Center of rotation automatic measurement for fan-beam CT system based on sinogram image features

Min Yang, Jing Pan, Jianhai Zhang, Sung-Jin Song, Fanyong Meng, Xingdong Li, Dongbo Wei

1. Introduction

Projection center of rotation (COR) is one of the important parameters for fan-beam CT system to guarantee the accuracy of the image reconstruction. And it should be strictly controlled with high precision, because the fan-beam reconstruction algorithms are derived on the basis of the assumption that the projection center of rotation is collinear with the midline of the fan-beam [1,2]. However, during the testing process, whether the usage of laboratory-built industrial CT systems or those commercially available CT systems, the tested object between the X-ray focus and the COR have been extensively used, such as the center-of-sinogram method, the geometrical method, the iterative method and the opposite-angle method [6-10]. In the center-of-sinogram method, two projections which are 180° opposite to each other are used, and the midpoint of the two projected feature locations on the detector is an estimate of the COR. This method is based on the assumption that the line connecting the X-ray focus and the COR is perpendicular to the detector. But the assumption is not always true, and during the calibration a specific straight metal wire is always required. The geometry method is the improvement of the center-of-sinogram method, which permits the line connecting the X-ray focus and the COR not to be perpendicular to the detector, while, the projected location of the central ray and the distance from X-ray focus to the detector need to be exactly calculated. The iterative reconstruction algorithm starting from the reconstructed image with errors of COR is employed in the iterative method. And the iterating times is controlled by the optimal criterion until getting the best image quality. Due to time-consuming of this method, it is seldom adopted in practice. The opposite-angle method is based on the fact that among all the 180° opposite angle projection pairs, only the ray passing the COR projects the same location on the detector in 0° and 180°. Recently, Li proposed a method to seek the projected location of the ray passing the COR by using the opposite

For fan-beam X-ray CT system, projection center of rotation (COR) is one of the most important parameters which should be strictly controlled with high precision to ensure the accuracy of the image reconstruction. In our research, a novel method is proposed to locate COR for fan-beam CT system based on sinogram features. In this method, getting a data set with regular symmetric shape by averaging the original sinogram along the direction of column, and another data set by flipping the averaged data along the direction of row, the cross correlation operation is applied to these two data sets and finally the position of COR is determined by locating the peak values of the cross correlation function. In our proposed method, only the sinogram of the scanned slice is used for locating COR based on the image cross correlation without the calibration sample. The experimental results prove that it is easy to implement with high accuracy and anti-noise ability.

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angle projection pairs [11]. But his algorithm will introduce large error when the sample is a circular shape.

In this paper, we propose a novel method to determine the COR for fan-beam CT system based on image cross correlation. Only the sinogram of the scanned slice is used for locating COR based on the image cross correlation without the calibration phantom in our developed method. Due to the image cross correlation operation and the unique coordinate of the peak value, it can be implemented with a good ability of anti-noise. In the following sections, the theory of the novel method based on image cross correlation is outlined and our experimental results are presented. In Section 3, anti-noise ability and accuracy analysis by computer simulation is conducted. In Section 4, the experiments are performed in a 2D CT system and the unique coordinate of the peak value, it can be implemented with a good ability of anti-noise. In the following sections, the experiments are performed in a 2D CT system and some experimental results are shown. Section 5 is devoted to conclusion.

2. Method based on image cross correlation

The scanning principle of fan-beam 2D CT system with linear detector array (LDA) is shown in Fig. 1. The X-ray focus is point F, the coordinate system of the scanned slice is XOY and the point O is the COR. Midline of the fan-beam (SO) passes COR and is perpendicular to the LDA. O2S2 is the coordinate system of LDA where the coordinate of COR is s0. The X-ray source and the detector remain stationary with respect to each other while the object rotates around the fixed center (point O) within 360°, which is equivalent to that X-ray source and the detector rotate around the fixed center synchronously while the object remain stationary as shown in Fig. 2. The LDA receives the X-ray photons penetrating through the object and converts them to a row of digital signals while the object is rotating step by step. All the row data constitute a two-dimensional matrix, which is named sinogram p(θ, s), where θ is the rotating angle, s is the detector unit coordinate on O2S2 axis. According to the requirements of 2D CT reconstruction algorithm, the position of COR, namely the value of s0, must be determined accurately as an important reconstruction parameter, otherwise, serious artifacts in the reconstructed image could be generated by the error of s0.

As shown in Fig. 2, the label of the ray connecting the focus and the detector unit defined by the angle between the ray and the midline of fan-beam, which is named as γ. When θ = β, β ∈ [0 2π], the ray γ passes the scanned slice and reaches the detector unit sγ with the projection value of p(β, sγ). Similarly, when θ = β + π + 2γ, its projected location is sβ+π+2γ and its projection value is p(β + π + 2γ, sβ+π+2γ). From the geometrical configuration, the ray γ penetrates scanned slice along the same path in β and β + π + 2γ rotating angle, so it can be deduced that point sβ and sβ+π+2γ are symmetrical about point s0 and the projection values at this two points are equal. Namely

\[ s_β - s_0 = s_β - s_0 \]

If we execute integral operation on both sides of Eq. (1), then it can be rewritten as

\[ \int_0^{2 \pi} p(\beta, s_β) d\beta = \int_0^{2 \pi} p(\beta + \pi + 2\gamma, s_β + \pi + 2\gamma) d\beta \]  

From Eq. (2), for any detector unit pair which are symmetrical to each other about s0, the sum of their projection value at all rotating angle are equal. Namely when we sum up the sinogram p(θ, s) along θ direction to get a row of data defined as p1(s), the curve line of the row data should be an even function with the symmetrical center of s0. And p2(s) can be shown as

\[ p_2(s) = 2N - p_1(s) \]

where N is the total number of the detector units which also equals to the length of the LDA. The cross correlation function Rp1p2(τ) between p1(s) and p2(s) can be formulated as

\[ R_{p_1, p_2}(τ) = \frac{1}{N} \int_0^N p_1(s)p_2(s + \tau) ds \]  

Because the symmetrical center of p1(s) is s0, so

\[ p_1(s) = p_1(2s_0 - s) \]

\[ p_2(s + \tau) = p_1(2s_0 - s - \tau) \]

Combining Eqs. (4) and (5), we get

\[ p_2(s + \tau) = p_1(2s_0 - N + s + \tau) \]

Substituting Eq. (6) to Eq. (2):

\[ R_{p_1, p_2}(τ) = \frac{1}{N} \int_0^N p_1(s)p_1(2s_0 - N + s + \tau) ds = R_{p_1, p_1}(2s_0 - N + \tau) \]

where \( R_{p_1, p_1} \) is the autocorrelation function of p1(s). According to the characteristic of autocorrelation function, when \( R_{p_1, p_1} \) reaches its peak value, the following condition should be satisfied:

\[ 2s_0 - N + \tau = 0, \tau = \tau_0. \]

Therefore,

\[ s_0 = \frac{N - \tau_0}{2} \]  

Fig. 1. Schematic diagram of fan-beam scanning.
By concluding the above analysis, the five-stepped methods are summarized to calculate the position of COR:

**Step 1**: When the tested object is rotating by a fixed angle step during $360^\circ$, the LDA receives the X-ray photons penetrating through the object and converts them to a row of digital signals. All the row data are arranged in sequence to constitute a matrix $p(\theta, s)$ named original sinogram.

**Step 2**: Conducting logarithmic transformation on each row of $p(\theta, s)$ as following:

$$p'(\theta_i, s) = \ln \frac{p^0_i}{p(\theta_i, s)}$$

where $p^0_i$ is the reference beam intensity which is the output signal converted by LDA units where X-ray photons do not penetrate the object. $p(\theta_i, s)$ represents each row data of the original sinogram $p(\theta, s)$.

**Step 3**: Summing up all the row data of the sinogram $p'(\theta_i, s)$ to get a row of data $p(s)$. As analyzed above, the curve line of the row data should be an even function with the symmetrical center of $s_0$. And $p(s)$ can be shown as

$$p(s) = \sum_{\theta_i = 0}^{2\pi} p'(\theta_i, s)$$

**Step 4**: Flipping $p(s)$ in right direction to create a new row of data labeled as $p_{flip}(s)$.

**Step 5**: Calculating the cross correlation between $p(s)$ and $p_{flip}(s)$ according to Eq. (3), and then statistic the coordinate of the peak value of the cross correlation function, the coordinate is recorded as $\tau'$. In order to ensure the accuracy of $\tau'$, we used data fitting method to get the value of $\tau'$ with decimal precision. According to Eq. (7), the position of COR is finally achieved.

### 3. Anti-noise ability and accuracy analysis by computer simulation

According to the characteristic of cross correlation function, all the data in $p(s)$ and $p_{flip}(s)$ are used to calculate the position of the peak value, so the errors caused by the noise in the projection data can be neglected in most cases. In addition, from Step 2, $p(s)$ is obtained by averaging the original sonogram $p(\theta, s)$ along the direction of column, which further decreases the affection of the random noise. To support the ratiocination, the noised sinogram of Shepp–Logan phantom is simulated numerically and the proposed method is applied to calculate $s_0$ with the true value of 255. Table 1 shows the calculation results from sinograms with different noise levels. The simulated noise is Gaus-type with mean value of zero while its variance is set as $\sigma$. The mean value of the zero-noised sinogram is 47.568. From the results, even the variance of the added noise is 10, which almost reaches 21% of the mean value of the original sinogram, $s_0$ still keeps an ideal precision of 0.217, which ensures a desired reconstruction result. And it proves that the proposed method is not sensitive to the random noise.
In addition, the proposed method is compared with two typical acknowledged calibration methods: Opposite-Angle Method (OAM) and Curve-Fitting Method (CFM) [10]. Here the Shepp–Logan phantom is still adopted to validate these methods. Through numerical simulations, the true value of $s_0$ is set to be 255. The calculation results of these three methods in Table 2 show that all the values of $s_0$ are very close to the true value. Liu and Malcolm [10] point out that OAM relies on the accuracy of the determination of the projections of the two nearest opposite-angle pairs severely. However, CFM uses the curve-fitting method to calculate $s_0$ based on a number of projections, therefore, it is a more reliable and accurate method which may be adopted in engineering application. The calibration result of our proposed method (CCM—Cross Correlation Method) is much closer to that of CFM than OAM. But CCM does not need opposite-angle projection pairs and is much easier to realize. In a sense, CCM can be regarded as an improved method of CFM, so it has a potential engineering application.

### 4. Experimental results

To verify the feasibility and accuracy of the proposed method, we performed experiments in a 2D CT system. The experimental conditions were set as

- X-ray energy: 220 kV, 5 mA
- LDA unit size: 0.4 mm
- Total detector units: 1600
- Distance from X-ray focus to LDA: 1200 mm
- Total number of the projections: 1800 frames

Under the above situations, the special resolution target was scanned and the original fan-beam sinogram was created as shown in Fig. 3. Fig. 4 is the curve plot of $p(s)$. It is apparent that $p(s)$ has a regular symmetrical shape with the center of $s_0$. Furthermore, we flipped $p(s)$ in right direction to create a new

<table>
<thead>
<tr>
<th>Noised sinogram</th>
<th>Curve line of $p(s)$</th>
<th>$s_0$</th>
<th>Errors</th>
<th>Reconstruction images</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Sinogram 1" /></td>
<td><img src="image2.png" alt="Curve 1" /></td>
<td>0</td>
<td>254.969</td>
<td><img src="image3.png" alt="Image 3" /></td>
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<tr>
<td><img src="image4.png" alt="Sinogram 2" /></td>
<td><img src="image5.png" alt="Curve 2" /></td>
<td>2.5</td>
<td>254.832</td>
<td><img src="image6.png" alt="Image 4" /></td>
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<tr>
<td><img src="image7.png" alt="Sinogram 3" /></td>
<td><img src="image8.png" alt="Curve 3" /></td>
<td>5</td>
<td>254.803</td>
<td><img src="image9.png" alt="Image 5" /></td>
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<tr>
<td><img src="image10.png" alt="Sinogram 4" /></td>
<td><img src="image11.png" alt="Curve 4" /></td>
<td>7.5</td>
<td>254.817</td>
<td><img src="image12.png" alt="Image 6" /></td>
</tr>
<tr>
<td><img src="image13.png" alt="Sinogram 5" /></td>
<td><img src="image14.png" alt="Curve 5" /></td>
<td>10</td>
<td>255.217</td>
<td><img src="image15.png" alt="Image 7" /></td>
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</tbody>
</table>

### Table 1

Calculated results of $s_0$ from sinograms with different noise levels (unit: Pixel).
row of data labeled as $\bar{p}_{\text{lip}}(s)$, and plotted $\bar{p}(s)$ and $\bar{p}_{\text{lip}}(s)$ in a figure together, as shown in Fig. 5, from which we can find that $\bar{p}_{\text{lip}}(s)$ and $\bar{p}(s)$ have the same shape except that there is a shift $\tau'$ between them along detector length direction.

In the experiments, the 2D CT scanning system had the initial value of $s_0 = 598$. But after a period of usage, the value would deviate from this value, which can decrease the special resolution of the reconstructed image. Fig. 6 shows the reconstructed image by $s_0 = 598$, from which the maximum discriminable line pair is 1.5 lp/mm. Through the method proposed in our work, the value of $s_0 = 602.446$. After inputting it to the reconstruction algorithm, a clearer image was obtained as shown in Fig. 7. According to the results, the special resolution of the image rose obviously and 2.0 lp/mm was reached. It is concluded that the proposed calibration method can effectively increase the accuracy of image reconstruction and improve the spatial resolution of 2D CT system.

In order to give a quantitative description of the experimental results, the entropy, the spatial frequency, and the standard deviation of the uncorrected and corrected CT images were compared. Basically, in image processing field these three parameters are commonly used to describe the detailed information of given images, such as texture and edge features [12–20]. Mathematically, their definitions are given as

<table>
<thead>
<tr>
<th>Methods</th>
<th>Reconstruction image</th>
<th>$s_0$</th>
<th>Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>OAM</td>
<td>![OAM Image]</td>
<td>255.333</td>
<td>0.333</td>
</tr>
<tr>
<td>CFM</td>
<td>![CFM Image]</td>
<td>254.766</td>
<td>0.234</td>
</tr>
<tr>
<td>CCM</td>
<td>![CCM Image]</td>
<td>255.144</td>
<td>0.144</td>
</tr>
</tbody>
</table>

Fig. 3. Sinogram after logarithmic transformation.

Fig. 4. Curve of $\bar{p}(s)$.

Fig. 5. Curves of $\bar{p}(s)$ and $\bar{p}_{\text{lip}}(s)$. 

Table 2
Comparison results among three calibration methods (unit: Pixel).
Entropy ($H$)

$$H = - \sum_{i=0}^{L-1} p(i) \log_2 p(i)$$  \hspace{1cm} (8)

Spatial frequency ($SF$)

$$RF = \sqrt{\frac{\sum_{m=1}^{M} \sum_{n=1}^{N} [p(m, n) - p(m, n-1)]^2}{M \times N}}$$  \hspace{1cm} (9)

$$CF = \sqrt{\frac{\sum_{n=1}^{N} \sum_{m=1}^{M} [p(m, n) - p(m-1, n)]^2}{M \times N}}$$  \hspace{1cm} (10)

$$SF = \sqrt{RF^2 + CF^2}$$  \hspace{1cm} (11)

Standard deviation ($Std$)

$$Std = \sqrt{\frac{1}{M \times N} \sum_{m=1}^{M} \sum_{n=1}^{N} (p(m, n) - \bar{p})^2}$$  \hspace{1cm} (12)

where $p(i) = N_i / N$, $N$ represents the total pixel number of the image, and $N_i$ represents the number of pixels whose gray value equals $i$. $L$ represents the total gray level steps of the image, and $p(m, n)$ represents the image. $M$ and $N$ represent the height and the width of the image, respectively. $SF$ is the spatial frequency, $RF$ and $CF$ are the row frequency and the column frequency, respectively. The calculated values of $H$, $SF$, and $Std$ before and after the correction are compared in Table 3.

Table 3: Quantitative comparison between the uncorrected and corrected CT images.

<table>
<thead>
<tr>
<th></th>
<th>Entropy</th>
<th>Variance</th>
<th>Space frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncorrected CT image</td>
<td>33.479</td>
<td>119.600</td>
<td>21.770</td>
</tr>
<tr>
<td>Corrected CT image</td>
<td>36.981</td>
<td>128.384</td>
<td>23.245</td>
</tr>
<tr>
<td>Increasing percentage</td>
<td>10.46</td>
<td>7.34</td>
<td>6.78</td>
</tr>
</tbody>
</table>

From these results, we can see that the proposed method can deblur the original CT image and enrich its detailed information effectively.

5. Conclusion

In this work, we have successfully developed a novel method to determine the location of COR in 2D CT system. Compared with the conventional calibration methods, only the sinogram of the scanned slice is used for locating COR based on the image cross correlation without the calibration phantom in our proposed method. Because the image cross correlation operation is adopted, and the coordinate of the peak value is unique according to the characteristic of cross correlation function, it is easy to implement and the automatic calibration can be realized with high accuracy and anti-noise ability. Meanwhile, our
developed method is also applicable to calibrate the position of COR in 3D CT system. Where, the central slice of the scanned object needs to be determined [21–26], because only the central slice satisfies the fan-beam scanning principle. After the central slice has been located, its sinogram can be generated and the proposed method can then be applied to locate COR. In addition, as shown in Figs. 1 and 2, the geometrical and mathematical prerequisite for the success of the proposed method is that the central X-ray must be perpendicular to the linear detector. If the object translates a small distance along the direction which is perpendicular to the central X-ray, the symmetrical shape of $p(s)$ will weaken and eventually the location accuracy of $s_0$ decreases. So before the calibration, mechanical adjustments are necessary to make central X-ray perpendicular to the linear detector as much as possible. And our practical experiments prove that for the objects with simple shape, especially circular shape, the affects of this kind of error are minor and even can be neglected sometimes.

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