A netting clustering analysis method under intuitionistic fuzzy environment

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A B S T R A C T

In this paper, we investigate the technique for clustering objects with intuitionistic fuzzy information. We first propose a formula to derive the intuitionistic fuzzy similarity degree between two intuitionistic fuzzy sets and develop an approach to constructing an intuitionistic fuzzy similarity matrix. Then, we present a netting method to make clustering analysis of intuitionistic fuzzy sets via intuitionistic fuzzy similarity matrix. Two numerical examples are given to illustrate and verify our method.

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1. Introduction

Atanassov [1] introduced the concept of intuitionistic fuzzy set (IFS), which is the generalization of fuzzy set originally introduced by Zadeh [2]. Since its appearance, IFS has been investigated by many researchers and applied to many fields, such as decision-making [3–6], pattern recognition [7–10], market prediction [11] and medical diagnosis [12,13]. Clustering analysis is a well-established technique for sorting observations into clusters such that each cluster is as homogeneous as possible [14]. How to cluster intuitionistic fuzzy information is an important research topic, which has been receiving great attention from researchers recently [15–17]. Zhang et al. [15] proposed a clustering technique of IFSs on the basis of the \( \lambda \)-cutting matrix of an interval-valued matrix. Xu and Yager [18,19] gave a clustering technique by transforming an association matrix into an equivalent association matrix, from which a \( k \)-cutting matrix is derived and used to cluster the given IFSs. Cai et al. [17] presented a clustering method based on the intuitionistic fuzzy equivalent dissimilarity matrix and \((\alpha,\beta)\)-cutting matrices. However, all these intuitionistic fuzzy clustering methods are on the basis of intuitionistic fuzzy equivalence matrices and the transitive closure technique, which needs a large amount of computational efforts and takes a lot of time to accomplish. In order to overcome this issue and make the clustering process more effective, in this paper, we shall develop a netting method for clustering the objects with intuitionistic fuzzy information.

The remainder of the paper is organized as follows. In Section 2, we give a brief review of some basic knowledge related to IFSs. Section 3 presents a formula to derive the intuitionistic fuzzy similarity degree between IFSs, and then develops an approach to constructing intuitionistic fuzzy similarity matrix. Based on the netting technique, in Section 4, we propose a method for clustering the objects which are represented by IFSs. Section 5 illustrates and verifies our method with two numerical examples, and Section 6 concludes the paper with a summary and some remarks.

Briefly, Fig. 1 provides a system diagram to show the main thought of this paper.

2. Preliminaries

Let a set \( X \) be fixed, then an intuitionistic fuzzy set (IFS) \( A \) on \( X \) is defined by Atanassov [1] as \( A = (X, \mu_A(x)v_A(x)) \) \( x \in X \), where \( \mu_A: X \rightarrow [0,1], x \rightarrow \mu_A(x) \in [0,1] \) and \( v_A: X \rightarrow [0,1], x \in X \rightarrow v_A(x) \in [0,1] \) denote a membership function and a non-membership function, respectively. Furthermore, \( \mu_A(x) + v_A(x) \leq 1 \), for any \( x \in X \), and \( \pi_A(x) = 1 - \mu_A(x) - v_A(x) \) is called an uncertainty (or hesitation) function of \( x \) to \( A \). Especially, if \( \pi_A(x) = 0 \), then \( A \) reduces to a fuzzy set. For convenience, Xu [18] called \( \alpha = (\mu_A, v_A) \) an intuitionistic fuzzy number (IFN), where \( \mu_A \in [0,1], v_A \in [0,1] \), \( \mu_A + v_A \leq 1 \), and \( \pi_A = 1 - \mu_A - v_A \).

Based on IFSs, we introduce some basic concepts as below:

Definition 1 ([15]).

Let \( Z = (z_{ij})_{m \times n} \) be an \( m \times n \) matrix, if all \( z_{ij} (i = 1, 2, \ldots, m ; j = 1, 2, \ldots, n) \) are IFNs, then \( Z \) is called an intuitionistic fuzzy matrix.
Definition 2 ([20]).
Let X and Y be two non-empty sets, then
\[
R = \{ \langle x, y \rangle, \quad \mu_R(x, y), \quad \nu_R(x, y) > 0 \mid x \in X, \quad y \in Y \}
\]
is called an intuitionistic fuzzy relation, where \( \mu_R : X \times Y \rightarrow [0, 1] \), \( \nu_R : X \times Y \rightarrow [0, 1] \), and \( 0 \leq \mu_R(x, y) + \nu_R(x, y) \leq 1 \), for any \( (x, y) \in X \times Y \).

Definition 3 ([20]). Let R be an intuitionistic fuzzy relation, if

1. (Reflexivity): \( \mu_{R(x,x)} = 1 \), \( \nu_{R(x,x)} = 0 \), for any \( x \in X \);
2. (Symmetry): \( \mu_{R(x,y)} = \mu_{R(y,x)} \), \( \nu_{R(x,y)} = \nu_{R(y,x)} \), for any \( (x, y) \in X \times Y \),
then R is called an intuitionistic fuzzy similarity relation.

Definition 4 ([33]). Let \( Z = (z_{ij})_{n \times n} \) be an \( n \times n \) intuitionistic fuzzy matrix, where \( z_{ij} = (\mu_{ij}, \nu_{ij}) \), \( i, j = 1, 2, \ldots, n \), if

1. (Reflexivity): \( z_{ii} = (1, 0) \), for all \( i = 1, 2, \ldots, n \);
2. (Symmetry): \( z_{ij} = z_{ji} \), i.e., \( \mu_{ij} = \mu_{ji}, \nu_{ij} = \nu_{ji} \), for all \( i, j = 1, 2, \ldots, n \),
then Z is called an intuitionistic fuzzy similarity matrix.

3. A new approach to constructing intuitionistic fuzzy similarity matrix

Considering that the aim of this paper is to construct the intuitionistic fuzzy similarity matrix, and then utilize it to derive a method for clustering analysis. To achieve this, we generally need to consider a multiple attribute decision making (MADM) problem, and then get an intuitionistic fuzzy matrix. After that, we shall seek for a method to construct an intuitionistic fuzzy similarity matrix so as to do clustering analysis.

Now we consider a MADM problem, let \( Y = \{ Y_1, Y_2, \ldots, Y_m \} \) be a set of alternatives, and \( G = \{ G_1, G_2, \ldots, G_n \} \) be a set of attributes. The characteristic of each alternative \( Y_i \) under all the attributes \( G_j \) (\( j = 1, 2, \ldots, n \)) is represented as an IFS:
\[
Y_i = \{ (G_j, \mu_{Y_i}(G_j), \nu_{Y_i}(G_j)) \mid G_j \in G \}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n
\]
where \( \mu_{Y_i}(G_j) \) denotes the membership degree of \( Y_i \) to \( G_j \), and \( \nu_{Y_i}(G_j) \) denotes the non-membership degree of \( Y_i \) to \( G_j \). Obviously, \( \pi_{Y_i}(G_j) = 1 - \mu_{Y_i}(G_j) - \nu_{Y_i}(G_j) \) is the uncertainty (or hesitation) degree of \( Y_i \) to \( G_j \).

Next, we shall develop an approach to constructing an intuitionistic fuzzy similarity matrix based on the intuitionistic fuzzy matrix \( Z = (z_{ij})_{m \times n} \).

For any two alternatives \( Y_i \) and \( Y_k \), we first use the normalized Hamming distance to get the average value of the absolute deviations of the non-membership degrees \( \nu_{ij} \) and \( \nu_{kj} \), for all \( i = 1, 2, \ldots, m \):
\[
d_1(Y_i, Y_k) = \frac{1}{n} \sum_{j=1}^{n} |\nu_{ij} - \nu_{kj}|, \quad i, k = 1, 2, \ldots, m
\]

Analogously, we get the average value of the absolute deviations of the membership degrees \( \mu_{ij} \) and \( \mu_{kj} \) for all \( i = 1, 2, \ldots, m \):
\[
d_2(Y_i, Y_k) = \frac{1}{n} \sum_{j=1}^{n} |\mu_{ij} - \mu_{kj}|, \quad i, k = 1, 2, \ldots, m
\]

Obviously, distances (2) and (3) show the closeness degrees of the characteristics of each two alternatives \( Y_i \) and \( Y_k \). The smaller the values of \( d_1(Y_i, Y_k) \) and \( d_2(Y_i, Y_k) \) are, the more similar the two alternatives \( Y_i \) and \( Y_k \).

In an intuitionistic fuzzy similarity matrix, each of its elements is an IFN. To get an intuitionistic fuzzy closeness degrees of \( Y_i \) and \( Y_k \), we may consider the value of \( d_1(Y_i, Y_k) \) as a non-membership degree \( \tilde{v}_{ik} \), and then it may be hopeful to define
\[
\tilde{\mu}_{ik} = 1 - \frac{1}{n} \sum_{j=1}^{n} |\mu_{ij} - \mu_{kj}|, \quad i, k = 1, 2, \ldots, m
\]
as a membership degree. Now we need to check whether \( 0 \leq \tilde{\mu}_{ik} + \tilde{v}_{ik} \leq 1 \) holds or not. However,
\[
\tilde{\mu}_{ik} + \tilde{v}_{ik} = 1 - \frac{1}{n} \sum_{j=1}^{n} |\mu_{ij} - \mu_{kj}| + \frac{1}{n} \sum_{j=1}^{n} |\nu_{ij} - \nu_{kj}| \geq 0
\]
\[
\tilde{\mu}_{ik} + \tilde{v}_{ik} = 1 - \frac{1}{n} \sum_{j=1}^{n} |\mu_{ij} - \mu_{kj}| + \frac{1}{n} \sum_{j=1}^{n} |\nu_{ij} - \nu_{kj}|
\]
\[
\geq 1 - \frac{1}{n} \sum_{j=1}^{n} (|\mu_{ij} - \mu_{kj}| + |\nu_{ij} - \nu_{kj}|) \geq 0
\]
\[
\tilde{\mu}_{ik} + \tilde{v}_{ik} = 1 - \frac{1}{n} \sum_{j=1}^{n} |\mu_{ij} - \mu_{kj}| + \frac{1}{n} \sum_{j=1}^{n} |\nu_{ij} - \nu_{kj}|
\]
\[
\geq 1 - \frac{1}{n} \sum_{j=1}^{n} \sum_{j=1}^{n} |\nu_{ij} - \nu_{kj}| \geq 0
\]
where \( \pi_{ij} = 1 - \mu_{ij} - \nu_{ij} \). Thus, \( 0 \leq \tilde{\mu}_{ik} + \tilde{v}_{ik} \leq 1 \) cannot be guaranteed.

Fig. 1. The main thought of this paper.
In the numerical analysis above, we can see that in an IFN, the membership degree is closely related to both the non-membership and the uncertainty degree. Motivated by this idea, we may modify (4) as:

\[ \hat{\mu}_{ik} = 1 - \frac{1}{n} \sum_{j=1}^{n} |v_{ij} - v_{kj}| - \frac{1}{n} \sum_{j=1}^{n} |\pi_{ij} - \pi_{kj}|, \quad i, k = 1, 2, \ldots, m \]  

(5)

with \( \hat{\mu}_{ik} = 1 \) if and only if \( v_{ij} = v_{kj} \) and \( \pi_{ij} = \pi_{kj} \), for all \( j = 1, 2, \ldots, n \).

Based on (2) and (5), we have the following concept:

**Definition 5.** Let \( Y_i \) and \( Y_k \) be two IFSs on \( X \), and \( R(Y_i, Y_k) \) be a binary relation on \( X \times X \), if

\[ R(Y_i, Y_k) = \begin{cases} (1, 0), & \text{if } Y_i = Y_k \\ \left( 1 - \frac{1}{n} \sum_{j=1}^{n} |v_{ij} - v_{kj}| - \frac{1}{n} \sum_{j=1}^{n} |\pi_{ij} - \pi_{kj}|, \frac{1}{n} \sum_{j=1}^{n} |v_{ij} - v_{kj}| \right), & \text{if } Y_i \neq Y_k \end{cases} \]

(6)

then \( R(Y_i, Y_k) \) is called a closeness degree of \( Y_i \) and \( Y_k \).

By (6), we have

**Theorem 1.** The closeness degree \( R(Y_i, Y_k) \) of \( Y_i \) and \( Y_k \) is an intuitionistic fuzzy similarity relation.

**Proof.**

(1) Let us first prove that \( R(Y_i, Y_k) \) is an IFN:

(a) If \( Y_i \neq Y_k \), then \( R(Y_i, Y_k) = (1, 0) \);

(b) If \( Y_i = Y_k \), then

\[ \mu_{ik} = 1 - \frac{1}{n} \sum_{j=1}^{n} |v_{ij} - v_{kj}| - \frac{1}{n} \sum_{j=1}^{n} |\pi_{ij} - \pi_{kj}| \]

\[ = 1 - \frac{1}{n} \sum_{j=1}^{n} |v_{ij} - v_{kj}| \]

Obviously, we have \( 0 \leq \hat{\mu}_{ik} \leq 1 \), with \( \hat{\mu}_{ik} = 1 \) if and only if \( v_{ij} = v_{kj} \), for all \( j = 1, 2, \ldots, n \), and with \( \hat{\mu}_{ik} = 0 \) if and only if \( v_{ij} = 1 \) and \( v_{kj} = 0 \), for all \( j = 1, 2, \ldots, n \), or \( \hat{\mu}_{ik} = 0 \) and \( \hat{\mu}_{kj} = 1 \), for all \( j = 1, 2, \ldots, n \).

Similarly, we have \( 0 \leq \hat{\pi}_{ik} = \frac{1}{n} \sum_{j=1}^{n} |v_{ij} - v_{kj}| \leq 1 \), with \( \hat{\pi}_{ik} = 1 \) if and only if \( v_{ij} = v_{kj} \), for all \( j = 1, 2, \ldots, n \), and with \( \hat{\pi}_{ik} = 0 \) if and only if \( v_{ij} = 1 \) and \( v_{kj} = 0 \), for all \( j = 1, 2, \ldots, n \), or \( v_{ij} = 0 \) and \( v_{kj} = 1 \), for all \( j = 1, 2, \ldots, n \).

Also since

\[ \hat{\mu}_{ik} + \hat{\pi}_{ik} = 1 - \frac{1}{n} \sum_{j=1}^{n} |v_{ij} - v_{kj}| - \frac{1}{n} \sum_{j=1}^{n} |\pi_{ij} - \pi_{kj}| + \frac{1}{n} \sum_{j=1}^{n} |v_{ij} - v_{kj}| \]

\[ = 1 - \frac{1}{n} \sum_{j=1}^{n} |\pi_{ij} - \pi_{kj}| \leq 1 \]

then we have \( 0 \leq \hat{\mu}_{ik} + \hat{\pi}_{ik} \leq 1 \), with \( \hat{\mu}_{ik} + \hat{\pi}_{ik} = 1 \) if and only if \( \pi_{ij} = \pi_{kj} \), for all \( j = 1, 2, \ldots, n \), and \( \hat{\mu}_{ik} + \hat{\pi}_{ik} = 0 \), if and only if \( \pi_{ij} = 1 \) and \( \pi_{kj} = 0 \), for all \( j = 1, 2, \ldots, n \), or \( \pi_{ij} = 0 \) and \( \pi_{kj} = 1 \), for all \( j = 1, 2, \ldots, n \).

(2) Since \( R(Y_i, Y_j) = (1, 0) \), then \( R \) is reflexive;

(3) Since \( |v_{ij} - v_{kj}| = |v_{ij} - v_{kj}| \) and \( |\pi_{ij} - \pi_{kj}| = |\pi_{ij} - \pi_{kj}| \), then \( R(Y_i, Y_j) = R(Y_j, Y_i) \), i.e., \( R \) is symmetrical. Thus, \( R(AB) \) is an intuitionistic fuzzy similarity relation.

From (6), we can see that if all the differences of the non-membership degrees and the differences of the uncertainty degrees of two alternatives \( Y_i \) and \( Y_j \), with respect to the attributes \( G_j \) \((j = 1, 2, \ldots, n)\) get smaller, then the two alternatives are more similar to each other. \( \square \)

In the following section, we shall use (6) to develop a new clustering method.

**4. A netting method for clustering the objects with intuitionistic fuzzy information**

The so-called netting means a simple process: firstly, for an intuitionistic fuzzy similarity matrix \( Z \), we should choose a confidence level \( \lambda \in [0, 1] \), and then get a \( \lambda \)-cutting matrix \( Z_\lambda \), and change the elements on the diagonal of \( Z_\lambda \) with the symbol of the alternatives. Under the diagonal, we replace ‘1’ with nodal point ‘*’ and ignore all the ‘0’ in \( Z_\lambda \). From the node ‘*’, we draw vertical line and horizontal line to the diagonal and the corresponding alternatives on the diagonal will be clustered into one class [21].

Netting method was first used to cluster data in the field of fuzzy mathematics [21]. With this method, we can get the clustering results by ‘netting’ the elements of similarity matrix directly. In the following, we propose a netting method for clustering the objects with intuitionistic fuzzy information.

**Step 1.** For a MADM problem, let \( Y = \{Y_1, Y_2, \ldots, Y_m\} \) and \( G = \{G_1, G_2, \ldots, G_n\} \) be defined as in Section 3, and assume that the characteristics of the alternatives \( Y_i \) \((i = 1, 2, \ldots, m)\) with respect to the attributes \( G_j \) \((j = 1, 2, \ldots, n)\) are represented as in (1).

**Step 2.** Construct the intuitionistic fuzzy similarity matrix \( Z = (z_{ij})_{m \times n} \) by using (6), where \( z_{ij} \) is an IFN, and \( z_{ij} = (\mu_{ij}, v_{ij}) = Z(Y_i, Y_j), i, j = 1, 2, \ldots, m \).

**Step 3.** Delete all the elements above the diagonal and replace the elements on the diagonal with the symbol of the alternatives.

**Step 4.** Choose the confidence level \( \lambda \) and construct the corresponding \( \lambda \)-cutting matrix. Replace ‘1’ with ‘*’ and delete all the ‘0’ in the matrix before drawing the vertical and horizontal line to the symbol of alternatives on the diagonal from ‘*’. Corresponding to each ‘*’, we have a class which contains two elements. Unit the classes together which have the common elements, and then we get the classes corresponding to the selected \( \lambda \). Update the values of \( \lambda \) before all the alternatives are clustered into one class.

The principal to choose \( \lambda \): Based on the idea of constructing the similarity degree matrix in this paper, we balance the similarity degree of two alternatives mainly through the membership degree of the corresponding IFN. We choose the confidence level \( \lambda \) from the biggest one to the smallest one. When we encounter that two membership degrees are equal, we firstly choose the one with the smaller non-membership degree. If both of them are equal, they are clustered into the same class. After that, in terms of the chosen \( \lambda \), we construct the corresponding \( \lambda \)-cutting matrix. With this principle, the clustering results will be more detailed.

**5. Illustrative examples**

**Example 1.** An auto market wants to classify five different cars \( Y_i \) \((i = 1, 2, 3, 4, 5)\) into several kinds [15]. Each car has six evaluation factors: (1) \( G_1 \) – oil consumption; (2) \( G_2 \) – coefficient of friction; (3) \( G_3 \) – price; (4) \( G_4 \) – comfortable degree; (5) \( G_5 \) – design; (6) \( G_6 \) – safety coefficient. The evaluation results of each car with respect to
the factors \( G_j (j = 1, 2, \ldots, 6) \) are represented by the IFSs, shown as in Table 1.

In the following, we utilize the intuitionistic fuzzy netting method to classify the five cars, which involves the following steps.

**Step 1.** By (6), we calculate
\[
\mu_{12} = 1 - \frac{1}{6} \sum_{j=1}^{6} |\nu_{1j} - \nu_{2j}| = 1 - \frac{1}{6}(0.2 + 0.1 + 0.2 + 0.0 + 0.0 + 0.1) = 0.8
\]
and then calculate the others in a similar way. Consequently, we get the intuitionistic fuzzy similarity matrix:
\[
Z = \begin{pmatrix}
(1.0, 0.8, 0.1) & (0.82, 0.12) & (0.75, 0.13) & (0.65, 0.22) & (0.68, 0.18) \\
(0.82, 0.12) & (1.0, 0.82, 0.08) & (0.72, 0.1) & (0.63, 0.25) & (0.68, 0.18) \\
(0.75, 0.13) & (0.72, 0.1) & (1.0, 0.7, 0.05) & (1.0, 0.68, 0.08) & (0.72, 0.1) \\
(0.65, 0.22) & (0.68, 0.18) & (0.63, 0.23) & (1.0, 0.68, 0.08) & (0.65, 0.22)
\end{pmatrix}
\]

**Step 2.** Delete all the elements above the diagonal and replace the elements on the diagonal in \( Z \) with the symbol of the alternatives \( Y_i (i = 1, 2, 3, 4, 5) \):

\[
Z' = \begin{pmatrix}
Y_1 & Y_2 & Y_3 & Y_4 & Y_5 \\
0.82, 0.12 & 0.72, 0.1 & 0.75, 0.13 & 0.65, 0.22 & 0.68, 0.18 \\
0.82, 0.08 & 0.72, 0.1 & 0.7, 0.05 & 0.68, 0.18 & 0.72, 0.1 \\
0.65, 0.22 & 0.68, 0.18 & 0.63, 0.23 & 1.0 & 0.65, 0.22 \\
0.68, 0.18 & 0.72, 0.1 & 0.75, 0.13 & 0.82, 0.08 & 0.82, 0.12 \\
0.65, 0.22 & 0.68, 0.18 & 0.63, 0.23 & 0.68, 0.18 & 0.82, 0.08 \\
0.65, 0.22 & 0.68, 0.18 & 0.63, 0.23 & 0.68, 0.18 & 0.65, 0.22
\end{pmatrix}
\]

**Step 3.** Choose the confidence level \( \lambda \) properly, and get the corresponding clustering results with intuitionistic fuzzy netting method:

1. When \( 0.82 < \lambda \leq 1.0 \), we have
\[
Z' = \begin{pmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
Y_4 \\
Y_5
\end{pmatrix}
\]
and then each car is clustered into a class: \{\( Y_1 \)\}, \{\( Y_2 \)\}, \{\( Y_3 \)\}, \{\( Y_4 \)\}, \{\( Y_5 \)\}.

2. When \( 0.8 < \lambda \leq 0.82 \), we have
\[
Z' = \begin{pmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
Y_4 \\
Y_5
\end{pmatrix}
\]
and then the cars \( Y_i (i = 1, 2, 3, 4, 5) \) are clustered into following four classes: \{\( Y_1 \)\}, \{\( Y_2, Y_5 \)\}, \{\( Y_4 \)\}, \{\( Y_3 \)\}..

(3) When \( 0.75 < \lambda \leq 0.8 \), we have
\[
Z' = \begin{pmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
Y_4 \\
Y_5
\end{pmatrix}
\]
and then the cars \( Y_i (i = 1, 2, 3, 4, 5) \) are clustered into three classes: \{\( Y_1 \), \( Y_2, Y_3 \)\}, \{\( Y_4 \)\}, \{\( Y_5 \)\}.

(4) When \( 0.72 < \lambda \leq 0.75 \), we have
\[
Z' = \begin{pmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
Y_4 \\
Y_5
\end{pmatrix}
\]
and then the cars \( Y_i (i = 1, 2, 3, 4, 5) \) are clustered into two classes: \{\( Y_1 \), \( Y_2, Y_3 \), \( Y_4 \)\}, \{\( Y_5 \)\}.

(5) When \( 0.68 < \lambda \leq 0.72 \), we have the following two cases:

(i) In this case, the cars \( Y_i (i = 1, 2, 3, 4, 5) \) are clustered into two classes: \{\( Y_1 \), \( Y_2, Y_3, Y_4 \)\}, \{\( Y_5 \)\}.

(ii) In this case, the cars \( Y_i (i = 1, 2, 3, 4, 5) \) are also clustered into two classes: \{\( Y_1 \), \( Y_2, Y_3, Y_4 \)\}, \{\( Y_5 \)\}.

(6) When \( 0.65 < \lambda \leq 0.68 \), we have
\[
Z' = \begin{pmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
Y_4 \\
Y_5
\end{pmatrix}
\]
and then the cars \( Y_i (i = 1, 2, 3, 4, 5) \) are clustered into one class: \{\( Y_1 \), \( Y_2, Y_3, Y_4, Y_5 \)\}.

In the following, let us make a simple comparison between the developed method in this paper and Zhang et al.'s method [15] in Table 2.

Through Table 2, we know that the intuitionistic fuzzy netting method has some desirable advantages over Zhang et al.'s method:

1. It does not need to calculate the equivalent matrix, and thus requires much less computational efforts; 2. It can derive more detailed clustering results. Therefore, Compared to Zhang et al.'s method, our method has more prospects for practical applications.
Table 2
Comparisons of the derived results.

<table>
<thead>
<tr>
<th>Classes</th>
<th>The result derived by intuitionistic fuzzy netting method</th>
<th>The result developed by Zhang et al.'s method [15]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>{Y_1, Y_2, (Y_3), (Y_4), (Y_5)}</td>
<td>{Y_1, Y_2, Y_3, (Y_4), (Y_5)}</td>
</tr>
<tr>
<td>4</td>
<td>{Y_1, Y_2, Y_3, (Y_4), (Y_5)}</td>
<td>{Y_1, Y_2, Y_3, (Y_4), (Y_5)}</td>
</tr>
<tr>
<td>3</td>
<td>{Y_1, Y_2, Y_3, (Y_4), (Y_5)}</td>
<td>{Y_1, Y_2, Y_3, (Y_4), (Y_5)}</td>
</tr>
<tr>
<td>2</td>
<td>{Y_1, Y_2, Y_3, Y_4, }</td>
<td>{Y_1, Y_2, Y_3, Y_4, }</td>
</tr>
<tr>
<td>1</td>
<td>{Y_1, Y_2, Y_3, Y_4, Y_5}</td>
<td>{Y_1, Y_2, y_3, Y_4, Y_5}</td>
</tr>
</tbody>
</table>

Why our method has these characteristics? For one thing, the proposed netting method can rely on similarity relation instead of equivalent relation as in fuzzy environment. For another, whether in [15] or in this work, the class stander are all based on \(\lambda\)-cutting matrix, so \(\lambda\) is an important parameter to decide the class scalar. In [15], before getting the \(\lambda\)-cutting matrix, Zhang et al. first transformed the intuitionistic fuzzy matrix into an intuitionistic fuzzy similarity matrix, and then calculated its equivalent matrix which needs lots of computational efforts. In this work, we not only get the \(\lambda\)-cutting matrix directly from the intuitionistic fuzzy similarity matrix, but also improve the principle of choosing \(\lambda\). Since Zhang et al.'s work needs to transform the intuitionistic fuzzy similarity matrix into an intuitionistic fuzzy equivalent matrix, some information maybe missing during this process. Namely, the intuitionistic fuzzy equivalent matrix cannot reflect all the information that the intuitionistic fuzzy similarity matrix contains. Considering the stated reasons above, it is not hard for us to comprehend why we can get more detailed classes than [15].

This work only makes a comparison with [15], because that the method in [15] is a representation of solving this class of problems, some closely related results can be found in [16,17].

In order to demonstrate the effectiveness of the proposed clustering algorithm, we further conduct experiments with simulated data through comparing these two methods.

Example 2. As we have explained above, the computational complexity is mainly related with the computations of intuitionistic fuzzy similarity matrix and intuitionistic fuzzy equivalent matrix. Next, we shall illustrate this with simulated experiments. Below we first introduce the experimental tool, the experimental data sets, and then make a comparison with other method [15].

(1) Experimental tool. In the experiments, we use the netting algorithm implemented by MATLAB. Note that if we let \(\pi(x) = 0\) for any \(x \in X\), then the netting algorithm reduces to the traditional fuzzy netting algorithm. Therefore, we can use this process to compare the performances of both the netting algorithm under intuitionistic fuzzy environment and the netting algorithm under fuzzy environment.

Table 3
The characteristics of the cars.

<table>
<thead>
<tr>
<th>(Y_1)</th>
<th>(Y_2)</th>
<th>(Y_3)</th>
<th>(Y_4)</th>
<th>(Y_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>{(0.8, 0.1)}</td>
<td>{(0.4, 0.1)}</td>
<td>{(0.6, 0.1)}</td>
<td>{(0.7, 0.3)}</td>
<td>{(0.6, 0.2)}</td>
</tr>
<tr>
<td>{(0.0, 0.3)}</td>
<td>{(0.1, 0.3)}</td>
<td>{(0.0, 0.6)}</td>
<td>{(0.0, 0.5)}</td>
<td>{(0.5, 0.3)}</td>
</tr>
<tr>
<td>{(0.2, 0.0)}</td>
<td>{(0.9, 0.1)}</td>
<td>{(0.0, 0.7)}</td>
<td>{(0.0, 0.1)}</td>
<td>{(0.3, 0.2)}</td>
</tr>
<tr>
<td>{(0.0, 0.5)}</td>
<td>{(0.3, 0.0)}</td>
<td>{(0.7, 0.1)}</td>
<td>{(0.6, 0.1)}</td>
<td>{(0.0, 0.7)}</td>
</tr>
<tr>
<td>{(0.4, 0.6)}</td>
<td>{(0.2, 0.4)}</td>
<td>{(0.9, 0.1)}</td>
<td>{(0.6, 0.1)}</td>
<td>{(0.7, 0.2)}</td>
</tr>
<tr>
<td>{(0.0, 0.2)}</td>
<td>{(0.0, 0.0)}</td>
<td>{(0.5, 0.4)}</td>
<td>{(0.5, 0.4)}</td>
<td>{(0.3, 0.6)}</td>
</tr>
<tr>
<td>{(0.8, 0.1)}</td>
<td>{(0.2, 0.1)}</td>
<td>{(0.1, 0.0)}</td>
<td>{(0.7, 0.0)}</td>
<td>{(0.6, 0.4)}</td>
</tr>
<tr>
<td>{(0.1, 0.7)}</td>
<td>{(0.0, 0.5)}</td>
<td>{(0.8, 0.1)}</td>
<td>{(0.7, 0.1)}</td>
<td>{(0.0, 0.0)}</td>
</tr>
<tr>
<td>{(0.0, 0.1)}</td>
<td>{(0.5, 0.1)}</td>
<td>{(0.3, 0.1)}</td>
<td>{(0.7, 0.3)}</td>
<td>{(0.1, 0.3)}</td>
</tr>
<tr>
<td>{(0.3, 0.2)}</td>
<td>{(0.7, 0.1)}</td>
<td>{(0.2, 0.2)}</td>
<td>{(0.2, 0.0)}</td>
<td>{(0.1, 0.9)}</td>
</tr>
</tbody>
</table>

Table 4
The clustering results with the netting method.

<table>
<thead>
<tr>
<th>(\lambda_{\text{net}})</th>
<th>Clustering</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.67 &lt; \lambda \leq 1)</td>
<td>{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}}</td>
</tr>
<tr>
<td>(0.63 &lt; \lambda \leq 0.67)</td>
<td>{A_4, A_5, A_6, A_7, A_8, A_9, A_{10}}</td>
</tr>
<tr>
<td>(0.55 &lt; \lambda \leq 0.57)</td>
<td>{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_{10}}</td>
</tr>
<tr>
<td>(0.49 &lt; \lambda \leq 0.55)</td>
<td>{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_{19}}</td>
</tr>
<tr>
<td>(0.16 &lt; \lambda \leq 0.49)</td>
<td>{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_{19}}</td>
</tr>
</tbody>
</table>

Table 5
The clustering results with Zhang et al.'s method.

<table>
<thead>
<tr>
<th>(\lambda_{\text{net}})</th>
<th>Clustering</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.67 &lt; \lambda \leq 1)</td>
<td>{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}}</td>
</tr>
<tr>
<td>(0.63 &lt; \lambda \leq 0.67)</td>
<td>{A_4, A_5, A_6, A_7, A_8, A_9, A_{10}}</td>
</tr>
<tr>
<td>(0.57 &lt; \lambda \leq 0.63)</td>
<td>{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_{10}}</td>
</tr>
<tr>
<td>(0.49 &lt; \lambda \leq 0.55)</td>
<td>{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_{10}}</td>
</tr>
<tr>
<td>(0 &lt; \lambda \leq 0.49)</td>
<td>{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_{10}}</td>
</tr>
</tbody>
</table>

(2) Experimental data sets. The car data set contains the information of ten new cars to be classified in an auto market. Let \(Y_i (i = 1, 2, \ldots, 10)\) be the cars, each of which is described by six attributes: (1) \(G_1\): oil consumption; (2) \(G_2\): coefficient of friction; (3) \(G_3\): price; (4) \(G_4\): comfortable degree; (5) \(G_5\): design; (6) \(G_6\): safety coefficient, as in Example 1 (for convenience, here we do not consider the weights of these attributes). The characteristics of the ten new cars under the six attributes, generated at random by MATLAB, are represented by the IFIs, as shown in Table 3.

In order to express the validity of the netting method, we shall make a comparison with Zhang et al.'s method [15].
With the netting method, we have the following clustering results as in Table 4.

Using Zhang et al.'s method, we first construct the intuitionistic fuzzy similarity matrix based on the data in Table 3:

\[
\begin{pmatrix}
1.0 & 0.41 & 0.41 & 0.3 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 \\
0.41 & 1.0 & 0.41 & 0.41 & 0.41 & 0.41 & 0.41 & 0.41 & 0.41 & 0.41 \\
0.33 & 0.41 & 1.0 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 \\
0.33 & 0.41 & 0.33 & 1.0 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 \\
0.41 & 0.41 & 0.41 & 0.41 & 1.0 & 0.41 & 0.41 & 0.41 & 0.41 & 0.41 \\
0.41 & 0.41 & 0.41 & 0.41 & 0.41 & 1.0 & 0.41 & 0.41 & 0.41 & 0.41 \\
0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 1.0 & 0.33 & 0.33 & 0.33 \\
0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 1.0 & 0.33 & 0.33 \\
0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 1.0 & 0.33 \\
0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 1.0
\end{pmatrix}
\]

In order to get the clustering result with Zhang et al.'s method, we should get the equivalent matrix. By the composition operation of similarity matrices, we have

\[
Z = Z \times Z = \begin{pmatrix}
1.0 & 0.41 & 0.41 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 \\
0.41 & 1.0 & 0.41 & 0.41 & 0.41 & 0.41 & 0.41 & 0.41 & 0.41 & 0.41 \\
0.33 & 0.41 & 1.0 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 \\
0.33 & 0.41 & 0.33 & 1.0 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 \\
0.41 & 0.41 & 0.41 & 0.41 & 1.0 & 0.41 & 0.41 & 0.41 & 0.41 & 0.41 \\
0.41 & 0.41 & 0.41 & 0.41 & 0.41 & 1.0 & 0.41 & 0.41 & 0.41 & 0.41 \\
0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 1.0 & 0.33 & 0.33 & 0.33 \\
0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 1.0 & 0.33 & 0.33 \\
0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 1.0 & 0.33 \\
0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 0.33 & 1.0
\end{pmatrix}
\]

After computation, we have \(Z^8 = Z^4\), thus we can make clustering analysis with Zhang et al.'s method. The clustering results are shown in Table 5.

We can see from Tables 5 and 6 that the netting method can make more detailed clustering results than Zhang et al.'s method.

In order to illustrate the computation complexity, we generate a mount of IFNs at random by MATLAB. Then we measure the computation time before we get the corresponding matrix that can make clustering analysis for the two methods, respectively. The elapsed time (seconds) for each method is shown in Table 6.

Let \(n\) and \(m\) represent the amount of alternatives and attributes, respectively. Then the computational complexities of our method and Zhang et al.'s method are \(O(n + m^2)\) and \(O(kn + m^2 + kn^2)\), respectively, where \(k(k \geq 2)\) represents the transfer times until we get the equivalent matrix. The elapsed time may become closed as \(n\) increased. Considering the practical application, we think the netting method can save much more time and computational efforts.

### 6. Concluding remarks

We have presented a new way to measure the intuitionistic fuzzy similarity degree between two intuitionistic fuzzy sets, and applied it to construct the intuitionistic fuzzy similarity matrix. Based on the netting technique, we have developed a method for clustering the objects which are represented by intuitionistic fuzzy information. In the process of clustering analysis, we have given the principle to choose the confidence levels so as to get much better clustering results. With the numerical examples, we have found that the intuitionistic fuzzy netting method can not only need less computational efforts, but also get more detailed clustering results than the existing intuitionistic fuzzy clustering method.

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References