Edge covering problem under hybrid uncertain environments

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ABSTRACT

Edge covering problem (ECP) is to find an edge cover with the minimum weight in a graph. By quantifying costs, time or opponent’s payoffs as weights on edges, ECP is employed to model many real-life problems in engineering and management. Since various types of uncertainty always exist in real world, this paper considers ECP under hybrid uncertain environments where randomness and fuzziness coexist. We propose three decision models and present a hybrid intelligent algorithm to solve the proposed models where genetic algorithm and random fuzzy simulation are embedded. Numerical experiments are performed to show the robustness of the proposed algorithm.

1. Introduction

Edge covering problem (ECP) is to find an edge cover with the minimum weight in a weighted graph, where an edge cover is a set of edges such that every vertex in the graph is an endpoint of at least one edge in the set. Since first introduced in 1950s [1], ECP has attracted a lot of attention from many areas such as computer science [2], management engineering [3], combinatorial mathematics [4], to name a few. The reason why ECP is popular is due to its ability to model many real-life situations. In real-life applications, the weights on edges usually represent costs, time or opponent’s payoffs. A simple example of the application of ECP is to monitor all the crossings in a transportation system. In order to keep a good transportation situation for each crossing in a city, police cars are assigned to some streets to monitor the crossings. Each police car is assigned to a single street to drive back and forth such that the two crossings as the endpoints of the street can be monitored. As assigning a police car to different streets leads to different costs, the problem is how to make the assignment such that the total costs are minimized. By representing each street as an edge, each crossing as a vertex and the cost of assigning a police car to each street as the weight on the corresponding edge, we can model the transportation system as a graph and this crossing monitoring problem is equivalent to an edge covering problem.

Many researchers focused on how to solve ECP efficiently [5–10], but they only investigate cases where the weights of edges are determinate. In real life, various types of uncertainties exist, such as randomness and fuzziness. Ignoring uncertainties in modelling real-life situations can lead to deviation and even errors in real-world applications. In this view, investigation on ECP under uncertain environments is practical and significant. Randomness is usually viewed as the most common type of uncertainty. In 2012, Ni [11] considered the randomness in ECP and introduced random variables to describe stochastic weights. In some real-life applications, historical data is not available, thus we can only depend on the experience of related experts, which however may lead to vagueness, or say fuzziness. In 2008, Ni [12] studied fuzzy minimum weight edge covering problem and introduced credibility theory to address fuzziness. However, the real world is usually much more complicated. Different types of uncertainty always coexist, so that real-life applications suffer from hybrid
uncertainties. In this paper, we focus on the edge covering problem under hybrid uncertain environments where randomness and fuzziness coexist.

The first work on hybrid uncertainty was proposed by Kwakernaak in late 1970s [13,14]. Kwakernaak proposed the concept of fuzzy random variable to describe hybrid uncertainty. In 2004, Liu [15] proposed the credibility theory to address fuzziness. Based on the credibility measure which is the core concept in credibility theory, Liu [15] proposed fuzzy random theory and random fuzzy theory to deal with hybrid uncertainties, which are differently defined in mathematics. The former is based on the fuzzy random variable that is a function from probability space to a set of fuzzy variables, while the core of the latter is the random fuzzy variable that is a function from credibility space to a set of random variables. In this paper, we employ random fuzzy theory to model and solve the edge covering problem under hybrid uncertain environments.

In this paper, based on random fuzzy theory, we propose three different concepts of optimal edge cover according to three different decision criteria, and thus formulate three uncertain programming models. Since traditional algorithms are usually unable to solve the ECP under hybrid uncertain environments, we combine random fuzzy simulation techniques and genetic algorithm to produce a hybrid intelligent algorithm for solving our problem. A numerical example is presented at the end of this paper to illustrate the effectiveness of the models and algorithm we propose.

This paper is organized as follows: In Section 2, we introduce the basic knowledge of random fuzzy theory. In Section 3, we describe the ECP under hybrid uncertain environments and propose three types of uncertain programming models. A hybrid intelligent algorithm is presented in Section 4, and a simple numerical example is given in Section 5. Finally, in Section 6, we draw some conclusions.

2. Preliminary knowledge of random fuzzy theory

In this section, we introduce some basic knowledge of random fuzzy theory. Random fuzzy theory is based on both probability theory and credibility theory. In order to deal with fuzziness in real-life applications, Liu and Liu [16] proposed the concept of credibility measure. For a fuzzy variable $\xi$ whose membership function is $\mu(x)$, for any set $B$ in $\mathbb{R}$, the credibility of the fuzzy event $(\xi \in B)$ is defined as

$$\text{Cr}(\xi \in B) = \frac{1}{2} \left( \sup_{x \in B} \mu(x) + 1 - \sup_{x \in B^c} \mu(x) \right).$$

Let $\Theta$ be a nonempty set, $\mathcal{P}(\Theta)$ a power set of $\Theta$, and $\text{Cr}$ a credibility measure, then the triplet $(\Theta, \mathcal{P}(\Theta), \text{Cr})$ is defined to be a credibility space. Note that there is a crisp connection between credibility measure $\text{Cr}$ and the possibility measure $\text{Pos}$ defined by Zadeh [17]. For each $\theta$ in the credibility space $(\Theta, \mathcal{P}(\Theta), \text{Cr})$,

$$\text{Pos} \{ \theta \} = 2\text{Cr} \{ \theta \} \wedge 1.
$$

**Definition 1.** (Liu [15]) A random fuzzy variable $\xi$ is defined as a function from a credibility space $(\Theta, \mathcal{P}(\Theta), \text{Cr})$ to a set of random variables.

In order to measure the hybrid uncertainty, the chance measure of a random fuzzy event is defined [15] as a function from $(0, 1]$ to $[0, 1]$, which is quite different from the probability measure and the credibility measure.

**Definition 2.** (Liu [15]) Let $\xi$ be a random fuzzy variable defined on a credibility space $(\Theta, \mathcal{P}(\Theta), \text{Cr})$, $B$ a Borel set on $\mathbb{R}$. The chance measure of a random fuzzy variable event $(\xi \in B)$ is a function from $(0, 1]$ to $[0, 1]$ defined as

$$\text{Ch} \{ \xi \in B \} (x) = \sup_{\text{Cr}(A) \geq x} \inf_{A \supseteq B} \Pr \{ \xi(\theta) \in B \}.$$

The expected value of a random fuzzy variable is defined as follows.

**Definition 3.** (Liu and Liu [18]) Let $\xi$ be a random fuzzy variable defined on a credibility space $(\Theta, \mathcal{P}(\Theta), \text{Cr})$. The expected value of $\xi$ is defined as

$$E[\xi] = \int_{-\infty}^{+\infty} \text{Cr} \{ \theta \in \Theta | E[\xi(\theta)] \geq r \} \, dr - \int_{-\infty}^{0} \text{Cr} \{ \theta \in \Theta | E[\xi(\theta)] \leq r \} \, dr,$$

provided that at least one of the two integrals is finite.

Note that for each $\theta$, $\xi(\theta)$ is a random variable, so the symbol $E$ on the right hand side of the above equation represents the operator of expectation in probability theory. Besides, we find that $E[\xi(\cdot)]$ is a fuzzy variable, and the expected value of the random fuzzy variable $\xi$ is actually the expected value of the fuzzy variable $E[\xi(\cdot)]$. Furthermore, Liu and Liu [18] presented the linearity of the expectation operator as shown in the following theorem.
Theorem 1. (Liu and Liu [18]). Let ξ and η be two random fuzzy variables with finite expected values. If \( E[ξ(·)] \) and \( E[η(·)] \) are independent fuzzy variables, then for any real numbers a and b,
\[
E[aξ + bη] = aE[ξ] + bE[η].
\]

As those in probability theory and credibility theory, variance is defined as follows.

Definition 4. (Liu and Liu [18]) Let ξ be a random fuzzy variable with finite expected value. Then the variance of ξ is defined as
\[
V[ξ] = E[(ξ - E[ξ])^2].
\]

Let us consider a specific type of random fuzzy variables. In probability theory, \( N(μ, σ^2) \) is usually used to denote a random variable with normal distribution whose expected value is \( μ \) and variance is \( σ^2 \). A random fuzzy variable \( ξ \) is written as \( ξ \sim N(ρ, s^2) \), where \( ρ \) is a fuzzy variable and \( s^2 \) is a determinate non-negative number, if for each \( θ \) in the corresponding credibility space \( ξ(θ) \) is a random variable with distribution \( N(ρ(θ), s^2) \). For this type of random fuzzy variables, we have the following theorem.

Theorem 2. Let a random fuzzy variable \( ξ \sim N(ρ, s^2) \), where \( ρ \) is a fuzzy variable and \( s^2 \) is a determinate non-negative number, then
\[
\]

Proof. Because \( E[ξ(θ)] = ρ(θ) \), according to the definition of the expected value of a random fuzzy variable, we have \( E[ξ] = E[ρ] \).

Let \( η = (ξ - E[ξ])^2 \), then \( η \) is a random fuzzy variable, and \( η(θ) = (ξ(θ) - E[ξ(θ)])^2 \) is a random variable. Then we have
\[
E[η(θ)] = E[(ξ(θ) - E[ξ(θ)])^2] = E[ξ(θ)]^2 - 2E[ξ(θ)]E[ξ] + E[ξ]^2 = E[ξ(θ)]^2 - 2E[ξ(θ)]E[ξ] + E[ξ]^2 = E[(ξ(θ) - E[ξ(θ)])^2].
\]

According to the definitions of the expectation and variance of a random fuzzy variable as well as the definition of the variance of a fuzzy variable, we have
\[
\]

The interested reader may refer to the book of Liu [15] to see more concepts and properties in random fuzzy theory.

3. Decision models for ECP under hybrid uncertain environments

Let \( G = (V, E) \) be an undirected graph, where \( V = \{1, 2, \ldots, n\} \) is the set of vertices and \( E = \{(i, j)|i < j \text{ and } i, j \in V\} \) is the set of edges. A subset \( S \subseteq E \) is called an edge cover if each vertex of \( V \) is an endpoint of at least one edge of \( S \). Under hybrid uncertain environments where randomness and fuzziness coexist, the weights on edges are assumed as random fuzzy variables. We represent the random fuzzy weights by a vector \( ξ = \{ξ_{ij}|(i, j) \in E\} \), where each random fuzzy variable \( ξ_{ij} \) gives the weight of the edge \( (i, j) \). Besides, we denote by \( x = \{x_{ij}|(i, j) \in E\} \) a subset \( S \) of \( E \), where \( x_{ij} = 1 \) implies that the edge \( (i, j) \in S \) and \( x_{ij} = 0 \) means that the edge \( (i, j) \notin S \). Then \( x = \{x_{ij}|(i, j) \in E\} \) is a cover if and only if
\[
\sum_{j \neq i} x_{ji} + \sum_{j \neq i} x_{ij} \geq 1, \quad \forall i \in V.
\]

The objective of ECP under hybrid uncertain environments is to find a cover \( x \) such that the following cover weight is minimum:
\[
W(x, ξ) = \sum_{(i, j) \in k} ξ_{ij}x_{ij}.
\]

It is not difficult to find that the weight of a cover \( W(x, ξ) \) is a random fuzzy variable, and it is meaningless to define directly a minimum random fuzzy variable. Different decision makers may view different covers as optimal, thus to find the optimal cover depends on the decision criterion the decision maker holds.
A natural decision criterion is to pick the cover with the minimum expected weight. According to this common decision criterion, we define the optimal cover as follows.

**Definition 5.** A cover \( x \) is called random fuzzy expected minimum weight edge cover if

\[
E[W(x, \xi)] \leq E[W(x', \xi)]
\]

for any cover \( x' \) of \( G \), where \( E[W(x, \xi)] \) is called the expected minimum cover weight.

In order to find the above optimal cover, we provide the following expected value model (EVM),

\[
\begin{align*}
\min \ E \sum_{(i,j) \in E} \xi_{ij} x_{ij} \\
\text{subject to:} \\
\quad \sum_{j \in V} x_{ij} + \sum_{j \in V} x_{ij} \geq 1, \quad \forall i \in V \\
\quad x_{ij} \in \{0, 1\}, \quad \forall (i,j) \in E.
\end{align*}
\]

(1)

In fact, the EVM model embodies the natural idea that decision makers would estimate the average level of the random fuzzy weight of each edge cover by its expected value. By this model, the decision makers would compare the edge covers according to their expected values and then choose the optimal one.

If each random fuzzy weight \( \xi_{ij} \) has a finite expected value and \( E[\xi_{ij}(\cdot)] \) is independent to each other, then the above expected value model has a crisp equivalent, in which all the parameters are determinate. This is interpreted in the following theorem.

**Theorem 3.** If for each \( (i,j) \in E, \xi_{ij} \) is a random fuzzy variable with a finite expected value and \( E[\xi_{ij}(\cdot)] \) is independent to each other. The crisp equivalent of the expected value model (1) is as follows.

\[
\begin{align*}
\min \ \sum_{(i,j) \in E} E[\xi_{ij}] x_{ij} \\
\text{subject to:} \\
\quad \sum_{j \in V} x_{ij} + \sum_{j \in V} x_{ij} \geq 1, \quad \forall i \in V \\
\quad x_{ij} \in \{0, 1\}, \quad \forall (i,j) \in E.
\end{align*}
\]

(2)

Although the above expected value model is often adopted in real-life applications, it sometimes can not be fully trusted. The expected value model depends only on the expected values of random fuzzy variables, but variance is not considered in making decisions, thus it is quite possible that a random fuzzy variable returns a value that is far from its expected value. This may lead to the risk of making a bad decision. In order to avoid such risk, many decision makers may first set a confidence level \( \beta \) as a safety margin, which is a real value in \((0, 1)\), and then attempt to find the minimum value such that the event that the objective is below this value holds with chance \( \beta \). This decision criterion sources from the idea of chance-constrained programming (CCP) which was proposed by Charnes and Cooper [19] and developed by many scholars [20]. We first define the following concept of the optimal cover.

**Definition 6.** A cover \( x \) is called a random fuzzy \((\alpha, \beta)\)-minimum weight edge cover, if

\[
\min \{\overline{W} | \text{Ch}\{W(x, \xi) \leq \overline{W}(\alpha) \geq \beta\} \} \leq \min \{\overline{W} | \text{Ch}\{W(x', \xi) \leq \overline{W}(\alpha) \geq \beta\} \},
\]

for any cover \( x' \) of \( G \), where \( \alpha \) and \( \beta \) are predetermined confidence levels and the minimum \( \overline{W} \) is called \((\alpha, \beta)\)-weight.

Then we propose a CCP model.

\[
\begin{align*}
\min \overline{W} \\
\text{s.t.} \\
\quad \text{Ch}\left\{ \sum_{(i,j) \in E} \xi_{ij} x_{ij} \leq \overline{W}(\alpha) \geq \beta \right\} \\
\quad \sum_{j \in V} x_{ij} + \sum_{j \in V} x_{ij} \geq 1, \quad \forall i \in V \\
\quad x_{ij} \in \{0, 1\}, \quad \forall (i,j) \in E.
\end{align*}
\]

(3)

where \( \alpha \) and \( \beta \) are predetermined confidence levels.

In some cases, decision makers may be interested in some predetermined value, which may denote the available budget or the amount of some limited resources in real-world applications, and expect that the cover weight is below this value. Because of the existence of uncertainty, the cover weight is not necessarily under the predetermined value. In this view, decision makers may be concerned with maximizing the chance of the event that the cover weight is smaller than or equal
to the predetermined value. This decision criterion comes from the third type of uncertain programming, dependent-chance programming (DCP) proposed by Liu [21]. By this decision criterion, we introduce the third concept of the optimal cover as follows.

**Definition 7.** A cover \( x \) is called the maximum chance minimum weight edge cover, if

\[
\text{Ch}(\{x_j : x_j \leq W_0\} | \alpha) \geq \text{Ch}(\{x_j : x_j \leq W_0\} | \alpha),
\]

for any cover \( x \) of \( G \), where \( W_0 \) is a predetermined value and \( \alpha \) is a predetermined confidence level.

We propose the following DCP model.

\[
\begin{align*}
\max \text{ Ch} & \left( \sum_{(i,j) \in E} x_{ij} \right) | W_0 \geq 0,
\text{ s.t. } \sum_{j \in V} x_{ij} + \sum_{i \in V} x_{ij} \geq 1, \forall i \in V, \\
x_{ij} \in \{0, 1\}, \forall (i,j) \in E,
\end{align*}
\]

(4)

where \( W_0 \) is a predetermined value and \( \alpha \) is a predetermined confidence level.

4. Hybrid intelligent algorithm

In order to solve the edge covering problem under hybrid uncertain environments, we need to deal with two obstacles first. One obstacle to solving the proposed decision models is the impossibility to get the analytical form of the related uncertain functions due to the coexistence of both randomness and fuzziness. We employ random fuzzy simulation to estimate such uncertain functions. The other obstacle is the complex structure of the ECP problem. The edge covering problem is a classic combinatorial optimization problem in graph theory, and the computational cost increases severely with the problem scale increasing. Although some efficient approaches have been proposed [5–10], the heuristic is dependent on the crisp weights on edges, which is not available for ECP under hybrid uncertain environments. In this view, traditional methods seem to be not applicable to the problem of our concern. We adopt genetic algorithm (GA) for solving the proposed decision models.

Random fuzzy simulation is a technique to estimate random fuzzy functions by performing sampling experiments [15], which we use to estimate three related uncertain functions. The first function we need to estimate is

\[
U_1 : x \rightarrow E[\mathcal{W}(x, \xi)].
\]

Based on the definition of the expected value of a random fuzzy variable, we have the following simulation procedure.

Step 1. Let \( E = 0 \).
Step 2. Randomly generate \( \theta^k \) from the credibility space, such that \( \text{Pos}(\theta^k) \geq \varepsilon \) for \( k = 1, 2, \ldots, N \), where \( \varepsilon \) is a sufficiently small number and \( N \) is a sufficiently large positive integer.
Step 3. Let \( a = \min_{1 \leq k \leq N} E[\mathcal{W}(x, \xi(\theta^k))] \) and \( b = \max_{1 \leq k \leq N} E[\mathcal{W}(x, \xi(\theta^k))] \) where \( E[\mathcal{W}(x, \xi(\theta^k))] \) is estimated by random simulation.
Step 4. Randomly generate a number \( r \) from interval \([a, b]\).
Step 5. If \( r \geq b \), then \( E \leftarrow E + Cr(\theta \in \Theta | E[\mathcal{W}(x, \xi(\theta))] \geq r) \); otherwise \( E \leftarrow E - Cr(\theta \in \Theta | E[\mathcal{W}(x, \xi(\theta))] \leq r) \).
Step 6. Repeat Step 4 and 5 for \( N \) times.
Step 7. Set \( E[\mathcal{W}(x, \xi)] = a \vee 0 + b \wedge 0 + E \cdot (b - a) / N \).

The second function we are interested in is

\[
U_2 : x \rightarrow \min \{ \mathcal{W} | \text{Ch}(\{x_j : x_j \leq \mathcal{W}\} | \alpha) \geq \beta \}.
\]

According to the definition of chance function, we need to find the minimum \( \mathcal{W} \) satisfying

\[
\text{Cr}(\theta \in \Theta | \text{Pr}(\{x_j : x_j \leq \mathcal{W}\} \geq \beta) \geq \alpha) \geq \beta.
\]

We have the following simulation procedure.

Step 1. Randomly generate \( \theta^k \) from the credibility space, such that \( \text{Pos}(\theta^k) \geq \varepsilon \) for \( k = 1, 2, \ldots, N \), where \( \varepsilon \) is a sufficiently small number and \( N \) is a sufficiently large positive integer. Set \( \mu^k = \text{Pos}(\theta^k) \).
Step 2. For \( k = 1, 2, \ldots, N \), get the minimum \( \mathcal{W}(x, \xi(\theta^k)) \) satisfying \( \text{Pr}(\{x_j : x_j \leq \mathcal{W}(x, \xi(\theta^k))\} \geq \beta \) by random simulation.
Step 3. Get the minimum \( r \) satisfying \( L(r) \geq \alpha \), where

\[
L(r) = \frac{1}{2} \left( \max_{1 \leq k \leq N} \{ \mu^k | \mathcal{W}(x, \xi(\theta^k)) \leq r \} + \min_{1 \leq k \leq N} \{ 1 - \mu^k | \mathcal{W}(x, \xi(\theta^k)) > r \} \right).
\]
Step 4. Return \( r \).
The last function we need to simulate is the chance function

$$U_2 : x \rightarrow \text{Ch}(W(x, \xi) \leq W_0)(x).$$

In other words, we need to generate the maximum $$\hat{b}$$ satisfying

$$\text{Cr}\{\theta \in \Theta | \Pr\{W(x, \xi(\theta)) \leq W_0\} \geq \hat{b}\} \geq \alpha.$$  

We have the following simulation procedure.

Step 1. Randomly generate $$\theta^k$$ from the credibility space, such that $$\text{Pos}(\theta^k) \geq \varepsilon$$ for $$k = 1, 2, \ldots, N$$, where $$\varepsilon$$ is a sufficiently small number and $$N$$ is a sufficiently large positive integer. Set $$\mu^k = \text{Pos}(\theta^k)$$.  

Step 2. For $$k = 1, 2, \ldots, N$$, get $$g(\theta^k) = \Pr\{W(x, \xi(\theta^k)) \leq W_0\}$$ by random simulation.  

Step 3. Get the minimum $$r$$ satisfying $$L(r) \geq \alpha$$, where

$$L(r) = \frac{1}{2} \left( \max_{1 \leq k \leq N} \left\{ \mu^k g(\theta^k) \geq r \right\} + \min_{1 \leq k \leq N} \left\{ 1 - \mu^k g(\theta^k) < r \right\} \right).$$

Step 4. Return $$r$$.

Genetic algorithm (GA) was developed by Holland [22] in 1975, which is viewed as one of the most common evolutionary approaches to solving complex optimization problems. In 1996, Beasley and Chu [23] designed a genetic algorithm for solving the covering problem, which we modify in this paper for solving the ECP under uncertainty.

For a graph with $$m$$ edges, we first rank the edges in set $$E$$ as $$(e_1, e_2, \ldots, e_m)$$ such that $$\forall i, j \in \{1, 2, \ldots, m\}$$,

$$E[\delta_i] \geq E[\delta_j],$$

if $$i < j,$$

$$\text{Var}[\delta_i] \geq \text{Var}[\delta_j],$$

if $$i < j$$ and $$E[\delta_i] = E[\delta_j],$$
in which $$\delta_i$$ and $$\delta_j$$ are the random fuzzy weights of $$e_i$$ and $$e_j$$, respectively. We represent each subset $$S$$ of $$E$$ as a vector $$\nu = (x_1, x_2, \ldots, x_m)$$, which is called a chromosome, where

$$x_i = \begin{cases} 1, & \text{if } e_i \in S, \\ 0, & \text{if } e_i \notin S. \end{cases}$$

for each $$i = 1, 2, \ldots, m$$.

In order to initialize the first generation of chromosomes, we randomly generate each chromosome, that is, a number either 0 or 1 is randomly generated for each $$x_i$$ in each chromosome $$\nu$$. According to the values of the objective function in the corresponding decision model, we re-rank the chromosomes in each generation as $$\{\nu_1, \nu_2, \ldots, \nu_{\text{pop.size}}\}$$ from good to bad, where pop.size is the size of the population. Then we define the rank-based evaluation function as

$$\text{Eval}(\nu_i) = a(1 - a)^{i - 1}, \quad i = 1, 2, \ldots, \text{pop.size},$$

where $$a \in (0, 1)$$ is a predetermined parameter. Each chromosome for a new population is selected by spinning the roulette wheel pop.size times.

Since the structure of ECP is complex, we employ the idea of Beasley and Chu [23] to adopt the fusion operator as the crossover operator. According to the fusion operator, offspring chromosome is generated based on both the fitness and the structure of the parent chromosomes. For parent chromosomes $$\nu_1 = (x_1^1, x_2^1, \ldots, x_m^1)$$ and $$\nu_2 = (x_1^2, x_2^2, \ldots, x_m^2)$$ whose values of objective function are $$f_1$$ and $$f_2$$, respectively, we select each gene $$x_i'$$ of their single offspring chromosome $$\nu' = (x_1', x_2', \ldots, x_m')$$ according to the following rule:

- if $$x_i^1 = x_i^2$$, then $$x_i' = x_i^1$$;
- if $$x_i^1 \neq x_i^2$$, then
  - $$x_i' = x_i^1$$ with probability $$p$$,
  - $$x_i' = x_i^2$$ with probability $$q$$,

where

$$p = \begin{cases} \frac{h_2}{h_1 + h_2}, & \text{for model (1) and (3)}, \\ \frac{h_1}{h_1 + h_2}, & \text{for model (4)}, \end{cases}$$

$$q = 1 - p.$$  

We keep the parent chromosome with greater fitness and replace the parent with less fitness by the offspring chromosome. In the mutation operation, we randomly choose $$b$$ genes of each chromosome for mutation and change them from 1 to 0 or from 0 to 1, where $$b$$ is a predetermined positive integer.
After the crossover and mutation operations, generated offspring chromosomes may not be an edge cover. We modify Beasley and Chu’s [23] method and propose a repair operation such that the feasibility of solutions and efficiency of the GA are both maintained. We denote by $N(V)$ and $N(E)$ the number of vertices and the number of edges, respectively. The repair operation is shown as in Table 1.

Table 1  
The repair operation.

<table>
<thead>
<tr>
<th>INPUT: $v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>define $U_i$ as the collection of the edges incident to vertex $i$</td>
</tr>
<tr>
<td>for $i = 1 : N(V)$</td>
</tr>
<tr>
<td>if none of the edges in $U_i$ is contained in $v$</td>
</tr>
<tr>
<td>find edge $x_j$ in $U_i$ whose index $j$ is maximum</td>
</tr>
<tr>
<td>set $x_j = 1$</td>
</tr>
<tr>
<td>for $j = 1 : N(E)$</td>
</tr>
<tr>
<td>if $x_j = 1$</td>
</tr>
<tr>
<td>set $x_j = 0$</td>
</tr>
<tr>
<td>if $v$ is infeasible</td>
</tr>
<tr>
<td>set $x_j = 1$</td>
</tr>
<tr>
<td>return $v$</td>
</tr>
</tbody>
</table>

To sum up, the hybrid intelligent algorithm is shown as in Table 2.

Table 2  
Hybrid intelligent algorithm.

| initialize pop_size chromosomes |
| for $i = 1 : N$ (N is a sufficiently large number) |
| calculate the objective values for all chromosomes by random fuzzy simulation |
| compute the fitness of the chromosomes by the evaluation function |
| select chromosomes for a new population |
| update chromosomes by crossover operation, mutation operation and repair operation |
| return the best chromosome $v' = (x_1, x_2, \ldots, x_n)$ |

After the crossover and mutation operations, generated offspring chromosomes may not be an edge cover. We modify Beasley and Chu’s [23] method and propose a repair operation such that the feasibility of solutions and efficiency of the GA are both maintained. We denote by $N(V)$ and $N(E)$ the number of vertices and the number of edges, respectively. The repair operation is shown as in Table 1.

To sum up, the hybrid intelligent algorithm is shown as in Table 2.

5. Numerical experiments

In this section, we perform several numerical experiments to show the effectiveness and robustness of the proposed algorithm. Consider the graph shown in Fig. 1. Assume all weights on the edges are random fuzzy variables in form of $N(\rho, s^2)$, where $\rho$ is a triangular fuzzy variable and $s^2$ is a determinate value. A triangular fuzzy variable is usually denoted by a triplet $(r_1, r_2, r_3)$ with $r_1 < r_2 < r_3$, whose membership function is given by

$$
\mu(x) = \begin{cases} 
0, & \text{if } x > r_3, \\
\frac{x - r_1}{r_2 - r_1}, & \text{if } r_2 \leq x \leq r_3, \\
\frac{x - r_3}{r_3 - r_1}, & \text{if } r_1 \leq x \leq r_2, \\
0, & \text{if } x < r_1.
\end{cases}
$$

Thus, the random fuzzy weights are shown in Table 3.

Fig. 1. A weighted graph.
Without loss of generality, we set the parameters in the hybrid intelligent algorithm as follows: the size of a population \( \text{pop.size} \) is 50, the number of generations is 500, the number of random fuzzy simulation cycles is 400, the number of related random simulation cycles is 500, the crossover probability \( p_c \) is 0.3, the mutation probability \( p_m \) is 0.1, the number of genes undergoing mutation \( b \) is 6, the parameter \( p \) in repair operation is 0.8, and the parameter \( a \) in the rank-based evaluation function is 0.02.

The first model (1) is transformed into the crisp equivalent (2). By the hybrid intelligent algorithm, we obtain the random fuzzy expected minimum weight edge cover

\[
\{(1,4), (2,6), (3,8), (5,12), (7,11), (9,16), (10,17), (13,19), (14,21), (15,19), (18,22), (20,23)\}
\]

with the expected minimum cover weight 174.25.

For model (3), we consider different parameters \( \alpha \) and \( \beta \) and report the results obtained by the hybrid intelligent algorithm in Table 4.

According to the definition of the chance function, the \((\alpha, \beta)\)-weight increases with \( \alpha \) increasing and \( \beta \) increasing, which is verified by results shown in Table 4.

For model (4), we employ the hybrid intelligent algorithm and report the results for different values of \( W_0 \) and \( \alpha \) in Table 5.

According to the definition of the chance function, \( \text{Ch} \{W(x, \xi) \leq W_0\}(\alpha) \) decreases with \( \alpha \) increasing and increases with \( W_0 \) increasing, which is verified by results shown in Table 5.

Finally, in order to illustrate the robustness of hybrid intelligent algorithm, we solve model (3) with \( \alpha = \beta = 0.9 \) by running the algorithm with different parameters. We are interested in the relative error defined as

Relative error = \((\text{actual value} - \text{minimum value})/\text{minimum value} \cdot 100\%\).
In this paper, we study the edge covering problem under hybrid uncertain environments for the first time. We employ random fuzzy theory to address the hybrid uncertainty and use random fuzzy variables to model the uncertain weights on edges. According to different decision criteria in the three types of decision models including EVM, CCP and DCP, we propose three decision models for solving the problem concerned. Because traditional methods are usually inapplicable to the ECP under hybrid uncertain environments, we integrate random fuzzy simulation and genetic algorithm to produce a hybrid intelligent algorithm to solve our problem. Numerical experiments are performed at last to reveal the effectiveness and robustness of the proposed models and algorithm.

This work points out some directions of future research. Fuzzy random theory is another mathematical theory that characterizes circumstances where hybrid uncertainty exists. It is worthwhile to study the edge covering problem by assuming the uncertain weights to be fuzzy random variables. It would be interesting to compare the optimal edge covers under different decision criteria with those obtained in this paper. Another direction of future work is to investigate other efficient algorithms for solving the models proposed in this paper. Although the hybrid intelligent algorithm is effective for our problem, its scalability is limited due to the computational burden of the random fuzzy simulation. In the future, we will attempt to save the computation of random fuzzy simulation by employing techniques such as artificial neural networks for estimating random fuzzy functions.

Acknowledgement

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References


Table 6

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<th>pop_size</th>
<th>$p_i$</th>
<th>$p_m$</th>
<th>$b$</th>
<th>$p$</th>
<th>(0.9, 0.9)-weight</th>
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<td>0.8</td>
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</table>

The results are shown in Table 6.

We find in Table 6 that the relative errors do not exceed 1%, so the hybrid intelligent algorithm is shown to be robust.

6. Conclusion

In this paper, we study the edge covering problem under hybrid uncertain environments for the first time. We employ random fuzzy theory to address the hybrid uncertainty and use random fuzzy variables to model the uncertain weights on edges. According to different decision criteria in the three types of decision models including EVM, CCP and DCP, we propose three decision models for solving the problem concerned. Because traditional methods are usually inapplicable to the ECP under hybrid uncertain environments, we integrate random fuzzy simulation and genetic algorithm to produce a hybrid intelligent algorithm to solve our problem. Numerical experiments are performed at last to reveal the effectiveness and robustness of the proposed models and algorithm.

This work points out some directions of future research. Fuzzy random theory is another mathematical theory that characterizes circumstances where hybrid uncertainty exists. It is worthwhile to study the edge covering problem by assuming the uncertain weights to be fuzzy random variables. It would be interesting to compare the optimal edge covers under different decision criteria with those obtained in this paper. Another direction of future work is to investigate other efficient algorithms for solving the models proposed in this paper. Although the hybrid intelligent algorithm is effective for our problem, its scalability is limited due to the computational burden of the random fuzzy simulation. In the future, we will attempt to save the computation of random fuzzy simulation by employing techniques such as artificial neural networks for estimating random fuzzy functions.