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Rotating spin–orbit coupled Bose–Einstein condensates in concentrically coupled annular traps

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Abstract

We study trapped 2D spin-1 Bose–Einstein condensates with isotropic spin–orbit coupling and rotation, showing rich ground state phases and phase transition results from the nontrivial interplay among spin–orbit coupling, rotation and confinement. We show that for experimentally feasible parameters and only in the presence of spin–orbit coupling, a double-ring structure for the longitudinal magnetization of the system can be formed within the minima of inner and/or outer rings. Moreover, the rotation effectively enhances the effect of the spin–orbit coupling and controls the location of atoms between both rings. Our results provide a scenario of controlling different phases of a spin–orbit coupled condensate by varying the spin–orbit coupling strength and rotation frequency, instead of using the conventional approach of changing the s-wave scattering length.

(Some figures may appear in colour only in the online journal)

1. Introduction

With the pioneering experimental realization of an artificial Abelian or non-Abelian gauge potential in neutral atoms, effective spin–orbit (SO) coupling has been realized in atomic Bose–Einstein condensates (BEC) systems based on appropriate laser arrangements and tailored dressed states [1–5]. The process on the SO coupled BECs’ side to stimulate a corresponding wave of activity has been studied both theoretically and numerically, with a number of papers focused on pseudo spin-1/2 condensates [6–8]. Thus opening up new possibilities to simulate the effect of SO coupling for a wide range of phenomena in ultracold Bose gases. A major merit of such a system lies in the unprecedented level of experimental control of the system’s parameters [6–8], thus providing new opportunities to simulate the effect of SO coupling for a wide range of phenomena in quantum systems, such as the topological insulator, exotic superconductivity or superfluidity, and quantum spin Hall effects [9, 10].

So far, most of the theoretical studies on SO coupled BECs have been restricted to a free space or harmonic potential, with or without rotation [11–29]. In particular, for a harmonically trapped pseudo spin-1/2 condensate, the combined effects of Rashba SO coupling and a rotating trap have been investigated, which show the appearance of some unusual topological patterns, including the skyrmion and giant vortex [12, 13, 17]. However, various external potentials, such as double-well, single-ring, or vertically or concentrically coupled double-ring traps are within the current experimental capacity as well [30–40]. Thus it is instructive to extend the study to different external traps as mentioned above. In previous work, we have investigated the ground state properties of pseudo spin-1/2 SO coupled BECs confined in concentrically coupled annular traps [41]. However, the effect of other parameters, such as spin-dependent interaction and rotation, on the ground state properties has not yet been
discussed. If we add these degrees of freedom, will that help?

The answer is yes and the aim of this work is to investigate the ground state and rotational properties of spin-1 BECs influenced by SO coupling and rotation respectively or jointly. We find that these parameters not only enrich the phase diagram of the system, but also enable us to explore a new regime in rotating ultracold gases confined in concentrically coupled annular traps.

The remainder of this paper is organized as follows. In section 2, a theoretical description of SO coupled spin-1 BECs confined in concentrically coupled annular traps is given. The ground state structure and the rotational properties of the system with ferromagnetic (FM) and antiferromagnetic (AFM) interactions are discussed in section 3. In particular, the effects of SO coupling, rotation, and the two combined are discussed. Finally, we conclude our results in section 4.

2. The theoretical model

We illustrate the physics using quasi-two-dimensional spin-1 BECs as an example. The two-dimensional potential can be realized by adding a very tight trapping potential along the z axis, which completely freezes out the degrees of freedom of the gases along this direction. The general procedure we use to attain SO coupled spin-1 BECs is identical to those discussed in [42, 43]. After completing the sample preparation, the condensate is adiabatically loaded into concentrically coupled annular traps, and the rotation is turned on by rotating the trap or imposing a synthetic Abelian field on top of the synthetic spin–orbit interactions [13, 17]. In the following discussions, we consider Rashba SO coupling whose particle’s spin couples its degree of motion in the x–y plane. The Hamiltonian for this system can be written as \( \hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} \) with \( \hat{H}_0 \) and \( \hat{H}_{\text{int}} \) given by

\[
\hat{H}_{\text{int}} = \int dx \, dy \left[ \frac{c_0}{2} \hat{n}^2 + \frac{c_2}{2} \hat{s}^2 \right],
\]

\( \hat{H}_0 = \int dx \, dy \hat{\psi}^\dagger \left[ -\frac{\hbar^2}{2m} \nabla^2 / (2m) + V_{\text{trap}}(x, y) \right. \]

\[- \left. \nabla \cdot \mathbf{L} \right] \hat{\psi}, \tag{1}\]

where \( \hat{\psi} = [\hat{\psi}_1, \hat{\psi}_0, \hat{\psi}_{-1}]^T \) denotes collectively the spinor Bose field operators, \( \hat{n} = \hat{n}_1 + \hat{n}_0 + \hat{n}_{-1} \) with \( \hat{n}_\ell \) being the density of the particle for each spin component, \( \hat{\sigma} = \hat{\psi}_\ell^\dagger \hat{\sigma}_{\alpha \beta} \hat{\psi}_\beta \). Rashba-type SO coupling \( V_{\text{SO}} = -i\gamma (\hat{\sigma}_x \hat{\sigma}_y - \hat{\sigma}_y \hat{\sigma}_x) \) with \( \sigma_{x,y,z} \) being the Pauli matrices and \( \gamma \) describing the SO coupling strength. The coefficients of the interaction part are given by

\[
c_0 = \frac{4\pi \hbar^2 a_0}{m} + \frac{2a_2}{3}, \tag{2}\]

and

\[
c_2 = \frac{4\pi \hbar^2 a_2 - a_0}{m}, \tag{3}\]

with \( m \) being the mass of the boson, and \( a_0 \) and \( a_2 \) are the scattering lengths describing binary elastic collisions in the channels of total spin 0 and 2. The system is FM if \( c_2 < 0 \) and AFM if \( c_2 > 0 \). Finally, the external trapping potential considered in this work is given as

\[
V(r) = \min \left\{ \frac{1}{2} m \omega_0^2 (r - R_0)^2, \frac{1}{2} m \omega_1^2 (r - R_1)^2 \right\}, \tag{4}\]

with \( r \) being the radial coordinate in cylindrical coordinates. It is easy to see that the two (overlapping) parabolas in \( V(r) \) with frequencies \( \omega_0 \) and \( \omega_1 \) are centered at the positions with \( r = R_0 \) and \( R_1 \). Moreover, to make the product of the ‘width’ of each annulus and the radius of each annulus to be comparable to each other, \( \omega_1 > \omega_0 \) should be satisfied. Throughout this paper, we set \( R_0 = 2a_0 \) and \( R_1 = 4a_0 \) with \( a_0 = [\hbar / (m \omega_0)]^{\frac{1}{2}} \) being the oscillator length, \( \omega = \omega_0 / 4 \), and \( \omega_1 / \omega_0 = 5 / 4 \).

3. Ground state structure and discussions

Within the framework of mean-field theory, the static and dynamical properties of our system can be described well by macroscopic wavefunctions, which are also called order parameters, obeying a set of dimensionless coupled Gross–Pitaevskii equations

\[
\frac{i}{\hbar} \partial \phi_1 \partial t = \left[ -\frac{\nabla^2}{2} + V_{\text{trap}} + c_0 n + c_2 (|\phi_1|^2 + |\phi_0|^2) - |\phi_{-1}|^2 - \Omega \cdot \mathbf{L} \right] \phi_1 + c_2 \phi_0^* \phi_{-1} - \gamma L_0 \phi_0, \tag{5}\]

\[
\frac{i}{\hbar} \partial \phi_0 \partial t = \left[ -\frac{\nabla^2}{2} + V_{\text{trap}} + c_0 n + c_2 (|\phi_1|^2 + |\phi_{-1}|^2) - \Omega \cdot \mathbf{L} \right] \phi_0 + 2c_2 \phi_{-1} \phi_0^* - \gamma (L_0 \phi_{-1} + L_1 \phi_1), \]

\[
\frac{i}{\hbar} \partial \phi_{-1} \partial t = \left[ -\frac{\nabla^2}{2} + V_{\text{trap}} + c_0 n + c_2 (|\phi_1|^2 + |\phi_{-1}|^2) - |\phi_{1}|^2 - \Omega \cdot \mathbf{L} \right] \phi_{-1} + c_2 \phi_0^* \phi_{-1} - \gamma L_1 \phi_0, \]

where \( L_0 = -i(\hat{\partial}_x \hat{\partial}_y - \hat{\partial}_y \hat{\partial}_x) \), \( L_0 = i \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \), and \( L_1 = i \frac{\partial}{\partial x} - \frac{\partial}{\partial y} \). Here all the lengths and time are rescaled by the unit length \( a_0 = \sqrt{\hbar / m \omega_0} \) and the unit time \( 1 / \omega_0 \), respectively, and the dimensionless wavefunction is rescaled by \( 1 / \sqrt{\hbar / m \omega_0} \). Solving these equations analytically in general is tedious and unrealistic, but fortunately we can use the normalized gradient flow with Jacobian iteration method within an imaginary-time propagation approach (\( \tau = it \)) [44]. This works well for finding the stationary ground state solution, to perform a detailed analysis of the ground state structure, rotational properties, and quantum phase transition of the system as a function of SO coupling and rotation.

3.1. The effect of SO coupling

To gain a physical understanding of the new degrees of freedom introduced above, it is instructive to begin our investigation with the FM (\( c_2 < 0 \)) condensate and only SO coupling is considered (in this case, the rotation being neglected (\( \Omega = 0 \)).
Figure 1. The ground state density distributions of each spin component for $\gamma = 0, 0.1, 0.3, 0.5, 0.7, 1.0, 2.0$, respectively. The first to seventh columns correspond to the SO coupling strength $\gamma = 0, 0.1, 0.3, 0.5, 0.7, 1.0, 2.0$, respectively. The fourth row is the density distribution for the whole condensate, and the fifth row is the spin density $S_z$ of the condensate.

Figure 1 displays the density distributions for $n_1(\rho, \theta) = |\phi_1(\rho, \theta)|^2, n_0(\rho, \theta) = |\phi_0(\rho, \theta)|^2, n_{-1}(\rho, \theta) = |\phi_{-1}(\rho, \theta)|^2$, and the total density of the condensate for FM condensate with $c_0 = 5, c_2 = -1$. The first column of figure 1 shows the simplest case without SO coupling. It is easy to find that the density distribution of each spin component is mainly on the inner ring, but with no regular pattern formation. For a small value of SO coupling, such as $\gamma = 0.1$, the essential picture is not changed. However, on further increasing the SO coupling, the spin-up and spin-down components show phase separation on the inner ring. Examples for such situations are shown in the third to seventh columns of figure 1 for $\gamma = 0.3, 0.5, 0.7, 1.0, 2.0$, respectively.

The density distribution is also reflected in the longitudinal magnetization of the system, which is described by $S_z = |\phi_1|^2 - |\phi_{-1}|^2$ and shown in the fifth row of figure 1. It is readily seen that with the growth of SO coupling, the longitudinal magnetization evolves from ‘azimuthal phase separation’ (i.e. $\gamma = 0.7$) to ‘radial phase separation’ (when $\gamma = 2.0$), and forms a double-ring structure. We notice that for the pseudo spin-1/2 BEC case, ‘azimuthal phase separation’ always manifest itself both on the inner and outer rings [41]. However, for the spin-1 case, with the strength of the SO coupling increased to a large value, such as $\gamma = 2.0$, most of the atoms for each spin component are still on the inner ring. Furthermore, the spin-up and spin-down components can undergo a phase transition from a single-ring structure showing azimuthal phase separation to a double-ring structure showing the radial phase separation within the minima for the inner ring.

3.2. The effect of rotation

We now turn to the investigation of the effect of rotation on the ground state properties in the absence of SO coupling ($\gamma = 0$). By fixing the values for the interatomic interactions to $c_0 = 50$ and $c_2 = -1$, the density distributions for each spin component, the spin density $S_z$ and the total density of the FM condensate are numerically analyzed and shown in figure 2. Using the same procedure as previously, we first look at the simplest case without rotation, which is shown in the first column of figure 2. In this case, both rings are...
occupied by each spin component, and no regular pattern forms. With a moderate rotation, it is interesting to find that each spin component rearranges itself on both the inner and outer rings, as shown in the second to fourth columns of figure 2. With further increasing rotation frequency, almost all of the atoms are relocated on the outer ring. In particular, the spin-up and spin-down components always alternatively arrange themselves and show azimuthal phase separation, whether on the inner or the outer ring. Here we want to mention that without the help of SO coupling, no double-ring structure forms for the spin density $\mathcal{S}_z$ within the minima of the inner or the outer parabolas for the confining trap.

To further highlight the effects of rotation on the ground state structure, it is helpful to study the phase distribution of such a system. Figure 3 shows the phase distributions of the spin-up component associated with the results for figure 2 (other spin components have a similar behavior). When the rotation frequency is small, no vortex appears. However, by mediating the values of the rotation frequency, such as $\Omega = 0.3, 0.5, 0.7$, a vortex appears on both rings, and the winding number increases with increasing rotation frequency. For even larger values of the rotation frequency, such as $\Omega = 0.9, 0.99$, the winding number also increases as the rotation frequency increases, but with no vortex appearance on the inner ring since almost all of the atoms are located in the outer ring.

3.3. The combined effect of SO coupling and rotation

Next we consider in detail the combined effect of SO coupling and rotation. We thus begin with both SO coupling and rotation, and select two typical sets of parameters: (i) $\Omega = 0.1, \gamma = 2.0$ and (ii) $\Omega = 0.9, \gamma = 0.3$ to find out the effects of modifying the strength of the SO coupling and rotation.

Typical density distributions, together with the spin density $\mathcal{S}_z$ and the phase distribution of the BEC atoms confined in the concentrically coupled traps are plotted in figure 4. For case (i), as shown in figure 4(a), each spin component is located on both rings, with the whole condensate illustrating phase coexistence. Moreover, the spin density $\mathcal{S}_z$ distribution shows a double-ring structure where both the spin-up and spin-down components show a radial phase separation on each ring. With regard to the phase distribution of the spin-up component, we also find the formation of a vortex and a hidden vortex within both rings.

Figure 2. The ground state density distributions of each spin component for $\gamma = 0, c_0 = 50, c_2 = -1$, and for rotation frequency $\Omega = 0, 0.1, 0.3, 0.5, 0.7, 0.9, 0.99$, corresponding to the first to seventh columns. Note that the fourth row is the density distribution for the whole condensate, and the fifth row is the spin density $\mathcal{S}_z$ of the condensate.
Figure 3. The ground state phase distributions of the spin-up component for $\gamma = 0, c_0 = 50, c_2 = -1$. The columns (a)–(f) correspond to a rotation frequency of $\Omega = 0.1, 0.3, 0.5, 0.7, 0.9, 0.99$, respectively. The other spin components have a similar behavior.

Figure 4. The ground state density profiles of each spin component for (a) $\Omega = 0.1, \gamma = 2.0$; (b) $\Omega = 0.9, \gamma = 0.3$ with the interatomic interaction parameters fixed to $c_0 = 50, c_2 = -1$. Note that the fourth and fifth columns are the spin density $S_z$ and phase distribution for the spin-up component, respectively. The other spin components have a similar phase distribution behavior.

and the central barrier region, respectively. These results are obviously different from previous results described in section 3.1 where only SO coupling is included. When both SO coupling and rotation take effect, even with a rotation frequency as small as $\Omega = 0.1$, part of the condensed atoms will be repelled from the initially located inner ring to the outer ring, leading to the formation of a double-ring structure and vortex on the outer ring.

Shown in figure 4(b) are the results associated with case (ii). It is clearly seen that the atoms are easily repelled to the outer ring with increasing rotation, which is partly similar to the discussion in section 3.2. However, if SO coupling is lacking, even if the rotation is large enough, such as $\Omega = 0.99$ in figure 2, each spin component is alternatively arranged on the outer ring (in particular, the spin-up and spin-down components show azimuthal phase separation). In the presence of SO coupling, even a small amount of coupling can dramatically enhance the effects of the interatomic interactions. In this case, the spin-up and spin-down components show radial phase separation and
show a double-ring structure within the outer minima of the external trap. Meanwhile, we observe the formation of the well-known giant vortex on the outer ring, which was also formed in a spin-1/2 SO coupled BEC under rotation when SO coupling and rotation are both strong enough [12].

So far, we have discussed $c_2 < 0$ for a FM condensate. We have also examined the AFM condensate ($c_2 > 0$), and this system shows a similar behavior. Last but not least, let us consider the feasibility of experimentally observing these interesting phenomena. For a FM condensate, we can choose a $^{87}$Rb condensate with $c_2 < 0$. Effective spin–orbit coupling, as previously discussed in section 2, can be created in spin-1 BECs using atom–light interactions based on appropriate laser arrangements and tailored dressed states, and its strength can be precisely tuned by optical means, which can vary from 1 to 10 in our dimensionless units. An effective quasi-two-dimensional system can be obtained in the limit of $\omega_z \gg \omega_z^L$. Hence, the reduced dimensionless parameters used in our numerical simulations are given as

$$\gamma = \frac{\tilde{g}}{\sqrt{\hbar m_o}}, \quad c_0 = 4\pi N(a_0 + 2a_2)/(3\sqrt{2}\pi \zeta),$$

$$\zeta = (\hbar/m_o)^{1/2}, \quad c_2 = 4\pi N(a_2 - a_0)/(3\sqrt{2}\pi \zeta).$$ \hspace{1cm} (6)

Since $a_0$ and $a_2$ are comparable, $c_2$ is actually small: $|c_2|/c_0 \sim 0.1$ for $^{23}$Na (AFM) and $\sim 1/35$ for $^{87}$Rb (FM) [45]. Moreover, by adjusting the two s-wave scattering lengths $a_0$ and $a_2$ through Feshbach resonances or the atom number, the spin-independent and spin-dependent interaction parameters $c_0$ and $c_2$ can be tuned. Combined with the unprecedented controllability of these interactions in ultracold atoms, we can reach a wider range of parameter space. Therefore, the dimensionless parameters used in this paper are realizable in current experiments.

Note that the present study can be extended to a dynamics investigation of the various nonlinear excitations for this system based on numerically simulating the time-dependent Gross–Pitaevskii equation using the split-operator method [46], and to the harmonic plus quartic potential which allows us to study a regime where the rotation frequency is equal to or larger than the harmonic trapping frequency [47].

4. Conclusions

In summary, we have studied numerically the effects of SO coupling, rotation, and their combination, on the ground state properties of spin-1 SO coupled BECs confined in concentrically coupled annular traps. Our results show that nontrivial new structures, such as the single- and double-ring, are formed by the introduction of SO coupling and rotation, enriching the phase diagram of the system. In the presence of SO coupling, the system can show a double-ring structure for a spin density $S_z$, within the single or both minima of the external trap; hence it can be regarded as an ‘internal switch’, which can be used to induce various ground state structures without changing the location of the atoms between the inner and outer rings. Moreover, the rotation can be regarded as an ‘external switch’, which can be used to control the location of atoms between the inner and outer rings. These phenomena offer us a more flexible method to manipulate the interaction between matter and gauge fields.

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