Application of grey linear control theory for price regulation in China’s real estate market

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Abstract
Purpose – The purpose of this paper is to propose a grey linear control system for regulating the price of China’s real estate and provide the necessary support to assist the relevant management departments with their policy making.
Design/methodology/approach – A grey state equation of the real estate market price that can reflect both the market supply-demand price mechanism and the production price mechanism is proposed based on the economic cybernetics. Also, the grey control linear theory is used to demonstrate the disequilibrium fluctuation control system for the China’s real estate market with uncertain parameters.
Findings – The price disequilibrium fluctuation control system for China’s real estate market has been in a critical state. The system would reach a balanced state in 2013, if the real estate price in 2010 and 2011 firstly decreased 244.41 yuan/m2 and 62.33 yuan/m2, respectively, and then increased 60.88 yuan/m2 in 2012. The disequilibrium state will continue for years before it reaches a balanced state.
Research limitations/implications – Due to the complexity of operation of grey numbers, the present technique still cannot analyze the properties of the grey control system exactly and further research is needed.
Practical implications – The modelled results can help the relevant management departments steady China’s real estate market by price regulation.
Originality/value – A new approach to study the price regulation system of a real estate market is proposed based on grey linear control theory.
 Keywords China, Real estate, Prices, Regulation, Economic cybernetics, Systems theory, Real estate market, Price regulation

1. Introduction
The Chinese real estate investment has entered a booming period since the abolition of the housing allocation system and the execution of the mortgage policy in 1998, and it plays an essential role in economic growth due to its close relations with building materials, transportation, energy, chemical industry and machinery, etc. The real estate sector has nowadays become one of the important industrial sectors of the national economy. However, a disequilibrium state of supply exceeding demand has also emerged. Reasons for this may be summarized as following: the rapid growth

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of the real estate industry mainly relies on investment, and excessive assets in this field accelerate the market supply. On the contrary, effective demands for the housing are limited due to factors such as people’s income although the potential demands are large. As a result, many commercial housing developments are idle.

Fair and Jaffee (1972) established the disequilibrium econometric model, and studied its basic form and estimation approaches comprehensively, pioneering the earliest research in this mode. Presently, most studies on the combination of the disequilibrium theory and concrete cases in China are mainly focused on the growth of the real estate market and the disequilibrium of the price. Wang and Gao (2004) employed dynamic quantitative analysis to demonstrate that people’s income affected housing demands more than the price and the rate. On the other hand, the price and the rate affected the housing supply more than the income. Ji (2005) calculated the gross of the disequilibrium degree by virtue of the nonlinear iterative regression of four variances. Tong and Liu (2008) empirically analyzed the price disequilibrium in the Chinese real estate market based on the semi-parametric regression model and the expanded semi-parametric regression model. Shi and Tian (2009) established three panel data models with different characteristics and investigated the effect of the price on the real estate market as well as on regional differences based upon 25 regional data sets. Chow and Niu (2010) investigated housing supply and demand since the housing commercialization of the late 1980s in China, and gave the supply-demand equation for urban housing with the framework of the simultaneous equation in terms of the supply-demand theory of durable goods. Summaries are given from the above bibliography that the disequilibrium of the real estate market focuses on both the growth and the price, whose disequilibrium interact with one another. In addition, the disequilibrium of the housing price affects the gross disequilibrium by influencing the effective demands.

To date, econometric models have been widely applied to study disequilibrium states of real estate markets, such as disequilibrium econometric models (Fair and Jaffee, 1972; Wang and Gao, 2004; Ji, 2005), semi-parametric regression models (Tong and Liu, 2008), panel data models (Shi and Tian, 2009), and simultaneous equations (Chow and Niu, 2010). However, econometric models present two disadvantages: first, a reliable econometric model cannot be established merely based on a large amount of historical data. The parameters with sample size of less than 30 models fail to fully show relative statistical characteristics; second, it is difficult for econometric models to adjust economic policy variables in the preset target or state-like cybernetics. The grey systems theory proposed by Deng (1982a) is an effective way to deal with the modeling of small sample data and has been used in many fields (Deng, 2002; Li and Liu, 2009; Kose et al., 2011; Zeng et al., 2010, etc.). Additionally, in economic cybernetics (Wang et al., 2008), the price regulation function is a feedback regulator; it can guarantee the market balance. Since the data employed in this study were released by China statistics departments in succession since 2000, the data amount collected is insufficient to employ the econometric modeling methods which are commonly applied in economics. Therefore, grey control theory (Deng, 1982b; Liu and Lin, 2006) can offer a new approach for the disequilibrium of the real estate market in China.

This paper attempts to apply grey linear control system theory to study the price disequilibrium in the real estate market in China. The rest of the paper mainly includes: an introduction to grey linear control system theory, construction of the grey
2. A brief introduction to grey linear control system theory

The so-called grey control means the control of essential grey systems, including general control systems with grey parameters and controls built on the analysis, modeling, prediction, and decision making of grey systems. The grey linear control system is one of the basic components of grey control theory. Before building the control system for China’s real estate market, there is a brief introduction to the grey linear control system (Liu and Lin, 2006).

Definition 1 (Liu and Lin, 2006) assume that:

\[ U = [u_1, u_2, \ldots, u_s]^T \]

is the control variable:

\[ X = [x_1, x_2, \ldots, x_n]^T \]

is the state variable, and:

\[ Y = [y_1, y_2, \ldots, y_m]^T \]

is the output variable. Then:

\[
\begin{align*}
\dot{X} &= \otimes AX + \otimes BU \\
Y &= \otimes CX
\end{align*}
\]

is called the mathematical model of a grey linear control system, where \( \otimes A \in G_{n \times n}^{G}, \otimes B \in G_{s \times s}^{G}, \otimes C \in G_{m \times n}^{G} \). Accordingly, \( \otimes A \) is called a grey state matrix, \( \otimes B \) a grey control matrix, and \( \otimes C \) a grey output matrix.

Theorem 1. (Liu and Lin, 2006) for the system:

\[
\begin{align*}
\dot{X}(t) &= \otimes AX(t) + \otimes BU(t) \\
Y(t) &= \otimes CX(t)
\end{align*}
\]

where:

\[
\begin{align*}
U(t) &= [u_1(t), u_2(t), \ldots, u_s(t)]^T, \\
X(t) &= [x_1(t), x_2(t), \ldots, x_n(t)]^T, \\
Y(t) &= [y_1(t), y_2(t), \ldots, y_m(t)]^T,
\end{align*}
\]

\( \otimes A \in G_{n \times n}^{G}, \otimes B \in G_{s \times s}^{G}, \otimes C \in G_{m \times n}^{G} \). Let:

\[
\otimes D = [\otimes C \otimes C \otimes A \otimes C(\otimes A)^2 \ldots \otimes C(\otimes A)^{n-1}]^T
\]

and:

\[
\otimes L = [\otimes B \otimes A \otimes B \otimes (\otimes A)^2 \otimes B \ldots (\otimes A)^{n-1} \otimes B].
\]
Then, we have the following:

- when \( \text{rank}(\otimes D) = n \), the system is observable;
- when \( \text{rank}(\otimes L) = n \), the system is controllable; and
- a necessary and sufficient condition for the system to be asymptotically stable is that the upper bounds of the grey elements of the real parts of the grey characteristic roots of the state grey matrix \( \otimes A \) are less than zero.

Where, “rank” means the number of linearly independent rows or columns for a matrix.

Observability and controllability, as the important properties of control system, are two basic concepts in cybernetics. Observability is a measure for how well internal states of a system can be inferred by knowledge of its external outputs. If each state variable of a system can influence its output, or the information about an arbitrary state variable can be obtained from the output, the system is observable. Otherwise, it is unobservable. In this study, observability is utilized to explain whether the price, supply and demand we selected as relative factors can effectively determine the system state. Controllability denotes an ability which moves a system around in its entire configuration space using only certain admissible manipulations. For an economic system, controllability analysis mainly considers whether the economic system can reach a preset state or target using a policy adjusting method. In this study, the equilibrium target of China real estate market will reach in 2013 is preset. Then by analyzing system controllability, how government adjusts prices in 2010-2012 is known.

3. Grey state model of price regulation for China's real estate market

The price mechanism of the Chinese real estate market is hybrid; it is combined with the planned economy and market regulation, mainly focused on the market supply-demand mechanism and the production price mechanism. In this section, the grey state space model is constructed for the real estate market, and the supply-demand function and the production price regulation function with grey parameters are employed to demonstrate the above two price mechanisms.

We assume that \( p(k) \) is the price of the \( k \)th term real estate market, \( p^*(k) \) the developers' scheduled price for the \( k \)th term, \( D(k) \) and \( S(k) \) are the demand and supply for the \( k \)th term, respectively. According to economic theories, the supply, demand and price have relations as following:

\[
\begin{align*}
D(k) &= \otimes \alpha - \otimes \beta p(k) \\
S(k) &= -\otimes \theta + \otimes \gamma p^*(k)
\end{align*}
\]

where grey parameters \( \otimes \alpha, \otimes \beta, \otimes \theta, \otimes \gamma \in [0, +\infty] \).

Assume that the developers’ scheduled price in the \( k \)th term to be the linear combination of the initial price, initial demand and the supplying difference (excess demands). \( \otimes \delta \) is a grey parameter for regulating the price. Then the price regulation function can be expressed as:

\[
p^*(k) = p(k - 1) + \otimes \delta [D(k - 1) - S(k - 1)]
\]

The above function can reflect the price regulation in terms of changes of the excess demands. Based upon the above three equations, the adjustment object is confirmed to
regulate the price fluctuation by virtue of the state feedback, achieving the balance between supply and demand.

**Theorem 2.** The balance between supply and demand is:

\[
D(k + 1) - S(k + 1) = (1 - \otimes \gamma \otimes \delta)[D(k) - S(k)] + \otimes \gamma \otimes \delta[D(k) - S(k)] \\
- S(k - 1) - \otimes \gamma[p(k) - p(k - 1)] - \otimes \beta[p(k + 1) - p(k)]
\]

**Proof.** By the state space model of the price disequilibrium of the real market, we have:

\[
D(k + 1) - S(k + 1) = \otimes \alpha - \otimes \beta p(k + 1) + \otimes \theta - \otimes \gamma p(k) - \otimes \gamma \otimes \delta[D(k) - S(k)] \\
= (1 - \otimes \gamma \otimes \delta)[D(k) - S(k)] + \otimes \alpha - \otimes \beta p(k + 1) + \otimes \theta - \otimes \gamma p(k) \\
- [D(k) - S(k)] = (1 - \otimes \gamma \otimes \delta)[D(k) - S(k)] \\
+ \otimes \alpha - \otimes \beta p(k + 1) + \otimes \theta - \otimes \gamma p(k) - \otimes \alpha + \otimes \beta p(k) \\
- \otimes \theta + \otimes \gamma p(k - 1) + \otimes \gamma \otimes \delta[D(k - 1) - S(k - 1)] \\
= (1 - \otimes \gamma \otimes \delta)[D(k) - S(k)] + \otimes \gamma \otimes \delta[D(k - 1) - S(k - 1)] \\
- \otimes \gamma[p(k) - p(k - 1)] - \otimes \beta[p(k + 1) - p(k)]
\]

The above state equation has removed grey parameters \( \otimes \alpha \) and \( \otimes \theta \), only containing \( \otimes \beta, \otimes \gamma \) and \( \otimes \delta \).

The state variables are assumed to be:

\[
x_1(k) = D(k) - S(k) \\
x_2(k) = D(k - 1) - S(k - 1) \\
x_3(k) = p(k) - p(k - 1)
\]

The price change is assumed to be the control variable:

\[
u(k) = p(k + 1) - p(k)
\]

The state equation can be written as:

\[
\begin{bmatrix}
x_1(k+1) \\
x_2(k+1) \\
x_3(k+1)
\end{bmatrix} = \begin{bmatrix} 1 - \otimes \gamma \otimes \delta & \otimes \gamma \otimes \delta & -\otimes \gamma \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\
x_2(k) \\
x_3(k) \end{bmatrix} + \begin{bmatrix} -\otimes \beta \\ 0 \\ 1 \end{bmatrix} u(k)
\]

The output equation is:

\[
y(k) = x_1(k) = D(k) - S(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\
x_2(k) \\
x_3(k) \end{bmatrix}.
\]
If we assume:
\[
\begin{bmatrix}
  x_1(k) \\
  x_2(k) \\
  x_3(k)
\end{bmatrix} = \begin{bmatrix} x_1(k) \\
  x_2(k) \\
  x_3(k)
\end{bmatrix}
\]
\[
\otimes A = \begin{bmatrix}
  1 - \otimes \gamma \otimes \delta & \otimes \gamma \otimes \delta & -\otimes \gamma \\
  1 & 0 & 0 \\
  0 & 0 & 0
\end{bmatrix},
\]
\[
\otimes B = \begin{bmatrix}
  -\otimes \beta \\
  0 \\
  1
\end{bmatrix},
\]
\[
C = \begin{bmatrix}
  1 \\
  0 \\
  0
\end{bmatrix}^T,
\]
then we obtain the following grey control system for the price disequilibrium in real estate market:
\[
\begin{cases}
  x(k+1) = \otimes A x(k) + \otimes B u(k) \\
  y(k) = C x(k)
\end{cases}
\]

4. Empirical analysis of the price disequilibrium for China’s real estate market

4.1 Whitenization of grey parameters in the state space model

The grey control system of the price fluctuation in Section 3 is an uncertain system with grey parameters. It can describe the relations of the variables comprehensively. And yet, the present technique can still not analyze the basic properties of this system precisely. Therefore, the grey parameter whitenization method is used in this section for the effective analysis of the disequilibrium fluctuation. The whitenization values of grey parameters are first calculated by the least squares method, then they are brought into the system model. Because data from the National Statistics Bureau (www.stats.gov.cn/) do not include the supply and demand of the real estate market, we regard the sales volume as the effective demand for the general housing, and construction volume as the effective supply. Data in Table I represent China’s general housing market.

According to the parameters whitening method for GM(1,1) model (Deng, 2002; Liu and Lin, 2006), the grey parameters in the state equation are whitened by the least-squares method. The goal of the least-squares method is to find a good estimation of parameters that fit a function. The least-squares method requires that the estimated function has to deviate as little as possible from the function in the sense of a 2-norm. This can be seen specifically in the book by Cheney and Kincaid (2004). In this study,

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand (billion m²)</td>
<td>12,169</td>
<td>15,039</td>
<td>18,451</td>
<td>24,310</td>
<td>28,235</td>
<td>43,564</td>
<td>18,414</td>
<td>62,047</td>
<td>52,788</td>
<td>78,500</td>
</tr>
<tr>
<td>Supply (billion m²)</td>
<td>17,919</td>
<td>23,280</td>
<td>27,162</td>
<td>36,174</td>
<td>40,716</td>
<td>48,837</td>
<td>55,966</td>
<td>69,071</td>
<td>73,683</td>
<td>84,294</td>
</tr>
<tr>
<td>Mean price (¥/m²)</td>
<td>1,557</td>
<td>1,651</td>
<td>1,708</td>
<td>1,805</td>
<td>2,058</td>
<td>2,416</td>
<td>2,718</td>
<td>3,303</td>
<td>3,518</td>
<td>4,176</td>
</tr>
</tbody>
</table>

Table I. Basic economic indicators of China’s real estate market in 2000-2009
letting $D(k), S(k)$ and $x_3(k) = p(k) - p(k - 1)$ as unknown functions, the whitenization values of grey parameters $\bar{\otimes} \beta, \bar{\otimes} \gamma$ and $\bar{\otimes} \delta$ are calculated basing on the data in Table I using least-squares method. We have:

\[
\bar{\otimes} \beta = 22.844135, \quad \bar{\otimes} \gamma = 28.311055, \quad \bar{\otimes} \delta = -0.020086.
\]

The above results reveal that $\bar{\otimes} \beta > 0, \bar{\otimes} \gamma > 0$, indicating that relations of supply and demand in the Chinese real estate market is consistent with the economic theories. That is, demand has a negative correlation with the price while supply has a positive correlation with the price. However, $\bar{\otimes} \delta < 0$ shows that the developers’ scheduled price will not fall but rise when supply exceeds demand in the market. By further calculation, it can be obtained that $\bar{\otimes} \gamma/\bar{\otimes} \beta = 1.238 > 1$. This suggests that the price system of real estate in China is diverging. In such a price system, the deviation from the equilibrium point can produce growing fluctuations and more unstable supply-demand relations as time goes on. If government does not put forward relative control measures, disequilibrium of market supply-demand relations can incur. Certainly, this phenomenon not only exists in the monopoly market of Chinese real estate. It can also be observed in some free markets, e.g. the economic phenomenon described in the cobweb model (Kaldor, 1938).

The whitenization values of grey parameters are brought into the grey control system, then the grey state matrix is obtained:

\[
\bar{\otimes} A = \begin{bmatrix} 1 - \bar{\otimes} \gamma \bar{\otimes} \delta & \bar{\otimes} \gamma \bar{\otimes} \delta & -\bar{\otimes} \gamma \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1.568650 & -0.568650 & -28.311055 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

The whitenized grey outputs matrix is:

\[
\bar{\otimes} B = \begin{bmatrix} -\bar{\otimes} \beta \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -22.844135 \\ 0 \\ 1 \end{bmatrix}
\]

The whitenized price fluctuation system of the real estate market can be expressed as:

\[
\begin{cases}
    x(k + 1) = \begin{bmatrix} 1.568650 & -0.568650 & -28.311055 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x(k) + \begin{bmatrix} -22.844135 \\ 0 \\ 1 \end{bmatrix} u(k) \\
    y(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(k)
\end{cases}
\]
4.2 Observability
Observability helps to ascertain the system state according to the system output; it reflects how much system information can be acquired from the system output. In terms of the economic system, observability describes whether the system’s economic variables can ascertain the state of the system operation. By Theorem 1, calculation of the price system for the real estate market can be expressed as:

\[
\mathcal{D} = \begin{bmatrix}
1 & -22.844135 & 445.662217 \\
0 & -58.594837 & 2,677.096733 \\
0 & -76.192290 & 5,173.901871
\end{bmatrix}, \quad \text{rank}(\mathcal{D}) = 3
\]

The above results show that the price disequilibrium fluctuation control system for the Chinese real estate market that is put forward in this paper is observable. Variables such as the price, supply and demand can effectively ascertain the system state.

4.3 Stability
Stability of the economic system in concrete operations will affect whether the scheduled object can be achieved. If an effective forecast on the economic system is given with proper measurements, the system operation would reach an ideal state.

By calculation, the characteristic roots of \( \mathcal{A} \) are \( \lambda_1 = 1, \lambda_2 = 0.5686 \) and \( \lambda_3 = 0 \), indicating that the price system of real estate in China has been in a critical state. In this case, necessary measurements must be taken to prevent economic loss.

4.4 Controllability
For the economic system, controllability helps adjust relevant polices so that the economic system can reach the scheduled object. By Theorem 1, calculation for the price system is expressed as:

\[
\mathcal{L} = \begin{bmatrix}
-22.844135 & -64.145513 & -87.63155 \\
0 & -22.844135 & -64.145513 \\
1 & 0 & 0
\end{bmatrix}, \quad \text{rank}(\mathcal{L}) = 3
\]

The above indicates that the price disequilibrium fluctuation control system for the Chinese real estate market that is put forward in this paper is controllable.

Let us discuss the control variable of the price \( u(k) = p(k + 1) - p(k) \) when the market becomes balanced in the scheduled years. In the hypothesis, the government plans to achieve the supply-demand balance in 2013 by relevant price regulation in 2010-2012.

The state variable in 2009 is assumed to be \( x(0) = [-5, 793.97 - 20, 895.44 \ 658.32]^T \), then, \( x(4) \) should be 0 if we want to achieve market balance in 2013.
By the definition of controllability, we can obtain:

\[
x(4) = \tilde{A}^4 x(0) + \sum_{i=0}^{3} \tilde{A}^{3-i} \tilde{B} u(i).
\]

Both sides of the above equation are multiplied by \(\tilde{A}^{-4}\), then:

\[
x(0) + \sum_{i=0}^{3} \tilde{A}^{-i-1} \tilde{B} u(i) = 0,
\]

Subsequently:

\[
x(0) = -\begin{bmatrix} \tilde{A}^{-1} \tilde{B} & \tilde{A}^{-2} \tilde{B} & \tilde{A}^{-3} \tilde{B} & \tilde{A}^{-4} \tilde{B} \end{bmatrix} \cdot \begin{bmatrix} u(0) \\ u(1) \\ u(3) \\ u(3) \end{bmatrix} = 0.
\]

For the sake of the calculation convenience, the recursive method can be also used for the control variable. By calculation, the following control variable sequences based on \(x(4) = 0\) are obtained:

\[
\begin{bmatrix} u(0) \\ u(1) \\ u(3) \\ u(3) \end{bmatrix} = \begin{bmatrix} p(2010) - p(2009) \\ p(2011) - p(2010) \\ p(2012) - p(2011) \\ p(2013) - p(2012) \end{bmatrix} = \begin{bmatrix} -244.41 \\ -62.33 \\ 60.88 \\ 0 \end{bmatrix}.
\]

The above results show that the economic system would reach a balanced state if the real estate price in 2010 and 2011 first decreased by 244.41 yuan/m² and 62.33 yuan/m², respectively, and then increased by 60.88 yuan/m² in 2012. However, this is not in fact the case. Therefore, the disequilibrium state of the Chinese real estate market will continue for years before it reaches to a balanced state.

5. Conclusion
The unbalanced price has affected the development of China’s real estate market, and it has negative effects on the stability of the capital market, finance and economy. This paper employed grey linear control system theory to analyze the disequilibrium regulation of the Chinese real estate market that includes both the supply-demand price mechanism and the production price mechanism. Empirical analysis shows that the disequilibrium fluctuation control system of the Chinese real estate market put forward in this paper is observable and controllable; the stability analysis also shows that this system has been in a critical state.

At present, the average price of China’s real estate market has been rising all the time, showing a state of disequilibrium in the long term. Therefore, the Chinese Government must effectively regulate the real estate market, standardize the land-trading activities and stabilize the price mechanism fundamentally.
References

Further reading

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