Revisiting the spectrum of baryonium in heavy baryon chiral perturbation theory

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In the framework of the heavy baryon perturbation theory, in which the two-pion exchange is considered, the physical properties of heavy baryon and anti-heavy baryon systems are revisited. The potentials between heavy baryon and anti-heavy baryon are extracted in a holonomic form. Based on the extracted potentials, the S-wave scattering phase shifts and scattering lengths of \( \Lambda_c - \Lambda_c \) and \( \Sigma_c - \Sigma_c \) are calculated. From these scattering features, it is found that the \( \Lambda_c - \Lambda_c \) system can be bound only when the value of the coupling constant at the baryon-Goldstone-boson vertex is larger than that calculated from the decay data of the \( \Sigma_c (\Sigma_c) \rightarrow \Lambda_c \pi \) process. The binding condition for the \( \Sigma_c - \Sigma_c \) system is also examined. The binding possibilities of these systems deduced from the scattering calculations are also checked by the bound state calculation and the binding energies are obtained if the system can be really bound. The binding possibility of the \( \Lambda_c - \Lambda_c \) system is investigated as well.

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I. INTRODUCTION

Charmonium and bottomonium are important objects in studying strong interactions and structures of hadrons. In the past decade, many new hadron states with heavy flavors, such as \( Y(4260), Y(4360), Y(4660), Z^+(4430), Y_6(10890), \) etc., have been found in the \( e^+e^- \) annihilation and the \( B \) meson decay experiments by the BABAR and Belle Collaborations [1]. The state \( Z_c (3900) \) has recently been observed in the \( e^+e^- \) annihilation by the BESIII and Belle [2,3] Collaborations. These new findings have attracted much attention on the structure of hadron all over the particle and nuclear physics communities [4–7]. However, some of the states cannot be identified as a conventional quarkonium with heavy flavor, such as the charmonium or its excited state because of their abnormal quantum numbers, masses, decay modes, and corresponding branching ratios in experiments. Therefore, as mentioned in our previous paper [8], to explain the peculiar data, many postulates for their structures have been proposed, but up to now, no definite conclusions could be drawn yet.

One of the striking pictures among the postulates is the baryonium with heavy flavor. In the extended heavy baryon picture used in our previous paper [8,9], an approximate SU(2) symmetry between \( \Lambda_c \) and \( \Sigma_c^0 \) is assumed, and \( \Lambda_c \) and \( \Sigma_c^0 \) are taken as the basis vectors in the two-dimensional “C-spin” representation, which is analogous to the isospin in the nucleon doublet case.

Apparently, these basis vectors can form a C-spin triplet and a “C-spin” singlet [9]. The key point is to verify if a heavy baryon and a heavy antibaryon can really form dynamically a bound state, the baryonium. The simplest way to achieve this aim is to calculate a potential between heavy baryons by using a theory, for instance, the so-called heavy baryon chiral perturbation theory (HBCPT) which can effectively provide a good description for the heavy baryon, and then solving the Schrödinger equation for the energy eigenvalue and consequently the mass spectrum.

In fact, in our previous investigation [8] we have studied the possibility of forming a heavy baryonium by using HBCPT. The result showed that there might exist a heavy baryonium as long as the adopted values of the coupling constant at the baryon-Goldstone-boson vertex and the cutoff parameter \( \Lambda \) are in special ranges, respectively, although the result is very sensitive to such values. This strong parameter dependence is not satisfactory. Such a dependence might come from the inappropriate approximation in deriving potentials, for instance, the premature truncation to the term with \( 1/r^{5/2} \) in the asymptotic expressions of the potentials expanded in \( \lambda \) [8,10]. This is because of the short range contribution of the two-pion-exchange potential. The expansion of the potential function in \( \lambda \) requires a relatively larger \( r \). Although the dominant contribution of the \( \lambda \) integral comes from small \( \lambda \) values, the expansion converges extremely slowly. Moreover, whether the physical value of the coupling constant, which can be extracted from relevant decay data, supports the existence of a heavy baryonium is still questionable and this problem should further be investigated carefully.

In this paper, we first rederive the potential between \( \Lambda_c(\Sigma_c) \) and \( \bar{\Lambda}_c(\bar{\Sigma}_c) \) in a holonomic form rather than in
the truncated expansion of Ref. [8]. Then, we study the \( \Lambda_c - \bar{\Lambda}_c \) scattering to get scattering characters, in particular, those closely related to the binding features. Based on the enlightenmenment from scattering information, we finally calculate the binding behavior of the system to confirm whether a heavy baryonium really exists. The paper is organized as follows. In Sec. II, the formalism of HBCPT is briefly recalled. The two-body interaction potentials in the \( \Lambda_c^+ - \bar{\Lambda}_c^+ \) and \( \Sigma_c^0 - \Sigma_c^0 \) systems are given in Sec. III. In Sec. IV, the numerical results for the scattering information and the mass spectra of possible heavy baryonia are presented. And the summary is given in Sec. V.

II. A BRIEF INTRODUCTION TO HBCPT

As commonly adopted, symbol \( q_1 q_2 Q \), where \( q_1(2) \) represents the light quark and \( Q \) denotes the heavy quark, describes a heavy baryon which contains one heavy quark and two light quarks. Assuming that two light quarks form a pair of diquarks, then in the flavor space, these three quarks can form a symmetric sextet and an antisymmetric triplet, i.e., \( 3 \otimes 3 = 6 \oplus \bar{3} \). Because the wave function of the hadron in the color space is totally antisymmetric, the wave function in the direct product space of orbit, flavor and spin must be symmetric. Consequently, for a ground state hadron, the wave function in the flavor and spin spaces should be symmetric since the orbital wave function is symmetric. For the light quark pair, we use Young tables \( 1F \) and \( 1S \) to denote the symmetric sextet and antisymmetric triplet in the flavor space, respectively, and \( 1S \) to represent the triplet and singlet in the spin space, respectively. Coupling these wave functions of a diquark to that of a heavy quark, denoted by \( \{ Q \} \), we have

\[
\begin{align*}
\left[ \left( \frac{3}{2} \frac{1}{2} \right)^{F} \times \left( \frac{3}{2} \frac{1}{2} \right)^{S} \right] & \oplus \left[ \left( \frac{3}{2} \frac{1}{2} \right)^{F} \times \left( \frac{0}{0} \frac{0}{0} \right)^{S} \right] \\
& \times \left( \left[ \left( \frac{3}{2} \frac{1}{2} \right)^{F} \times \left( \frac{0}{0} \frac{0}{0} \right)^{S} \right] \oplus \left[ \left( \frac{0}{0} \frac{0}{0} \right)^{F} \times \left( \frac{0}{0} \frac{0}{0} \right)^{S} \right] \right) \\
& \times \left[ \left( \frac{3}{2} \frac{1}{2} \right)^{F} \times \left( \frac{0}{0} \frac{0}{0} \right)^{S} \right]
\end{align*}
\]

Equation (1) suggests that the sextet \( 6 \) has spin-1/2 and spin-3/2 states, while the triplet \( 3 \) has only spin-1/2 states. Writing them explicitly in the matrix form, we have

\[
B_6 = \left( \begin{array}{ccc} \Sigma_c^+ & \frac{1}{\sqrt{2}} \Sigma_c^0 & \frac{1}{\sqrt{2}} \Xi_c^+ \\ \frac{1}{\sqrt{2}} \Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}} \Xi_c^0 \\ \frac{1}{\sqrt{2}} \Xi_c^+ & \frac{1}{\sqrt{2}} \Xi_c^0 & \Omega_c^0 \end{array} \right)
\]

(2)

and

\[
B_3 = \left( \begin{array}{ccc} 0 & \Lambda_c & \Xi_c^+ \\ -\Lambda_c & 0 & \Xi_c^- \\ -\Xi_c^+ & -\Xi_c^- & 0 \end{array} \right)
\]

(3)

for the sextet and triplet of the charmed heavy baryon, respectively, and the same form of Eq. (2) for the spin-3/2 \( \Lambda_c^0 \) multiple. These forms are also applicable to the bottomed heavy baryon multiple by substituting \( c \) with \( b \).

On the other hand, in terms of the chiral perturbation theory, one has the leading order vector and axial vector fields in \( f_\pi \) [8,11,12]

\[
V_\mu = \frac{1}{f_\pi} M \partial_\mu M,
\]

(4)

\[
A_\mu = -\frac{1}{f_\pi} \partial_\mu M,
\]

(5)

where \( M \) is the Goldstone boson matrix

\[
M = \left( \begin{array}{ccc} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^- & \bar{K}^0 \\ K^0 & -\frac{1}{\sqrt{6}} \eta & \bar{K}^0 \end{array} \right).
\]

(6)

Then, the general form of the chiral-invariant Lagrangian can be written as [13]

\[
\mathcal{L} = \frac{1}{2} \left[ \partial_\mu (\partial^\mu M \partial_\nu M) + M \partial_\mu M \partial^\mu M^\dagger \right] + \frac{1}{2} (\partial_\mu M \partial_\nu M - g_{\mu\nu} \frac{1}{f_\pi} M M^\dagger)
\]

(7)

\[
- \gamma_\mu (i\partial_\nu + M^\dagger_\nu) \gamma_\nu B_6^\nu \right) + g_1 \operatorname{tr}(\bar{B}_6 \gamma_{\mu} \gamma_{5} A^\mu B_6) + g_2 \operatorname{tr}(\bar{B}_6 \gamma_{\mu} \gamma_{5} A^\mu B_3) + \bar{B}_6 \gamma_{\mu} \gamma_{5} A^\mu B_6 + \bar{B}_6 \gamma_{\mu} \gamma_{5} A^\mu B_3.
\]

(7)
where the chiral covariant derivative $D_{\mu}$ satisfies

$$D_{\mu}B_6 = \partial_{\mu}B_6 + V_{\mu}B_6 + B_6V^{\tau}_{\mu}, \quad (8)$$

$$D_{\mu}B_3 = \partial_{\mu}B_3 + V_{\mu}B_3 + B_3V^{\tau}_{\mu}. \quad (9)$$

According to the heavy quark symmetry, the above six coupling constants obey approximately the following relations:

$$g_1 = \frac{2\sqrt{3}}{3}g_3 = -\frac{2}{3}g_5, \quad g_2 = -\frac{\sqrt{3}}{3}g_4, \quad g_6 = 0; \quad (10)$$

thus, we have only two free parameters $g_1$ and $g_2$ in the numerical calculation [13].

### III. THE FORMULATION FOR TWO-BODY SCATTERING POTENTIAL

To derive the two-body scattering kernel and further the potential, as carried out in Ref. [8], we follow the technique developed in Refs. [14,15]. We first write down the scattering amplitude to get the interaction kernel, and then make the nonrelativistic reduction. Further making the Fourier transformation, we obtain the potential in the configuration space. Then, acting the operators onto a considered channel, we finally obtain the potential for such a particular system. Again, in this continuation paper, we calculate the potentials corresponding to the four $2\pi$-exchange diagrams shown in Fig. 1.

In the center of mass system, we define

$$p_a = -p_b = p, \quad p'_a = -p'_b = p',$$

$$P = p_a + p_b = (E_a + E_b, 0) = (E, 0),$$

$$P' = p'_a + p'_b = (E'_a + E'_b, 0) = (E', 0),$$

$$p = \frac{1}{2}(p_a - p_b) = (0, p),$$

$$p' = \frac{1}{2}(p'_a - p'_b) = (0, p').$$

as the three-momenta of the initial and final states, the total four-momenta of the initial and final states, and the relative four-momenta of the initial and final states, respectively.

For the $\Lambda_c^-\bar{\Lambda}_c$ interaction, we first calculate the potential of the box diagram. Following the prescription given in Refs. [14,15], we obtain the potential in the configuration space (a detailed calculation can be found in Ref. [8])

$$V_B(r_1, r_2) = -\left(\frac{g_1^2}{f^3}\right)\int d^3k_1d^3k_2\int \frac{d^3r_1}{(2\pi)^6}\int \frac{d^3r_2}{(2\pi)^6}\int \frac{d\lambda}{2\pi}f(k_1^2)f(k_2^2)f(k_1^2 + \Delta)(E_k + \Delta)(E_k + E_{k_2}). \quad (11)$$

As commonly used, we take a Gaussian form for the form factor $f(k_i^2) = \exp(-k_i^2/\Lambda^2), i = 1, 2$, which regulates the integral with the cutoff parameter $\Lambda$. In Eq. (11) $E_{k_i} = \sqrt{k_i^2 + m_\pi^2}, m_\pi$ being the pion mass. Further, using the integral factorization technique, we get the nonlocal potential

$$V_B(r_1, r_2) = -\left(\frac{g_1^2}{f^3}\right)\frac{1}{\pi}O_1(k_1, k_2)\left[\int_0^{\infty} \frac{d\lambda}{\lambda^2 + \lambda^2}F(\lambda, r_1)\int_0^{\infty} \frac{d\lambda}{\lambda^2 + \lambda^2}F(\lambda, r_2)\right]. \quad (12)$$

![FIG. 1. 2π-exchange diagrams: (a) Box diagram, (b) crossed diagram, (c) triangle diagram, (d) two-pion-loop diagram.](image-url)
where $\Delta = M_{\Sigma} - M_{\Lambda}$, with $\Sigma$ being either $\Sigma^+_c$ or $\Sigma^{'+}_{c}$ as the intermediate state in the $\Lambda_{c}^+ - \Lambda_{c}^+$ interaction, and the form of $F(\lambda, r)$ can be found in Ref. [8].

In the same way, we can calculate the potential of the crossed diagram and get the nonlocal potential

$$V_C(r_1, r_2) = -\left(\frac{g_s^4}{f_\pi^2}\right) \frac{1}{\pi} \mathcal{O}_1(k_1, k_2) \int_0^\infty d\lambda \frac{\Delta^2 - \lambda^2}{(\Delta^2 + \lambda^2)^2} F(\lambda, r_1)F(\lambda, r_2).$$

(13)

Similarly, we obtain for the nonlocal potential of the triangle diagram

$$V_{\text{triangle}}(r_1, r_2) = \frac{g_s^2}{2f_\pi^2} \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \mathcal{O}_i(k_1, k_2)(E_{k_1} + E_{k_2})e^{ik_1 r_1}e^{ik_2 r_2} f(k_1^2) f(k_2^2),$$

(14)

and the $2\pi$-loop diagram potential

$$V_{2\pi\text{-loop}}(r_1, r_2) = \frac{1}{16f_\pi^4} \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} e^{ik_1 r_1} e^{ik_2 r_2} f(k_1^2) f(k_2^2) A,$$

(15)

where $A = -\frac{1}{4r_{k_1}} - \frac{1}{4r_{k_2}} + \frac{1}{4r_{k_1} + 4r_{k_2}}$. In the above potentials, the operators $\mathcal{O}_i(k_1, k_2)$ with $i = 1, 2$ come from the nonrelativistic reduction for the interaction vertices. Their general forms are

$$\mathcal{O}_1(k_1, k_2) = c_1(k_1 \cdot k_2)^2 + c_2(\sigma_1 \cdot k_1 \times k_2)(\sigma_2 \cdot k_1 \times k_2),$$

(16)

$$\mathcal{O}_2(k_1, k_2) = (k_1 \cdot k_2).$$

(17)

Note that in the right side of Eq. (16), the first term will generate a central potential and the second term will produce a spin-spin potential and a tensor potential.

Finally, acting $\mathcal{O}_i(k_1, k_2)$ onto the concerned channel, making a local approximation and working out a detailed derivation, for the box diagram, we obtain a central potential

$$V_{BC}(r) = -\left[\frac{1}{\pi} \int_0^\infty \frac{d\lambda}{\Delta^2 + \lambda^2} F_C(\lambda, r) - \frac{4\Delta}{\pi} \left(\int_0^\infty \frac{d\lambda}{\Delta^2 + \lambda^2} F'(\lambda, r)\right)^2 - \frac{2\Delta}{\pi^2} \left(\int_0^\infty \frac{d\lambda}{\Delta^2 + \lambda^2} F''(\lambda, r)\right)^2\right].$$

(18)

a spin-spin potential

$$V_{BS}(r)(\sigma_1 \cdot \sigma_2) = -\left[\frac{2}{3\pi} \int_0^\infty \frac{d\lambda}{\Delta^2 + \lambda^2} F_S(\lambda, r) - \frac{4\Delta}{3\pi^2} \left(\frac{1}{r} \int_0^\infty \frac{d\lambda}{\Delta^2 + \lambda^2} F'(\lambda, r) \int_0^\infty \frac{d\beta}{\Delta^2 + \beta^2} F'(\beta, r)\right)\right] (\sigma_1 \cdot \sigma_2),$$

(19)

and a tensor potential

$$V_{BT}(r)S_{12} = -\left[\frac{2}{3\pi} \int_0^\infty \frac{d\lambda}{\Delta^2 + \lambda^2} F_T(\lambda, r) - \frac{4\Delta}{3\pi^2} \left(\frac{1}{r} \int_0^\infty \frac{d\lambda}{\Delta^2 + \lambda^2} F'(\lambda, r) \int_0^\infty \frac{d\beta}{\Delta^2 + \beta^2} F'(\beta, r)\right)\right] S_{12},$$

(20)

For the crossed diagram, the central, spin-spin, and tensor potentials are

$$V_{CC}(r) = -\left[\frac{1}{\pi} \int_0^\infty \frac{d\lambda}{\Delta^2 + \lambda^2} F_C(\lambda, r)\right].$$

(21)

$$V_{CS}(r) = -\left[\frac{1}{\pi} \int_0^\infty \frac{d\lambda}{\Delta^2 + \lambda^2} F_S(\lambda, r)\right].$$

(22)

$$V_{CT}(r) = -\left[\frac{1}{\pi} \int_0^\infty \frac{d\lambda}{(\Delta^2 + \lambda^2)^2} F_T(\lambda, r)\right].$$

(23)

respectively. In the above equations [(18) to (23)] one has

$$F_C(\lambda, r) = F'(\lambda, r)F'(\lambda, r) + F''(\lambda, r)F''(\lambda, r),$$

(24)

$$F_S(\lambda, r) = \frac{F'(\lambda, r)F'(\lambda, r)}{r} + 2F''(\lambda, r),$$

(25)
\[ F_T(\lambda, r) = \frac{F'((\lambda, r))}{r} \left( \frac{F((\lambda, r))}{r} - F''((\lambda, r)) \right). \]  

(26)

For the triangle diagram, at the order of \(O(1/m_H)\) (\(M_H\) denoting the heavy \(\Lambda_c\) or \(\Sigma_c^*\) baryon mass), we have only a central potential

\[ V_{TC}(r) = \frac{4\Delta}{\pi} \int_0^\infty d\lambda \frac{\lambda^2}{\Delta^2 + \lambda^2} F'((\lambda, r)) \times \int_0^\infty d\lambda \frac{\lambda}{\Delta^2 + \lambda^2} F((\lambda, r)). \]  

(27)

Similarly, for the \(2\pi\)-loop diagram, only a central potential contributes

\[ V_{2\pi\text{-loop}}(r) = -\frac{2}{\pi} \left[ \int_0^\infty d\lambda F((\lambda, r)) \right. \times \left( \frac{\lambda^3}{8\pi} \exp \left( -\frac{1}{4}\lambda^2 r^2 \right) - 2\lambda^2 F((\lambda, r)) \right]. \]  

(28)

Summing up all the potentials, we eventually obtain the two-pion-exchange potential for the heavy baryon–anti-heavy baryon interaction

\[ V(r) = V_C(r) + V_S(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T(r)S_{12}. \]  

(29)

where \(V_C(r), V_S(r),\) and \(V_T(r)\) are the radial parts of the central, spin-spin, and tensor potentials, respectively. From the above potential forms, we see that the longest range of the obtained potentials is, as expected, that of the two-pion exchange, because they have a quadratic product of \(F(\lambda, r)\) (or derivatives), and thus have their longest range terms proportional to \(\exp(-2m_\pi r)\). In addition, we would point out that because the kernels in a different channel are the same except the coefficients, for simplicity, we can derive the kernel itself first, and then add the coefficient later for the particular system.

Now, we go to specific systems.

**A. \(\Lambda_c^+ - \bar{\Lambda}_{c}^+\) potential**

In the \(\Lambda_c^+ - \bar{\Lambda}_{c}^+\) interaction, we assume that both \(\Sigma_c^+\) and \(\Sigma^*_c\) could be the intermediate state. The Lagrangian for the spin-\(1/2\) \(\Sigma_c^* - \pi - \Lambda_c\) interaction reads

\[ L_{\Sigma_c^* - \pi - \Lambda_c} = -\frac{g_2^2}{f_\pi} \Sigma_c^{+,+,0} \gamma^\mu \gamma_5 \partial_\mu \left( \pi^{+,0} - \Lambda_c^+ \right) + \text{H.c.} \]  

(30)

where the strong coupling constant \(g_2\) can be extracted from the \(\Sigma_c^+ \rightarrow \Lambda_c^+ + \pi^+\) decay process (see Fig. 2) by

\[ \Gamma = \frac{g_2^2 |\mathbf{k}|}{8f_\pi^2 M_{\Sigma_c^*}} (M_{\Sigma_c^*}^2 + M_{\Lambda_c^+}^2) [(M_{\Sigma_c^*} - M_{\Lambda_c^+})^2 - m_\pi^2]. \]  

(31)

where \(|\mathbf{k}| = 94\text{ MeV}\) is the momentum of the pion in the \(\Sigma_c^*\) rest frame, \(f_\pi = 0.132\text{ GeV}, M_{\Sigma_c^*} = 0.245\text{ GeV}, M_{\Lambda_c^+} = 0.229\text{ GeV},\) and \(\Gamma = 2.23 \pm 0.30\text{ MeV}\) [16].

The resultant phenomenological coupling constant is \(g_2 = 0.5 \pm 0.07\).

Based on the Lagrangian in Eq. (30), we have an explicit form of \(O_j(\mathbf{k}_1, \mathbf{k}_2)\) for both box and crossed diagrams

\[ O_j(\mathbf{k}_1, \mathbf{k}_2) = (\mathbf{k}_1 \cdot \mathbf{k}_2)^2 + (\mathbf{\sigma}_1 \cdot \mathbf{k}_1 \times \mathbf{k}_2)(\mathbf{\sigma}_2 \cdot \mathbf{k}_1 \times \mathbf{k}_2). \]  

(32)

It leads to a \(\Lambda_c^+ - \bar{\Lambda}_{c}^+\) potential, caused by the \(2\pi\) exchange with \(\Sigma_c^*\) as the intermediate state,

\[ V_{1\Lambda_c^+ - \bar{\Lambda}_{c}^+}(r) = \frac{g_4^2}{f_\pi^2} \left[ V_{BC}(r) + V_{CC}(r) + V_{TC}(r) \right. \]  

\[ + \frac{1}{f_\pi} V_{2\pi\text{-loop}}(r) + \frac{g_4^2}{f_\pi^2} \left. \left[ V_{BS}(r) + V_{CS}(r) \right] (\mathbf{\sigma}_1 \cdot \mathbf{\sigma}_2) + \frac{g_4^2}{f_\pi^2} \left[ V_{BT}(r) + V_{CT}(r) \right] S_{12} \right]. \]  

(33)

The Lagrangian for the spin-\(3/2\) \(\Sigma_c^* - \pi - \Lambda_c\) interaction can be written as

\[ L_{\Sigma_c^* - \pi - \Lambda_c} = -\frac{g_4^2}{f_\pi^2} (\Sigma_c^{++,+,0})\gamma^\mu \gamma_5 \partial_\mu \left( \pi^{+,0} - \Lambda_c^+ \right) + \text{H.c.} \]  

(34)

Similarly, the coupling constant \(g_4\) can be extracted from the \(\Sigma_c^* \rightarrow \Lambda_c + \pi\) decay process (see Fig. 3) by

\[ \Gamma = \frac{g_4^2 |\mathbf{k}|}{8f_\pi^2 M_{\Sigma_c^*}} (M_{\Sigma_c^*}^2 + M_{\Lambda_c}^2) [(M_{\Sigma_c^*} - M_{\Lambda_c})^2 - m_\pi^2]\]  

(35)

\[ + \frac{1}{f_\pi} \left( \Sigma_c^{++,+,0} \right) \gamma_5 \partial_\mu \left( \pi^{+,0} - \Lambda_c^+ \right) + \text{H.c.}. \]  

(36)
\[
\Gamma = \frac{g_5^2|k| M_{V_{\Sigma}}^2}{96 f_{\pi}^2 \Sigma} \left[ \left( 1 - \frac{M_{\Lambda_{c}}} {M_{\Sigma}} \right)^2 - \frac{m_{\pi}^2} {M_{\Sigma}^2} \right]
\times \left[ \left( 1 + \frac{M_{\Lambda_{c}}} {M_{\Sigma}} \right)^2 - \frac{m_{\pi}^2} {M_{\Sigma}^2} \right]^2,
\]

with \(|k| = 180 \text{ MeV}\) being the momentum of the pion in the rest frame of \(\Sigma_{c}^{+}\), \(M_{\Sigma_{c}^{+}} = 0.252 \text{ GeV}\), \(M_{\Lambda_{c}} = 0.229 \text{ GeV}\), and \(\Gamma = 14.9 \pm 1.9 \text{ MeV}\) [16]. From Eq. (35) we get \(g_{4} = 0.57 \pm 0.07\). The values of the coupling constants \(g_{2}\) and \(g_{4}\) calculated from the \(\Sigma_{c}\) decays, respectively, do not satisfy the symmetry relation given in Eq. (10). This implies that the heavy quark symmetry is broken.

Similar to the above case, using Lagrangian in Eq. (34) we can explicitly write out \(O_{1}(k_{1}, k_{2})\) for the box diagram as

\[
O_{1}(k_{1}, k_{2}) = \frac{4}{9} (k_{1} \cdot k_{2})^2 - \frac{1}{9} (\sigma_{1} \cdot k_{1}) (\sigma_{2} \cdot k_{1}) (\sigma_{1} \cdot k_{2}),
\]

and for the crossed diagram as

\[
O_{1}(k_{1}, k_{2}) = \frac{4}{9} (k_{1} \cdot k_{2})^2 + \frac{1}{9} (\sigma_{1} \cdot k_{1}) (\sigma_{2} \cdot k_{1}) (\sigma_{1} \cdot k_{2}).
\]

These lead to a \(\Lambda_{c}^{+} - \bar{\Lambda}_{c}^{+}\) potential arising from the 2\(\pi\) exchange with \(\Sigma_{c}^{*}\) as the intermediate state,

\[
V_{2\Lambda_{c}^{+} \bar{\Lambda}_{c}^{+}}(r) = \frac{4 g_{4}^2}{9 f_{\pi}^2} [V_{BC}(r) + V_{CC}(r)] + \frac{2 g_{4}^2}{3 f_{\pi}^2} V_{TC}(r)
\]

\[
+ \frac{g_{4}^2}{9 f_{\pi}^2} [-V_{BS}(r) + V_{CS}(r)] (\sigma_{1} \cdot \sigma_{2})
+ \frac{g_{4}^2}{9 f_{\pi}^2} [-V_{BT}(r) + V_{CT}(r)] S_{12}.
\]

Putting the contributions given in Eqs. (32) and (37) together, we finally obtain the \(\Lambda_{c}^{+} - \bar{\Lambda}_{c}^{+}\) potential

\[
V_{\Lambda_{c}^{+} \bar{\Lambda}_{c}^{+}}(r) = V_{1\Lambda_{c}^{+} \bar{\Lambda}_{c}^{+}}(r) + V_{2\Lambda_{c}^{+} \bar{\Lambda}_{c}^{+}}(r).
\]

**B. \(\Sigma_{c}^{0} - \bar{\Sigma}_{c}^{0}\) potential**

In the \(\Sigma_{c}^{0} - \bar{\Sigma}_{c}^{0}\) interaction, both the one-pion exchange and the two-pion exchange are allowed. In the one-pion-exchange case, the Lagrangian of the \(\Sigma_{c}^{0} - \pi - \Sigma_{c}^{0}\) interaction can be written as

\[
\mathcal{L}_{\Sigma_{c}^{0} - \pi - \Sigma_{c}^{0}} = - \frac{g_{1}}{\sqrt{2} f_{\pi}} \Sigma_{c}^{0} \gamma^{\mu} \gamma_{5} \partial_{\mu} \pi^{0} - \Sigma_{c}^{0}. \tag{40}
\]

The axial current interaction (one-pion exchange) leads to a spin-spin potential

\[
V_{\text{ops}}(r)(\sigma_{1} \cdot \sigma_{2})
= - \frac{g_{1}^2}{3 f_{\pi}^2} \left[ I_{2}^0(m_{\pi}, r) + \frac{1}{r} I_{2}^1(m_{\pi}, r) \right] (\sigma_{1} \cdot \sigma_{2}), \tag{41}
\]

and to a tensor potential

\[
V_{\text{opt}}(r) S_{12} = - \frac{g_{1}^2}{3 f_{\pi}^2} \left[ I_{2}^0(m_{\pi}, r) - \frac{1}{r} I_{2}^1(m_{\pi}, r) \right] S_{12}. \tag{42}
\]

where the function \(I_{2}(m_{\pi}, r)\) is given in the Appendix of Ref. [8]: \(I_{2}^0(r)\) and \(I_{2}^1(r)\) are the first and second order derivatives of \(I_{2}(r)\).

\[
V_{1\Sigma_{c}^{0} \bar{\Sigma}_{c}^{0}}(r) = V_{\text{ops}}(r)(\sigma_{1} \cdot \sigma_{2}) + V_{\text{opt}}(r) S_{12}. \tag{43}
\]

In the two-pion-exchange case, both spin-1/2 \(\Sigma_{c}^{0}\) and \(\Lambda_{c}^{+}\) can be the intermediate state. So, we have the spin-1/2 intermediate state potential

\[
V_{2\Sigma_{c}^{0} \bar{\Sigma}_{c}^{0}}(r) = \left[ \left\{ \frac{g_{1}^4}{4 f_{\pi}^2} + \frac{g_{4}^2}{f_{\pi}^2} \right\} (V_{BC}(r) + V_{CC}(r)) \right.
\]

\[
+ \left\{ \frac{g_{1}^2}{2 f_{\pi}^2} + \frac{g_{4}^2}{f_{\pi}^2} \right\} V_{TC}(r) + \frac{1}{f_{\pi}^2} V_{2\pi-\text{loop}}(r) \right]
\]

\[
\left. + \left\{ \frac{g_{1}^2}{2 f_{\pi}^2} + \frac{g_{4}^2}{f_{\pi}^2} \right\} (V_{BS}(r) + V_{CS}(r))(\sigma_{1} \cdot \sigma_{2}) \right]
\]

\[
+ \left\{ \frac{g_{1}^2}{2 f_{\pi}^2} + \frac{g_{4}^2}{f_{\pi}^2} \right\} (V_{BT}(r) + V_{CT}(r)) S_{12}. \tag{44}
\]

where \(g_{1}\) stands for the coupling constant in the case where spin-1/2 \(\Sigma_{c}^{0}\) is an intermediate state.

Moreover, the spin-3/2 \(\Sigma_{c}^{*}\), as an intermediate state, would also contribute. The Lagrangian of the \(\Sigma_{c}^{0} - \pi - \Sigma_{c}^{0}\) interaction reads

\[
\mathcal{L}_{\Sigma_{c}^{0} - \pi - \Sigma_{c}^{0}} = - \frac{g_{3}}{\sqrt{2} f_{\pi}} \Sigma_{c}^{0} \mu \partial_{\mu} \pi^{0} \Sigma_{c}^{0}, \tag{45}
\]

with \(g_{3}\) being the coupling constant. Based on this Lagrangian, following the same procedure as that used above, we have the following potential:

\[
V_{2\Sigma_{c}^{0} \bar{\Sigma}_{c}^{0}}(r) = \frac{g_{3}^4}{9 f_{\pi}^2} (V_{BC}(r) + V_{CC}(r)) + \frac{g_{3}^2}{3 f_{\pi}^2} V_{TC}
\]

\[
+ \frac{g_{3}^4}{36 f_{\pi}^2} [-V_{BS}(r) + V_{CS}(r)] (\sigma_{1} \cdot \sigma_{2})
\]

\[
+ \frac{g_{3}^4}{36 f_{\pi}^2} [-V_{BT}(r) + V_{CT}(r)] S_{12}. \tag{46}
\]

It should be mentioned that in solving the Schrödinger equation with the one-\(\pi^{0}\) exchange, one would generate in an automatic way a nonrelativistic 2\(\pi\)-exchange box potential with intermediate \(\Sigma_{c}^{0}\) states.
in which \(O_3(\mathbf{k}_1, \mathbf{k}_2) = (\sigma_1 \cdot \mathbf{k}_1 \times \mathbf{k}_2)(\sigma_2 \cdot \mathbf{k}_1 \times \mathbf{k}_2)\) and \(W = E(\mathbf{k}_1) + E(\mathbf{k}_2)\). Completing the integration in the \(\mathbf{r}_2 \to \mathbf{r}_1\) limit, one obtains the spin-spin and tensor terms of such a potential in the configuration space in the forms of

\[
V^{ll}(r)(\mathbf{r}_1 \cdot \mathbf{r}_2) = -\frac{g_s^4}{6f_{\pi}^4} \left[ \frac{1}{r} I_2^s(r)I_3^t(r) + \frac{1}{r^2} (I_2^t(r)I_3^s(r) + I_2^s(r)I_3^t(r)) \right]
\times (\mathbf{r}_1 \cdot \mathbf{r}_2),
\]

and

\[
V^{ll}(r)S_{12} = -\frac{g_s^4}{12f_{\pi}^4} \left[ \frac{1}{r} I_2^s(r) \left( \frac{1}{r} I_3^t(r) - I_3^s(r) \right) + \frac{1}{r^2} (I_2^t(r) - I_2^s(r)) \right] S_{12},
\]

in which \(I_2^s(r)\) and \(I_3^t(r)\) are the first and second order derivatives of \(I_3^s(r)\), respectively, with

\[
I_3^s(r) = \frac{2}{\pi} \int_0^\infty d\lambda \left( I_2(m_\pi, r) - e^{-\lambda/\Lambda^2} I_2(\sqrt{m_\pi^2 + \lambda^2}, r) \right).
\]

Taking this effect into account to avoid the double counting [15,17], one should write the \(\Sigma^0_c - \Sigma^0_c\) potential as

\[
V_{\Sigma^0_c - \Sigma^0_c}(r) = V_{1\Sigma^0_c - \Sigma^0_c}(r) + V_{2\Sigma^0_c - \Sigma^0_c}(r) + V_{3\Sigma^0_c - \Sigma^0_c}(r) - V^{ll}(r).
\]

It should also be mentioned that the spin-1/2 \(\Sigma^0_c\) and spin-3/2 \(\Sigma^c\) heavy baryons have both isospin 1. Consequently, for the \(2\pi\)-exchange box and crossed diagrams, there could be mixture of spin-1/2 \(\Lambda_c^+\), spin-1/2 \(\Sigma_c\), and spin-3/2 \(\Sigma^c\) intermediate states, i.e., additional potential contributions proportional to coupling constants, \(g_1^2 g_2^2, g_1^2 g_3^2\) and \(g_2^2 g_3^2\). Since for the box and crossed diagrams, the forms of these potentials are the same as those in Eqs. (12) and (13), respectively, except the coupling constant, we do not give their expression here for simplicity, but do include them in the numerical calculation.

### C. \(\Lambda^0_b - \bar{\Lambda}_b^0\) potential

The same formulas can also be applied to the \(\Lambda^0_b - \bar{\Lambda}_b^0\) interaction except that the \(c\)-flavored heavy baryon (antibaryon) is replaced by the \(b\)-flavored heavy baryon (antibaryon).

### IV. NUMERICAL RESULT AND DISCUSSION

In the numerical calculation, we take \(m_\pi = 0.135\) GeV and \(f_\pi = 0.132\) GeV. We also choose the cutoff parameter \(\Lambda = 0.6\)–1.2 GeV, because in the chiral perturbation theory, the momentum transfer is usually less than 1.0 GeV.

In the \(\Lambda_c^+ - \bar{\Lambda}_c^+\) system, the averaged mass difference between \(\Sigma^0_c\) and \(\Lambda_c\) is about \(\Delta = 0.234\) GeV. The resultant potentials for the spin-singlet and spin-triplet \(\Lambda_c - \bar{\Lambda}_c\) states are plotted in Figs. 4 and 5, respectively. From these figures, we see that comparing with our previous result [8], no matter in which states, the singular behavior of the potential around the origin is greatly reduced. This indicates that the contribution from large \(\Lambda\) values is also important in the two-pion-exchange process. Moreover, the potentials become more attractive with increasing values of \(g_2\) and \(\Lambda\). This is reasonable because the larger \(g_2\) value provides stronger coupling and consequently stronger potential. And the value of the cutoff \(\Lambda\) largely affects the depth of the potential, the smaller value of \(\Lambda\) makes the shorter distance interaction even more suppressed. It partly prevents the \(\Lambda_c\) and \(\bar{\Lambda}_c\) from getting too close, and thus matches our treatment of omitting the \(s\)-channel interaction. The line shape of these potentials also tells us that the interaction between \(\Lambda_c\) and \(\bar{\Lambda}_c\) is attractive and might bind these particles together.

**FIG. 4.** The \(\Lambda_c - \bar{\Lambda}_c\) potential in the singlet state with different \(g_2\) but fixed \(\Lambda\) (left figure) and different \(\Lambda\) but fixed \(g_2\) (right figure).
With these potentials, we can study the $\Lambda_c^-\bar{\Lambda}_c$ scattering property. The partial wave Schrödinger equation that the $\Lambda_c^-\bar{\Lambda}_c$ scattering obeys reads

$$\frac{d^2u_l(r)}{dr^2} + \left[ k^2 - \frac{l(l+1)}{r^2} - U(r) \right] u_l(r) = 0,$$

with the boundary condition

$$u_l(r) = krj_l(kr) + \int_0^\infty G_l(r, r')U(r')u_l(r')dr',$$

where $G_l(r, r')$ is the Green function in the form of

$$G_l(r, r') = krr'j_l(kr)n_l(kr'),$$

and $U(r) = 2\mu V(r)$ with the reduced mass $\mu$ being $M_{\Lambda_c}/2$. The functions $j_l(kr)$ and $n_l(kr)$ are the spherical Bessel function and the spherical Neumann function, respectively [18]. Then, the scattering phase shift $\delta_l(k)$ and the potential $V(r)$ have the relation

$$\tan \delta_l(k) = -\int_0^\infty r'j_l(kr')U(r')u_l(r')dr'.$$

Solving the above equations numerically, we obtain scattering phase shifts and plot them in Fig. 6. From this figure, one sees that although all the potentials are attractive, in some cases, the potential does not support a binding character, especially in the case where the value of $g_2$ is extracted from the $\Sigma_c^-\rightarrow \Lambda_c^-\pi$ decay data (the thick solid curve). It means that the $\Lambda_c^-\bar{\Lambda}_c$ could be bound (the dashed curve) only when the coupling constant $g_2$ takes a value larger than that from the data fitting, and the cutoff $\Lambda$ is larger than that in the light baryon sector.

FIG. 5. The $\Lambda_c^-\bar{\Lambda}_c$ potential in the triplet state with different $g_2$ but fixed $\Lambda$ (left figure) and different $\Lambda$ but fixed $g_2$ (right figure).

FIG. 6. Phase shifts of the $\Lambda_c^-\bar{\Lambda}_c$ system in the spin-singlet (left figure) and the spin-triplet (right figure) states.

### TABLE I. Scattering length for the spin-singlet and spin-triplet states in the $\Lambda_c^-\bar{\Lambda}_c$ and $\Sigma_c^-\bar{\Sigma}_c$ systems.

<table>
<thead>
<tr>
<th>$S = 0$ state for $\Lambda_c^-\bar{\Lambda}_c$</th>
<th>$S = 1$ state for $\Lambda_c^-\bar{\Lambda}_c$</th>
<th>$S = 0$ state for $\Sigma_c^-\bar{\Sigma}_c$</th>
<th>$S = 1$ state for $\Sigma_c^-\bar{\Sigma}_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_2$ (GeV)</td>
<td>$\Lambda$ (GeV)</td>
<td>$a$ (fm)</td>
<td>$g_1$ (GeV)</td>
</tr>
<tr>
<td>0.95</td>
<td>0.9</td>
<td>3.5</td>
<td>0.85</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7</td>
<td>-2.7</td>
<td>0.7</td>
</tr>
<tr>
<td>0.85</td>
<td>1.1</td>
<td>3.9</td>
<td>0.95</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>-2.1</td>
<td>0.8</td>
</tr>
</tbody>
</table>
The results are tabulated in Table I.

The scattering lengths in this table also tell us that only those \( g_2 \) and \( \Lambda \) values, with which the attractive potential is much stronger (denoted by dashed curve in Figs. 4 and 5), can produce an appropriate positive scattering length, which denotes a bound \( \Lambda - \Lambda_c \) system; otherwise the system is unbound.

Based on the enlightenment from the scattering study, we further perform the bound state calculation to check the condition for forming a \( \Lambda - \Lambda_c \) bound state. Since we have the spin-spin interaction in the \( \Lambda_c - \Lambda_c \) potential, Eq. (39), substituting such a potential into the Schrödinger equation and solving the equation numerically, we get the binding energy for the spin split \( S = 0 \) and \( S = 1 \) states, respectively. The results are tabulated in Table II.

From this table, we see that with the extracted \( g_2 \) value of 0.5–0.57 from the decay data, the \( \Lambda_c - \Lambda_c \) system would not be bound. If we wish \( \Lambda_c \) and \( \Lambda_c \) to be bound, no matter in the spin-singlet state or the spin-triplet state, the coupling constant should be much larger than the value extracted phenomenologically, namely, \( g_2 > 0.8 \) for the spin-singlet state and \( g_2 > 0.78 \) for the spin-triplet state.

This is in coincidence with those learned from above scattering study. The result also shows the required ranges of \( g_2 \) and \( \Lambda \) for \( \Lambda_c - \Lambda_c \) binding: \( 0.8 < g_2 \leq 1.1 \) and \( 0.8 \text{ GeV} < \Lambda \leq 1.0 \text{ GeV} \) for the spin-singlet state and \( 0.7 < g_2 \leq 1.0 \) and \( 0.7 \text{ GeV} < \Lambda \leq 0.95 \text{ GeV} \) for the spin-triplet state, respectively. The mass of the corresponding baryonium is in the region of \((4.406, 4.572) \text{ GeV}\) and \((4.287, 4.572) \text{ GeV}\) for the spin-singlet and spin-triplet states, respectively, where 4.572 GeV is the threshold of \( \Lambda_c \) system; otherwise the system is unbound. Since we have

\[
\Delta V = \sum_{i=1}^{N} -\frac{e^2}{r_i} \cos \theta_i
\]

The potentials between the system are plotted in Fig. 9. Again, the system in some cases could be bound (dashed curve) and in the other cases would be unbound (solid curve). However, due to lack of experimental data to fix the \( g_1 \) value, it is necessary to examine the marginal condition for its binding.

Same as before, substituting the obtained \( \Sigma_c - \Sigma_c \) potential, Eq. (51), into the Schrödinger equation and solving it

\[
\alpha = -\lim_{k \to 0} \frac{\tan \delta(k)}{k}
\]

The results are tabulated in Table II.

Moreover, we can also calculate the scattering length for concerned states by

[Table II. Binding energies (BE), as well as the masses of heavy baryonium (\( M_{\Lambda_c \Lambda_c} \)), in the \( \Lambda_c - \Lambda_c \) system in various parameter cases.]

| \( |g_2| \) | \( \Lambda \) (GeV) | BE (MeV) | \( M_{\Lambda_c \Lambda_c} \) (GeV) | \( |g_2| \) | \( \Lambda \) (GeV) | BE (MeV) | \( M_{\Lambda_c \Lambda_c} \) (GeV) |
|---|---|---|---|---|---|---|---|
| <0.8 | <0.8 | \( \cdots \) | \( \cdots \) | <0.7 | <0.7 | \( \cdots \) | \( \cdots \) |
| 0.9 | 0.9 | 34 | 4.538 | 0.9 | 0.85 | 75 | 4.497 |
| 0.9 | 1.0 | 118 | 4.45 | 0.9 | 0.95 | 285 | 4.287 |
| 0.8 | 0.9 | 3.25 | 4.568 | 0.8 | 0.85 | 14 | 4.558 |
| 1.1 | 0.9 | 166.2 | 4.406 | 1.0 | 0.85 | 199 | 4.373 |

FIG. 7. \( \Sigma_c - \Sigma_c \) potential in the spin-singlet state.
numerically, we have the binding character for the $\Sigma_c\bar{\Sigma}_c$ system. The resultant binding energies for the $S = 0$ and $S = 1$ states are tabulated in Table III.

From this table, we find that as long as $g_1 > 0.8$ and $\Lambda > 1.0$ GeV in the spin-singlet state and $g_1 > 0.9$ and $\Lambda > 1.0$ GeV in the spin-triplet state, the $\Sigma_c\bar{\Sigma}_c$ system could be bound. And also the spin-triplet state is slightly easier to be bound than the spin-triplet state. The result also shows the required ranges of $g_1$ and $\Lambda$ for the $\Sigma_c\bar{\Sigma}_c$ binding: $0.8 < g_1 \leq 0.85$ and $0.95$ GeV $\leq \Lambda \leq 1.1$ GeV for the spin-singlet state and $0.8 < g_1 \leq 0.9$ and $0.95$ GeV $< \Lambda \leq 1.1$ GeV for the spin-triplet state, respectively. The mass of the corresponding baryonium is in the region of (4.853, 4.910] and (4.875, 4.910] GeV for the spin-singlet and spin-triplet states, respectively, where 4.910 GeV is the threshold of the $\Sigma_c\bar{\Sigma}_c$ channel.

The similar study can be done for the systems with the bottom flavor. In the $\Lambda_b^-\bar{\Lambda}_b^+$ system, the averaged mass difference between $\Sigma_b^*$ and $\Lambda_b$ is about $\Delta = 0.114$ GeV. With the same reason in the charm flavor sector, namely, due to lack of the experimental data to fix the $g_b$ value, we also examine the marginal condition for its binding. Carrying out the same procedure, we obtain the binding character of the $\Lambda_b^-\bar{\Lambda}_b^+$ system.

<table>
<thead>
<tr>
<th>$S = 0$ state</th>
<th>$S = 1$ state</th>
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<tbody>
<tr>
<td>$g_1$</td>
<td>$\Lambda$ (GeV)</td>
</tr>
<tr>
<td>$&lt;0.8$</td>
<td>$&lt;0.95$</td>
</tr>
<tr>
<td>0.85</td>
<td>1</td>
</tr>
<tr>
<td>0.8</td>
<td>1.1</td>
</tr>
<tr>
<td>0.85</td>
<td>1.1</td>
</tr>
</tbody>
</table>
binding energies in the $S = 0$ and $S = 1$ states are tabulated in Table IV. The result shows that for the $\Lambda_b - \bar{\Lambda}_b$ system, a relatively smaller $g_b$ value can make the spin-triplet state bound. The result also presents the required ranges of $g_b$ and $\Lambda$ for the $\Lambda_b - \bar{\Lambda}_b$ binding: $0.65 \leq g_b \leq 0.8$ and $0.8 \leq \Lambda \leq 0.9$ GeV for the spin-singlet state and $0.55 \leq g_b \leq 0.6$ and $0.85 \leq \Lambda \leq 0.9$ GeV for the spin-triplet state, respectively. The mass of the corresponding baryonium is in the region of $(11.21, 11.24]$ GeV for the spin-singlet state and $(11.2, 11.24]$ GeV for the spin-triplet state, respectively. The mass of the corresponding baryonium is in the region of $(11.21, 11.24]$ GeV for the spin-singlet state and $(11.2, 11.24]$ GeV for the spin-triplet state, respectively. The mass of the corresponding baryonium is in the region of $(11.21, 11.24]$ GeV for the spin-singlet state and $(11.2, 11.24]$ GeV for the spin-triplet state, respectively. The mass of the corresponding baryonium is in the region of $(11.21, 11.24]$ GeV for the spin-singlet state and $(11.2, 11.24]$ GeV for the spin-triplet state, respectively.

Using lattice QCD, the value of $g_b$ has recently been calculated in Ref. [19]. They give $|g_b| = 0.475 \pm 0.050$ for the $\Sigma_b^0 - \pi - \Lambda_b$ coupling. This value seems too small to support a bound $\Lambda_b - \bar{\Lambda}_b$ state. However, the final conclusion should not be made before some issues are clarified, like, whether or not the $\Sigma_b^0 - \pi - \Lambda_b$ coupling can be straightforwardly applied to the loop calculation. When future decay data of the $b$-flavored baryon become available, we would be able to extract a physical value of $g_b$. If the extracted $g_b$ is consistent with the marginal $g_1$ value for binding in this calculation, one might confirm such a $b$-flavored heavy baryonium.

### V. Conclusion

The heavy baryon–anti-heavy baryon systems are studied in the framework of heavy baryon chiral perturbation theory. The potentials for the $\Lambda_c - \bar{\Lambda}_c$, $\Sigma_c - \bar{\Sigma}_c$, and $\Lambda_b - \bar{\Lambda}_b$ interactions are derived with the two-pion-exchange mechanism. Unlike our previous work, we use the holonomic potential to investigate the scattering and binding characters in this paper. The scattering characters of these systems are calculated by solving the partial Schrödinger equation. From the obtained phase shifts and the scattering lengths, it is found that the $\Lambda_c - \bar{\Lambda}_c$ system could be bound with a $g_2$ value larger than that extracted phenomenologically from the decay data of charmed baryons or estimated by Ref. [13]. For the $\Sigma_c - \bar{\Sigma}_c$ system, since we do not have available decay data to fix $g_1$, whether the system is bound depends on the selected value of $g_1$. To confirm these results, the bound state calculations are further performed. It is shown that marginal $g_2$ value for binding is about 0.8 which is larger than the physical value of about 0.5–0.57. In the $\Sigma_c - \bar{\Sigma}_c$ system, the marginal $g_1$ value for binding is also estimated. The minimum $g_1$ value is about 0.85. This value should be compared with that extracted from the future data to affirm whether the $\Sigma_c - \bar{\Sigma}_c$ system could be bound. The $\Lambda_b - \bar{\Lambda}_b$ system is studied as well. It is found that the minimum $g_b$ value for binding is much smaller than that for the $\Lambda_c - \bar{\Lambda}_c$ system. If the $g_b$ value extracted from the future decay data of the $b$-flavored baryon can meet this value, one might confirm such a $\Lambda_b - \bar{\Lambda}_b$ heavy baryonium.

It should be mentioned that the above conclusions are also related to the cutoff value which is assumed to be similar to that for the light hadron sector in chiral perturbation theory. A similar situation was met in Ref. [20], where a relative large cutoff is also required for a possible molecule state in the $\Lambda_c - \bar{\Lambda}_c$ system in the one-pion-exchange model. Furthermore, it is worthwhile to emphasize that in order to more realistically affirm whether the heavy baryon–anti-heavy baryon system could have a bound state, namely, a heavy baryonium, the annihilation channel and coupled channel effects on the heavy baryonium potential should also be taken into account. In particular, a study in the quark-gluon degree of freedom is necessary.

Even if some corrections should be further considered, our results are much more reliable and stable than those in our early calculation [8]. From the regions of possible heavy baryonium masses, we conjecture that up to this stage, $Y(4260)$ and $Y(4360)$ could be a spin-triplet $\Lambda_c - \bar{\Lambda}_c$ baryonium, but $Y(4660)$ could not be a $\Lambda_c - \bar{\Lambda}_c$ baryonium in either spin-singlet or spin-triplet states, and $Y(10890)$ could not be a $\Lambda_b - \bar{\Lambda}_b$ baryonium either. Moreover, because $Z^0(4430)$ is an electrically charged state and $Z_c(3900)$ is out of the possible binding range, these states are not related to the $\Lambda_c - \bar{\Lambda}_c$ baryonium.

### Acknowledgments

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<table>
<thead>
<tr>
<th>$S = 0$ state</th>
<th>$S = 1$ state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>g_b</td>
</tr>
<tr>
<td>&lt;0.65</td>
<td>&lt;0.8</td>
</tr>
<tr>
<td>0.8</td>
<td>0.85</td>
</tr>
<tr>
<td>0.68</td>
<td>0.8</td>
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