<td>9</td>
</tr>
</tbody>
</table>
demand is \( D \) would be the EOQ, \( Q = \sqrt{\frac{2DK}{hP}} \). The number she would order per year is then
\[
f = \frac{D}{Q} = \sqrt{\frac{hPD}{2K}}
\] (13)

The sum of purchasing cost and inventory-related cost is given by
\[
PD + \sqrt{2DKhP}
\]

In this situation the expected annual total cost of all retailers will be
\[
m \int \{ PD + \sqrt{2DKhP} \} \varphi(D) \, dD
\] (14)

The above order behavior of the retailer with the annual demand \( D \) will lead to the supplier’s order processing cost as \((A_s + A)f\). So before coordination the supplier’s expected annual order processing cost is
\[
m \int \{ (A_s + A) \sqrt{hPD/(2K)} \varphi(D) \} \, dD
\] (15)

After the UNAO strategy stated in (12) is declared, if the retailer with annual demand \( D \) accepts the UNAO, the sum of her purchasing cost and inventory-related cost would be
\[
(1-d)PD + Kf_o + hPD/(2f_o)
\] (16)

According to assumption (4), for any given UNAO strategy \((f_o,d)\), this retailer is willing to accept it only if the following condition is met.
\[
PD + \sqrt{2DKhP} - [(1-d)PD + Kf_o + hPD/(2f_o)]
\geq S\sqrt{2KPhD}
\]

which gives
\[
(h/(2f_o) - d)PD - (1-S)\sqrt{2DKhP} + Kf_o \leq 0
\] (17)

From (17) one can derive that a retailer’s reaction (accept or not) to the given UNAO strategy \((f_o,d)\) will depend upon the range her annual demand falls into, as shown in Lemma 3.

Define the following notations before Lemma 3:
\[
D_1 = \{2Kf_o/[1-(S \sqrt{2DKhP} + \sqrt{2KPhD} - S(2-S))]\}^2 \quad \text{and} \quad D_2 = \{2Kf_o/[1-(S \sqrt{2DKhP} - \sqrt{2KPhD} - S(2-S))]\}^2
\]

Lemma 3. (1) If \( h > 2df_{o}/S(2-S) \), all the retailers will not accept the UNAO strategy rather than implement their initial ordering policies instead.
(2) If \( 2df_{o}/S(2-S) \), only the retailers with \( D_1 < D < D_2 \) are willing to accept the UNAO and other retailers still implement their initial ordering policies.
(3) If \( h \leq 2df_{o} \), only the retailers with \( D > D_1 \) accept the UNAO but other retailers do not accept the UNAO.

4.3. Supplier’s expected annual gain

In order to obtain the optimal \((f_o^*,d^*)\), we first need to compute the supplier’s expected annual gain (or savings in cost) from any pair of parameters \((f_o,d)\).

For any given \((f_o,d)\), based on Lemma 3 above, the supplier’s expected annual gain from implementing this UNAO strategy can be derived through steps as explained below.

1. If \( 2df_{o}/S(2-S) \), there will be no change in each retailer’s ordering behavior because no retailer is willing to accept this UNAO proposed by the supplier. Hence, the supplier’s expected gain from this UNAO strategy is zero.

2. If \( hS(2-S) \leq 2df_{o}/h \), since only the retailers with \( D_1 < D < D_2 \) place their orders at the UNAOs \( f_o \), the supplier’s expected annual order processing cost for orders from these retailers is given by
\[
Af_o + m \int_{D_1}^{D_2} A f \varphi(D) \, dD
\]

where \( f \) is given in (13).

The expected annual compensation cost that the supplier pays these retailers in order to attract them to accept the UNAO is \( m \int_{D_1}^{D_2} PD f \varphi(D) \, dD \).

If these retailers did not change their ordering behavior, the supplier’s expected cost for dealing with orders from them would have been
\[
m \int_{D_1}^{D_2} (A_s + A f) \varphi(D) \, dD
\]

where \( f \) is given in (13).

Therefore, the supplier’s expected annual gain from this UNAO will be
\[
G_{s1} = m \int_{D_1}^{D_2} (A_s + A f) \varphi(D) \, dD - \left\{ Af_o + m \int_{D_1}^{D_2} A f \varphi(D) \, dD \right\}
\]
\[
+ m \int_{D_1}^{D_2} PD f \varphi(D) \, dD
\]
\[
= m \int_{D_1}^{D_2} \left\{ (A_s + A) \sqrt{hPD/2K} - (Af_o + PDf) \right\} \varphi(D) \, dD
\]
\[
- Af_o
\] (19)

Note that the bolded symbol here emphasizes that it is an “expected value”.

Similarly, if \( 2df_{o}/h \), the supplier’s expected annual gain from this UNAO will be given by
\[
G_{s2} = m \int_{D_1}^{D_2} \left\{ (A_s + A) \sqrt{hPD/2K} - (Af_o + PDf) \right\} \varphi(D) \, dD
\]
\[
- Af_o
\] (20)

To sum up, from any given \((f_o,d)\), the supplier’s expected annual gain from the UNAO strategy \((f_o,d)\) can be expressed as
\[
G_s(d,f_o) = \begin{cases} 
0 & 2df_o < hS(2-S) \\
G_{s1} & hS(2-S) < 2df_o < h \\
G_{s2} & h \leq 2df_o
\end{cases}
\] (21)
Table 3
The effect of parameter $\sigma$ on the optimal solution.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$d$ (%)</th>
<th>$f_0$ (in times per year)</th>
<th>Supplier's expected annual gain</th>
<th>Supplier's savings (%)</th>
<th>System's savings (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 000</td>
<td>0.205</td>
<td>18.09</td>
<td>40 360 691.97</td>
<td>87.61</td>
<td>3.93</td>
</tr>
<tr>
<td>20 000</td>
<td>0.199</td>
<td>18.03</td>
<td>40 586 124.58</td>
<td>87.72</td>
<td>3.94</td>
</tr>
<tr>
<td>15 000</td>
<td>0.195</td>
<td>17.85</td>
<td>40 781 715.52</td>
<td>87.92</td>
<td>3.96</td>
</tr>
<tr>
<td>10 000</td>
<td>0.187</td>
<td>17.76</td>
<td>40 956 653.31</td>
<td>88.24</td>
<td>3.96</td>
</tr>
<tr>
<td>5 000</td>
<td>0.180</td>
<td>17.57</td>
<td>41 118 517.22</td>
<td>88.50</td>
<td>3.97</td>
</tr>
</tbody>
</table>

Table 4
The effect of parameter $S$ on the optimal solution.

<table>
<thead>
<tr>
<th>$S$</th>
<th>$d$ (%)</th>
<th>$f_0$ (in times per year)</th>
<th>Supplier's expected annual gain</th>
<th>Supplier's savings (%)</th>
<th>System's savings (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.185</td>
<td>17.98</td>
<td>40 736 623.64</td>
<td>88.04</td>
<td>3.94</td>
</tr>
<tr>
<td>0.10</td>
<td>0.199</td>
<td>18.03</td>
<td>40 586 124.58</td>
<td>87.72</td>
<td>3.94</td>
</tr>
<tr>
<td>0.15</td>
<td>0.216</td>
<td>17.99</td>
<td>40 435 845.69</td>
<td>87.39</td>
<td>3.94</td>
</tr>
<tr>
<td>0.20</td>
<td>0.230</td>
<td>18.05</td>
<td>40 285 853.95</td>
<td>87.07</td>
<td>3.94</td>
</tr>
<tr>
<td>0.25</td>
<td>0.245</td>
<td>18.03</td>
<td>40 136 100.60</td>
<td>86.74</td>
<td>3.94</td>
</tr>
</tbody>
</table>

Table 5
The effect of parameter $h$ on the optimal solution.

<table>
<thead>
<tr>
<th>$h$</th>
<th>$d$ (%)</th>
<th>$f_0$ (in times per year)</th>
<th>Supplier's expected annual gain</th>
<th>Supplier's savings (%)</th>
<th>System's savings (%)</th>
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<tbody>
<tr>
<td>0.05</td>
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<td>2.37</td>
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<td>18.03</td>
<td>40 586 124.58</td>
<td>87.72</td>
<td>3.94</td>
</tr>
<tr>
<td>0.20</td>
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<td>4.51</td>
</tr>
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<td>52 396 357.97</td>
<td>87.72</td>
<td>5.01</td>
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Table 6
The effect of parameter $A_s$ on the optimal solution.

<table>
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<th>$A_s$</th>
<th>$d$ (%)</th>
<th>$f_0$ (in times per year)</th>
<th>Supplier's expected annual gain</th>
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<td>79 078 143.13</td>
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<tr>
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<td>0.202</td>
<td>17.95</td>
<td>59 830 455.34</td>
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<td>5.66</td>
</tr>
<tr>
<td>1000</td>
<td>0.199</td>
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<td>40 586 124.58</td>
<td>87.72</td>
<td>3.94</td>
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<td>13 661 822.83</td>
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<td>1.41</td>
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Table 7
The effect of parameter $A$ on the optimal solution.

<table>
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<tr>
<th>$A$</th>
<th>$d$ (%)</th>
<th>$f_0$ (in times per year)</th>
<th>Supplier's expected annual gain</th>
<th>Supplier's savings (%)</th>
<th>System's savings (%)</th>
</tr>
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<tr>
<td>100</td>
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<td>3.77</td>
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<td>39 615 690.47</td>
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<tr>
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<td>40 586 124.58</td>
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<td>3.94</td>
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<tr>
<td>250</td>
<td>0.237</td>
<td>16.37</td>
<td>41 655 280.93</td>
<td>86.43</td>
<td>4.03</td>
</tr>
<tr>
<td>300</td>
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<td>42 797 729.07</td>
<td>85.38</td>
<td>4.14</td>
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Table 8
The effect of parameter $K$ on the optimal solution.

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<th>$K$</th>
<th>$d$ (%)</th>
<th>$f_0$ (in times per year)</th>
<th>Supplier's expected annual gain</th>
<th>Supplier's savings (%)</th>
<th>System's savings (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
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<td>18.49</td>
<td>53 643 343.77</td>
<td>89.80</td>
<td>5.11</td>
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<tr>
<td>60</td>
<td>0.189</td>
<td>17.77</td>
<td>36 711 230.29</td>
<td>86.91</td>
<td>3.58</td>
</tr>
<tr>
<td>90</td>
<td>0.167</td>
<td>17.11</td>
<td>29 324 224.46</td>
<td>85.03</td>
<td>2.91</td>
</tr>
<tr>
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<td>16.44</td>
<td>24 968 958.48</td>
<td>83.60</td>
<td>2.50</td>
</tr>
<tr>
<td>150</td>
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<td>15.82</td>
<td>22 019 460.12</td>
<td>82.43</td>
<td>2.23</td>
</tr>
</tbody>
</table>
The supplier’s problem of determining the values of $d$ and $f_o$ that maximize $G_s$ can be formulated as below.

\[
\begin{align*}
\text{Max } & G_s(d, f_o) \\
\text{s.t. } & 0 < d < 1 \\
& f_o > 0 \\
& 2df_o > hS(2 - S) 
\end{align*}
\]  
(22)

For the optimal UNAO strategy ($f_o^*, d^*$) declared by the supplier, the expected annual savings in cost for all retailers who accept the UNAO will be

\[
G_F = \begin{cases} 
\int_{df_o^*}^{d_F} \left\{ \sqrt{2KhPD + d'PD - \left( \frac{Kf_o^* + hD}{d'} \right) \phi(D) } \right\} \phi(D) dD \\
\int_{2df_o^*}^{\infty} \left\{ \sqrt{2KhPD + d'PD - \left( \frac{Kf_o^* + hD}{d'} \right) \phi(D) } \right\} \phi(D) dD 
\end{cases}
\]  
(23)

4.4. Solution procedure

Due to the complexity of problem (22), it is difficult to obtain its closed-form optimal solution. However, one can easily find the numerical optimal solution for problem (22) by using the IMSL subroutine DBCPOL (IMSL Math/ Library, 1994). This procedure does not require a smooth objective function. Considering $G_s$ may be not unimodal, we execute the search subroutine DBCPOL at least 60 times in numerical examples below, each time with a different initial point. The computation time required for obtaining a final solution is trivial.

Next, consider a numerical example to illustrate the benefit of implementing the UNAO strategy when the supplier faces a very large number of retailers. In the example, we assume retailers’ annual demand $D$’s to be a random variable of normal distribution with mean $\mu$ and standard deviation $\sigma$. The values of parameters in the model are as follows:

Example: Let $A_i = 1000$, $A = 200$, $K = 50$, $S = 0.1$, $h = 0.15$, $P = 10$, $\mu = 100000$, $\sigma = 20000$ and $m = 1000$.

Without implementing coordination, the supplier’s expected annual order processing cost is 46269590.16 (obtained from (15)); and retailers’ expected total cost is 1004800965.44 (obtained from (14)). As a result, the expected annual cost for the whole system is 1051070555.6. If the supplier wants to implement the UNAO strategy, following the above solution procedure will give the optimal UNAO scheme as $f_o^* = 18.03$ and $d^* = 0.199%$. Under this optimal UNAO scheme, the supplier’s cost savings or gain is $40586124.58$ and the cost savings for retailers who accept the UNAO is $786298.38$ (computed from (23)), which yields the cost savings for the whole system as $41372422.96$. The corresponding percentage cost savings for the supplier and the system are, respectively, 87.72% and 3.94%.

To further study the effect of changing values of parameters on the optimal UNAO strategy and the savings from the UNAO strategy, the above numerical example is used to make sensitivity analysis. The analysis is implemented by changing one parameter at a time and keeping the remaining parameters at their true values. The results of this analysis are shown in Tables 3–10. From Table 3, it can be observed that as $\sigma$ increases, both the price discount $d$ and UNAOs offered by the supplier will increase while the supplier’s and system percentage cost saving decrease. Therefore, the supplier should set the price discount and specified number of annual orders according to the deviation of retailers’ demands from their mean value. The greater deviation, the higher price discount and specified number of annual orders. Table 4 shows that the price discount $d$ increases and the UNAOs slightly changes as $S$ increases, whereas the system’s percentage cost saving keeps the same but the supplier’s percentage cost saving decreases. It can be seen from
Tables 5 and 9 that the change of parameters $h$ and $P$ only affect system percentage cost saving but not the supplier’s percentage cost saving, which is unlike parameter $S$. Tables 6–8 show that both supplier’s and system percentage cost saving increase as $K$ and $A_i$ increase, whereas the system’s percentage cost saving increases but the supplier’s decreases as $A$ increases. Know from Table 10, the benefits gained by the supplier from the UNAO strategy increase as the number of retailers increases, while the change of parameter $m$ slightly affects the UNAO strategy.

5. Conclusions

The present paper proposes a new strategy for coordination of inventory policies in a one-supplier, multi-retailer supply chain. Under this strategy the supplier requires all retailers or some retailers (if beneficial) to place their replenishment orders with a unified number of annual orders. First, the paper considers a supply chain where a supplier supplies a relatively small number of retailers. A heuristic for finding the supplier’s optimal UNAO strategy is proposed. A numerical study is used to demonstrate the solution procedure and the benefits of this new strategy. A counter-intuitive insight is that the UNAO strategy dominates the existing CRE strategies in the supplier’s cost saving for lower fixed order processing cost rather than for the opposite case. Second, the paper extends the analysis to cover the situation with a large number of retailers. The numerical study further reveals that when the supplier faces a large number of retailers, implementing the UNAO strategy is still profitable for the supplier and the system. In this extended model, the cost parameters for all retailers are assumed to be the same. This assumption will be relaxed in next paper.

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Appendix A

Proof of Lemma 1. From (6) and (8), one has

$$g_i(f_o) - g_i(f_o) = (V_i f_o^2 - U_i f_o + h_i P D_i)/(P D_i f_o),$$

for each $i = 1, 2, \ldots, m$ \hfill (A.1)

Since there is some subscript $j$ such that $\delta_i < 0$, the discriminant of quadratic function with respect to $f_o$ in the brace at the right hand side of (A.1) is not larger than zero for this subscript $j$. It implies that this quadratic function is always no less than zero for every $f_o$ due to $V_i > 0$. Thus, from (A.1), one has $g_i(f_o) > g_i(f_o)$. Therefore, any UNAO strategy $(f_o, d)$ that is favorable to retailer $j$ is harmful to the supplier because $d$ will be not smaller than $g_i(f_o)$. This indicates that there is not the UNAO strategy favorable to both retailer $j$ and the supplier. Of course, there is no feasible UNAO strategy accepted by all partners. \hfill $\Box$

Proof of Lemma 2. Recall that a feasible UNAO scheme $(f_o, d)$ should simultaneously satisfy the conditions (6) and (8) in Section 3.2. From views of the geometric interpretation, condition (8) requires parameters $(f_o, d)$ chosen below line $d = g_i(f_o)$, whereas condition (6) means that parameters $(f_o, d)$ should be selected above curve $d = g_i(f_o)$ for each $i$. So a feasible UNAO scheme that can be ultimately implemented requires $(f_o, d)$ chosen only in the common area enclosed by line $d = g_i(f_o)$ and each curve $d = g_i(f_o)$. Known from definitions of $g_i(f_o)$ and $g_i(f_o)$, each $d = g_i(f_o)$ is a convex curve with $\lim_{x \to -\infty} g_i(f_o) = \lim_{x \to \infty} g_i(f_o) = +\infty$ while $d = g_i(f_o)$ linearly decreases from $0$ to infinity. Since $\delta_i > 0$ for each $i = 1, 2, \ldots, m$, there are two intersection points between line $d = g_i(f_o)$ and each curve $d = g_i(f_o)$, as sketched in Fig. 2. As a matter of fact, if not, then each curve $d = g_i(f_o)$ might locate at above line $d = g_i(f_o)$, i.e., $g_i(f_o) > g_i(f_o)$ for each $i$. Thus, from (A.1) one has $\{V_i f_o^2 - U_i f_o + h_i P D_i/2\} > 0$ for each $i$, which means its discriminant (i.e., $\delta_i$) less than or equal to $0$. It contradicts with the given condition.

Define $t_{i_1}$ and $t_{i_2}$ be, respectively, coordinates on the $f_o$ axis of two points (sketched in Fig. 2). Then solving equation $g_i(f_o) - g_i(f_o) = 0$ will give $t_{i_1} = (U_i - \sqrt{\delta_i})/(2V_i)$ and $t_{i_2} = (U_i + \sqrt{\delta_i})/(2V_i)$. Based on the analysis above, a UNAO scheme $(f_o, d)$ favorable to both the supplier and retailer $i$ requires the UNAOs $f_o$ chosen only in the interval $[t_{i_1}, t_{i_2}]$ ($i = 1, 2, \ldots, m$). Therefore, for any feasible UNAO scheme $(f_o, d)$, $f_o$ should fall into the intersection of the above $m$ intervals. Noting that the definitions of $\alpha$ and $\beta$, one has $\alpha = \Max(t_{i_1}, i = 1, 2, \ldots, m)$ and $\beta = \Min(t_{i_2}, i = 1, 2, \ldots, m)$. Thus,

1. when $\alpha > \beta$, this intersection is clearly empty. Hence, there is no feasible UNAO.
2. If $\alpha \leq \beta$, the above intersection will be $[\alpha, \beta]$. Thus, the proof of the second part of Lemma 2 is completed. \hfill $\Box$

Proof of Lemma 3. (1) Define a quadratic function:

$$y(x) = (h_x/(2f_o) - d) x^2 - x (1 - S) \sqrt{2Kh_o} + Kf_o$$

(A.2)

The discriminant of $y(x)$ is $A = 2P[K^2d_o - hS(2-S)]$. Since $h > 2d_o/\sqrt{2(2-S)}$ and $0 < S < 1$, $A < 0$ and $h_x/(2f_o) - d > 0$. It indicates that $y(x) > 0$ for any real number $x$. Therefore, setting $x = \sqrt{D}$, one has

$$(h_x/(2f_o) - d)PD - (1 - S)\sqrt{2DKh_o} + Kf_o > 0,$$

for any $D > 0$

which means that the condition (17) will not hold for the retailer with any annual demand $D$. So no retailer is willing to accept the UNAO strategy $(f_o, d)$ given by the supplier.
(2) If $hS(2–S) < 2D_0$, then $\Delta > 0$. Hence, there are two roots to equation $y(x) = 0$. Let the two roots be, respectively, $x_1$ and $x_2$. After solving $y(x) = 0$ and some simple algebraic operations, one has

$$x_1 = 2Kf_o/[((1 – S)/\sqrt{2KhP} + \sqrt{2Kp(2D_0 – S(2 – S))})$$

and

$$x_2 = 2Kf_o/[((1 – S)/\sqrt{2Kp(2D_0 – S(2 – S))})].$$

Then (A.2) can be rewritten as

$$y(x) = (h/2D_0) – dp(x – x_1)(x – x_2) \quad (A.3)$$

Due to $2D_0 < hS(2–S)$, one can easily derive from (A.3) that $y(x) \leq 0$ if and only if $0 < x_1 < x < x_2$. Thus, setting $x = \sqrt{D}$ and noting definitions of $D_1$ and $D_2$ will give that $y(\sqrt{D}) \leq 0$ if and only if $\sqrt{D_1} \leq \sqrt{D} \leq \sqrt{D_2}$. That is to say, the condition (17) keeps only for retailers whose annual demand $D$s belong to the interval $[D_1, D_2]$. The proof of the second part of Lemma 3 is completed.

(3) If $h > 2D_0$, then it is clear that $2D_0 > hS(2–S)$. At this moment, $\Delta$ is still not negative. Through similar analysis as used in the proof of the second part of Lemma 3, one can derive that $y(x) \leq 0$ if and only if $x > x_1 > 0$ or $x < x_2 < 0$. Therefore, $y(\sqrt{D}) < 0$, i.e., the condition (17) holds only if $D > D_1$. \(\Box\)

References


