Transient Lightning Response of Grounding Grid Buried in Multilayered Earth with the Hybrid method with “T” Typical Elementary Element

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Abstract—Combined with Fast Fourier Transform (FFT), a mathematical hybrid method for accurately computing the lightning current flowing along the grounding grid buried in multilayered earth model has been developed in this paper. In the hybrid method, electrical circuits consist with “T” typical elementary element. To accelerate the calculation, both dynamic state and quasi-static complex image methods, and closed form of Green’s function were introduced into this model. The model can be used for study performance of transient lightning response to grounding grid.

Index Terms—Closed form of Green’s function, Dynamic state and Quasi-static complex image methods, FFT, lightning current.

I. INTRODUCTION

Hybrid methods combine the advantages of circuit theory and electromagnetic field theory methods. The hybrid method based on quasi-static electromagnetic field theory has been developed to calculate the lightning response of a grounding grid buried in a uniform earth model [1]. The hybrid method based on high frequency electromagnetic field theory has also been developed, but again it only considered a uniform earth model [2]–[3]. Horizontal multilayered earth models have been described [4], but they are based on quasi-static electromagnetic field theory. Thus, a hybrid method based on high frequency electromagnetic field theory with a horizontal multilayered earth model needs to be developed, which was the focus of the present study.

The “π” typical elementary element (seen (a) in Fig. 1) of the electrical circuit of the hybrid method is used for all these papers [1]- [4]. However, the typical elementary element can be further displaced with “T” typical elementary element (seen (b) in Fig. 1), which owns better physical meanings. [5]

The dynamic state complex image method (DSCIM) has been proposed for lightning response of grounding grid in Refs. [2], [3]. However, only half homogenous earth model has been considered. A improved two-level DCIM [6] is adopted in the study for multilayered earth model, meanwhile, the quasi-static term of the Green’s function has been extracted easily.

In this paper, based on works in [1]- [4] [5], combined with FFT, an accurate mathematical hybrid method with “T” typical elementary element is developed for calculating the harmonic wave currents of lightning current distribution along the grounding grid buried in multilayered earth model within frequency domain. Both leakage currents and network currents within the grounding grid and their mutual coupling influence are considered in the calculation. To accelerate the calculation, DSCIM and quasi-static complex image method (QSCIM) and closed form of Green’s function were introduced. The mutual inductive and conductive coefficients have analytical formule through Maclaurin series expansion, and have been adopted in the mathematic model.

II. MATHEMATICAL MODEL OF EQUIVALENT CIRCUIT OF THE GROUNDING GRID

The transient problem is first solved by a formulation in the frequency domain. The time-domain response is then obtained by application the FFT. The response to a steady state, time harmonic excitation is computed for wide range of frequencies starting at zero Hz.

The grounding grid is assumed to be completely buried in a conductive \( N_b \) layer earth. The electrode is divided in \( N_p \) pieces of segments, and the discrete grounding grid has \( N_b \) end nodes and \( N_b \) middle nodes.

A. Mathematical model of a grounding grid

For single frequency case, “T” typical of basic element (see (b) in Fig. 1) is used, the obtained electric circuit may be studied using the conventional nodal analysis method [7]. For interaction of mutual induction among these discrete conductors, \( 2N_b \) pieces of conductors must be considered, which means each conductor has been separated into two parts due to its middle point. Meanwhile, for the interaction of mutual impedance, only \( N_b \) pieces of conductors should be considered, which means each conductor don’t be required to be separated into two parts.

Refer to [5], the equation can be written as

\[
\begin{bmatrix}
\vec{F} \\
0
\end{bmatrix} = \begin{bmatrix}
\tilde{A}_{pb}\tilde{Y}_{bb}\tilde{A}_{pb}^{-1} & \tilde{A}_{pb}\tilde{Y}_{bb}\tilde{A}_{bb}^{-1} \\
\tilde{A}_{bb}\tilde{Y}_{bb}\tilde{A}_{bb}^{-1} & \tilde{A}_{bb}\tilde{Y}_{bb}\tilde{A}_{bb}^{-1} + \tilde{Z}_s
\end{bmatrix} \begin{bmatrix}
\vec{V}_p \\
\vec{V}_b
\end{bmatrix}
\]

(1)

where \([\vec{F}]\) is a \( N_p \times 1 \) vector of external currents sources; \([\tilde{Y}_{bb}]\) is the \( N_b \times N_b \) branch admittance matrix; \(\vec{V}_p\) and \(\vec{V}_b\) are,
respectively, $N_p \times 1$ and $N_b \times 1$ vectors of SEP of the end and middle points of conductors; The sub-matrices $[\hat{A}_{pp}], [\hat{A}_{pb}]$ and $[\hat{A}_{bb}]$ are, respectively, $N_p \times N_p$, $N_p \times N_b$ and $N_b \times N_b$ matrices. The $[Z_s]$ is a $N_b \times N_b$ mutual impedance matrix. At last we have

$$[F] = [\hat{A}_{pb} \hat{V}_{bb} \hat{A}_{bp}]$$

$$- [\hat{A}_{pb} \hat{Y}_{bb} \hat{A}_{bb} \hat{A}_{bp} + Z_s^{-1}]^{-1} [\hat{A}_{bb} \hat{V}_{bb} \hat{A}_{bp}] [V_p]$$

(2)

The vector of nodal SEP $[V_p]$ may be calculated through solving the Eq. (2). The SEP of middle point $[V_b]$ can be calculated by

$$[V_b] = - [\hat{A}_{bb} \hat{Y}_{bb} \hat{A}_{bb} + Z_s^{-1}]^{-1} [\hat{A}_{bb} \hat{Y}_{bb} \hat{A}_{bp}] [V_p].$$

Both leakage and branch currents can also be calculated [1]-[4]. Meanwhile, the input impedance $Z_{in}$ of the grid at the frequency case can be calculated as, $Z_{in} = \frac{V_{in}}{I_{in}}$, where $V_{in}$ and $I_{in}$ are, respectively, SEP and external current at the injection point.

The study of the grounding grids performance in the frequency domain has been reduced to the computation of $[Z_s]$ and $[Y_{bb}]$ matrices. The elements of the two matrices for a horizontal dipole or monopole in homogenous conductive medium can be analytically calculated with the Maclaurin series expansion, more details can be referred to [3].

III. THE CLOSED FORM OF GREEN’S FUNCTION

The main task when simulating grounding grids is calculating the mutual induction and impedance coefficients for matrices $[Z_s]$ and $[Y_{bb}]^{-1}$. The elements of the two matrices for a point source in homogenous conductive medium can be analytically calculated [3]. In practice case, the earth is considered to be a multilayered conductive medium and an infinite integral about a Bessel function in Green’s function for a point source must be calculated for the dynamic state of an electromagnetic wave, so rapid calculation techniques must be used to avoid the integral. In the present study, the Green’s function is introduced for a two-layer horizontal earth model as an example, but Green’s function for other multilayered earth models can be obtained in the same way.

A. Green’s Function for a Monopole and a Horizontal Dipole

According to a previous study [8], we have Green’s function for a monopole, meanwhile, from [9], we have the x (or y) components of Green’s function for an horizontal dipole. For simplification, we have a unified formula as below:

$$G_{D\nu}^\varphi (x, y, z; x', y', z') = \frac{\vartheta}{4\pi} \int_0^\infty k_p \left( e^{-jk_{k_2} |z-z'|} + C_{D\nu} e^{-jk_{k_1} z} + C_{D\nu}^+ e^{+jk_{k_1} z} \right) J_0 (k_p \rho) dk_p$$

(3)

where for $G_{D\nu}^\varphi$, $g = \varphi$, $D = \varphi$, $\vartheta = \frac{1}{\rho}$; for $G_{A\nu}^{xx}$, $g = xx$, $D = A$, $\vartheta = \mu$; $\rho = ((x-x')^2 + (y-y')^2)^{1/2}$, and the coordinate values $(x, y, z)$ and $(x', y', z')$ are the field and source points, respectively; $k_p$ is the integral variable for (3) and $k_{z_1} = \sqrt{k_1^2 - k_p^2}$, while $k_1$ is the propagation constant of the first layer conductive medium, and $J_0 (k_p \rho)$ is the Bessel function of the first type of order zero. $C_D^-$ and $C_D^+$ are reflectivity functions of the generalized reflection coefficients in the plus and negative z directions, respectively. $C_D^- = \frac{-\kappa_p \phi e^{-jk_{k_1} z} - \kappa_p \phi e^{jk_{k_1}}}{1 + \kappa_p \phi e^{2jk_{k_1}}}$; for $G_{D\nu}^\varphi$,

$$\kappa_p \phi = \frac{\sigma_e e_{k_{z_1} - k_p} + \sigma_p e_{k_{z_1} + k_p}}{\sigma_e e_{k_{z_1} - k_p} + \sigma_p e_{k_{z_1} + k_p}} (o = 0, 1; p = o + 1),$$

for $G_{A\nu}^{xx}$, $\kappa_p^{xx} = \frac{k_{z_1} - k_p}{k_{z_1} + k_p}$, $k_p$ is the propagation constant of the pth layer conductive medium, $\sigma_p = \sigma_p + j \omega \varepsilon_p$ is the complex conductivity of the pth layer conductive medium, $h_1$ is the thickness of the first earth layer.

B. Closed Form of Green’s Function

Based on (3), we can find that all $\kappa_p \phi$, $\kappa_p^{xx}$ are functions of the complex variable $k_{z_1}$, so we can obtain the closed form of Green’s function $G_{D\nu}^\varphi$ and $G_{A\nu}^{xx}$ by defining $f_{w}^\nu (j k_{z_1}, k_p)$, $w = 1, 2, 3$, as given follows

$$f_{w}^\nu (j k_{z_1}, k_p) = \frac{(\kappa_{w_1}^\nu)^{\varpi_1} (\kappa_{w_2}^\nu)^{\varpi_2}}{1 + \kappa_{w_1}^\nu \kappa_{w_2}^\nu e^{-2jk_{z_1} h_1}}$$

(4)

where $\varpi_1 = 1$, $\varpi_2 = 0$ for $w = 1$, and $\varpi_1 = 0$, $\varpi_2 = 1$ for $w = 2$, and $\varpi_1 = 1$, $\varpi_2 = 1$ for $w = 3$.

In the present study, the $f_{w}^\nu (j k_{z_1}, k_p)$ will be developed as:

$$f_{w}^\nu (j k_{z_1}, k_p) = \tilde{f}_{w}^\nu (j k_{z_1}, k_p) + \sum_{d_w = 1}^{M_{d_w}} \alpha_{d_w}^\nu e^{\beta_{d_w}^\nu j k_{z_1}}$$

(5)

where $M_{d_w}^\nu$, $\alpha_{d_w}^\nu$ and $\beta_{d_w}^\nu$ are the dynamic state complex image’s coefficients; and $\alpha_{d_w}^\nu$ and $\beta_{d_w}^\nu$ are constants that need to be determined by selecting sample points for the function.
\((\kappa_{11}^q)^{\alpha_1}(\kappa_{11}^q)^{\beta_1}\), which can be obtained using the improved two-level DCIM [6].

It is also noted that \(f_w^\alpha(jk_{z1}, k_p)\) is the quasi-static term of \(f_w^\alpha(jk_{z1}, k_p)\), and for \(G_{op}^\varphi\), \(\tilde{k}_{op} = \kappa_{op} = \frac{\varphi_{op}}{\varphi_{op}}\); for \(G_{op}^{xx}\), \(f_{xx}(jk_{z1}, k_p) = 0\). Only the quasi-static term of \(G_{op}^{\alpha_1}\) needs to be considered, so we have

\[
\frac{1}{\kappa_{op}^q} = 1 + \sum_{q=1}^{M_q} \alpha_q^g e^{\beta q k_p} ,\]

where \(M_q^g\), \(\alpha_q^g\), and \(\beta_q^g\) are the quasi-static complex image’s coefficients, which can be obtained using the quasi-static complex image method (QSCIM) [8]. Thus,

\[
f_w^\alpha(jk_{z1}, k_p) = (\kappa_{11}^g)^{\alpha_1}(\kappa_{11}^g)^{\beta_1}\sum_{q=1}^{M_q^g} \alpha_q^g e^{\beta q k_p} + \sum_{d=1}^{M_d^g} \alpha_d^g jk_{z1} \kappa_{d1}^g k_p\]

(6)

Using the expression developed in Eqs (3) and (6), as well as the Sommerfeld identity

\[
\int_0^R e^{-jkr}J_0(k_p)dk_p = \frac{(\kappa_{11}^g)^{\alpha_1}(\kappa_{11}^g)^{\beta_1}}{(c^2 + \rho^2)^{\frac{1}{2}}},
\]

and the Lipschitz integration

\[
\int_0^R e^{-c \cdot k_{z1} k_{w1}} = \frac{1}{(c^2 + \rho^2)^{\frac{1}{2}}},
\]

so

\[
G_{D11}(\bar{F}_1, \bar{r}_1) = \frac{1}{4\pi \sigma_1} \left[ \frac{e^{-jk_1 R}}{R} - \frac{\bar{\kappa}_{01}^\varphi}{\bar{R}_1} + \frac{\bar{\kappa}_{12}^\varphi}{\bar{R}_2} - \frac{\bar{\kappa}_{01}\bar{\kappa}_{12}^\varphi}{\bar{R}_3} - \frac{\bar{\kappa}_{01}\bar{\kappa}_{12}^\varphi}{\bar{R}_4} \right]
\]

\[
+ \sum_{q=1}^{M_q^g} \alpha_q \left( \frac{\bar{\kappa}_{01}^g}{\bar{R}_{q1}} - \frac{\bar{\kappa}_{12}^g}{\bar{R}_{q2}} - \frac{\bar{\kappa}_{01}\bar{\kappa}_{12}^g}{\bar{R}_{q3}} - \frac{\bar{\kappa}_{01}\bar{\kappa}_{12}^g}{\bar{R}_{q4}} \right) - \sum_{d=1}^{M_d^g} \alpha_d \left( e^{-jk_1 R_{d1}} - \frac{\bar{\kappa}_{d1}^g}{\bar{R}_{d1}} \right)
\]

(7)

where for \(G_{\varphi_1, \varphi_1}^\varphi\), \(D = \varphi, g = \varphi; \) for \(G_{xx}^{xx}\), \(D = A, g = xx;\)

\[
R = (\rho^2 + (z^2 + z^2)^2)^{\frac{1}{2}}, R_c = (\rho^2 + (signa_z z + signb_z z^2 + z^2)^2)^{\frac{1}{2}},
\]

here \(\varsigma = 1, 2, 3, 4, R_{q_i} = (\rho^2 + (signa_z z + signb_z z^2 + z^2)^2)^{\frac{1}{2}}, R_{d_i} = (\rho^2 + (signa_z z + signb_z z^2 + z^2)^2)^{\frac{1}{2}},\)

in which \(signa_z = 1\) for \(R_1\) and \(R_2\), \(R_{q1}\) and \(R_{q2}\), \(R_{d1}\) and \(R_{d2}\), \(signb_z = 1\) for \(R_3\) and \(R_4\), \(R_{q3}\) and \(R_{q4}\), \(R_{d3}\) and \(R_{d4}\), \(signa_z = -1\) for others, \(z_0 = (2h)s, z_0 = (2h)s - \beta_0^q, z_0 = (2h)s - \beta_0^q, z_0 = (2h)s - \beta_0^0, \]

\(\beta_0^q = (2h)s - \beta_0^0, \) in which \(s = 0\) for \(\varsigma = 1, \) otherwise, \(s = 1.\)

Furthermore, \(M_d^q = M_d^g, \alpha_d^q = \alpha_d^g, \beta_d^q = \beta_d^g.\)

We can see that each term, except the first term of Eq. (7), can be regarded as an image point source, the location of which is indicated by \(R_{q_i}, R_{d_i}\), and \(R_{d_i}\), with the amplitudes \(\alpha_q^g\) and \(\alpha_d^g.\) However, \(\alpha_q^g, \alpha_d^g, \) and \(R_{q_i}, R_{d_i}\). In Eq. (7) are complex numbers and the electromagnetic field is regarded as a high frequency dynamic state electrical field, so this approach is referred to as DSCIM. [3]

For a quasi-static electrical field, because the electromagnetic wave’s propagation effect is neglected, we have \(M_q^g = 0, e^{-jk_1 R} = 1\) for \(G_{\varphi_1, \varphi_1}^\varphi;\) for \(G_{xx}^{xx}\), \(\bar{\kappa}_{op} = 0, \) and \(M_q^g = 0.\)

\(G_{\varphi_1, \varphi_1}(\bar{r}, \bar{r}'\) and \(G_{xx}^{xx}(\bar{r}, \bar{r}')\) in a quasi-static electrical field are based on [8].

IV. NUMERICAL RESULTS AND ANALYSIS

Numerical results about transient response of lightning from a grounding grid in a multilayered earth will be discussed. The impulse impedance is defined following expression in [12].

A. VERIFICATION OF MODEL

To verify the proposed method, some cases solved by other authors [11] [12], are studied.

1) BRANCH CURRENTS IN FREQUENCY DOMAIN: The verified case from [11], in which an horizontal conductors with 100.0m lengths, and 5mm radius was buried at \(H = 0.5m\) and \(H = 1.0m\) depth within a two layered earth, which is characterized by relative permittivities \(\varepsilon_r = 10.0,\) and its upper and bottom layers conductivities are, respectively, \(\sigma_1 = 0.01S/m\) and \(\sigma_2 = 0.19S/m\) and its first layer thick is \(d = 1.0m\) and \(d = 2.0m.\) The distribution of branch currents can be seen in Figs. 2 and 3, we can see that the numerical results are well agreed with the distribution of branch currents in Fig. 4 from [11].
2) **TIME DOMAIN:** The case from [12], a typical grounding grid with a square grid and the size of 10 × 10m is considered, which was made of round copper conductors with a 50mm² cross section. The grounding grid was buried at 0.5 m depth in two-layer horizontal earth, whose resistivity’s ratio for the upper and the lower soil layers is \( \rho_1/\rho_2 = 50/20 \) Ωm/Ωm, the upper layer thickness being \( H = 0.6 \) m, and the all soil’s permittivities is set as 10. The inject lightning current parameter was set at \( T_1 = 65 \) μs, \( T_2 = 185 \) μs, and \( I_m = 6.86 \) A. The feed point is at the middle of one border of the grid. The transient SEP curves from quasi-static or dynamic state electromagnetic field cases can be seen in Fig. 4, both of the two curves almost agree with the measured curve in Fig. 7 in [12]; meanwhile, the impulse grounding impedance was 1.664 Ω, as given by [12], and it is \((1.681, -2.033 \times 10^{-8})\) Ω and \((1.625, -4.555 \times 10^{-8})\) Ω from quasi-static and dynamic state electromagnetic field cases for our model.

![Fig. 4. Transient scalar electrical potential at injection point](image)

**B. SIMULATION RESULT ANALYSIS**

A grounding grid consisted of 3 × 3 grids with 60m × 60m is used, which is buried below the ground surface at a depth of 0.5m. The earth is modeled as a horizontal two-layer conductive medium, parameters of the earth model are, \( \sigma_1 = 100 \) S/m, \( \sigma_2 = 800 \) S/m, and \( \varepsilon_1 = 10 \varepsilon_0, \varepsilon_2 = 10 \varepsilon_0 \), respectively. For thickness of the upper earth, to obtain a sensitivity analysis about the multilayered earth, different thickness of the upper earth are considered as \( h_1 = 1.5, 3, 6, 12 \) m. The material of the grounding grid conductor is Cu with conductivity \( \sigma_{Cu} = 5.8 \times 10^7 \) S/m. The conductor radius is 5 mm. The external excited lightning current is injected from the corner of the grounding grid, which is described by a double-exponential function as \( I(t) = 10.956 \times (e^{-0.00958563t} - e^{-0.3767054t}) \) A, which means that the parameters of the lightning current are \( T_1 = 10 \) μs, \( T_2 = 85 \) μs and \( I_m = 9.5 \) A.

1) Discussion influence of thickness of upper earth and QSCIM and DSCIM methods: In Fig. 5, eight pieces of curves for transient SEP at injection point from different thickness of upper layer earth model is given. We know that maximum peak value of these curves will descend with the thickness of upper layer earth increasing for both QSCIM and DSCIM case, this is because that for the reflection factor \( k = 0.78 \), resistivity of the second layer earth is bigger than the one of upper layer earth, and the grid is buried at 0.5m, always be placed in upper layer earth, so currents along the grid is difficult to penetrate into second layer earth, and second layer earth is more resistive, who can influence on distribution of the current along the grid, further influence on maximum peak value of transient SEP at injection point; however, with the thickness of upper layer earth increases, the influence on the maximum peak value will weaken; On the other hand, curve from QSCIM is higher than the one from DSCIM for the same thickness, which is due to electromagnetic wave propagation effect.

The impulse grounding impedance from QSCIM and DYCIM can be seen in Table I. We know that impulse impedances from no matter QSCIM or DSCIM always obey a rule, which is the impulse impedance will decrease with the thickness of upper layer earth increasing, this is because the influence from much resistive of second layer earth weaken.

![Fig. 5. The transient SEP at injection point](image)

Both QSCIM method and DSCIM methods have been used to calculate the grounding resistance \( Re(Z(j\omega)) \) of the grid opposite to frequency. The numerical results can be seen Fig. 6, we can see that no matter which thickness of upper layer earth case, grounding resistance \( Re(Z(j\omega)) \) from QSCIM method is bigger than the one from DSCIM method at same frequency, the error between the two method is small for low frequency, and become bigger with increase of the frequency. On the other hand, curves of grounding resistance will decrease with the thickness of the upper layer earth increasing, this is because the influence from much resistive of second layer earth weaken. Furthermore, we find that for QSCIM method, grounding resistance \( Re(Z(j\omega)) \) is independence on the frequency below 0.01 MHz and equal to the low frequency grounding impedance; However, since propagation effects of electromagnetic wave have been considered in DSCIM method, this phenomenon don’t occurs.
Fig. 6. The grounding impedance of the grid correspondence to frequency

TABLE I

<table>
<thead>
<tr>
<th>Model</th>
<th>QSCIM case</th>
<th>DSCIM case</th>
</tr>
</thead>
<tbody>
<tr>
<td>h=1.5</td>
<td>(6.547, +1.022 x 10^{-4})</td>
<td>(4.108, +7.587 x 10^{-4})</td>
</tr>
<tr>
<td>h=3.0</td>
<td>(5.939, -6.433 x 10^{-4})</td>
<td>(3.123, -6.137 x 10^{-4})</td>
</tr>
<tr>
<td>h=6.0</td>
<td>(6.197, -1.000 x 10^{-4})</td>
<td>(3.052, -5.535 x 10^{-4})</td>
</tr>
<tr>
<td>h=12.0</td>
<td>(4.481, -2.579 x 10^{-4})</td>
<td>(2.884, -5.046 x 10^{-4})</td>
</tr>
</tbody>
</table>

V. CONCLUSION

With the FFT, based on the theory of dynamic state electromagnetic field, combined with the rapid Galerkin MoM and the conventional nodal analysis, a new mathematical method for calculating the transient lightning current distribution along a grounding grid buried in the multilayered earth was developed. Through some numerical results were discussed, some conclusions have been obtained:

1) The QSCIM assumption provides overestimated results in comparison with the DSCIM case.
2) Numerical calculation has been replaced with analytical calculations introduced in our method under the DSCIM approximation for the non-far field case.
3) No matter DSCIM or QSCIM approximation, the coefficients of mutual induction and impedance can be fast calculated through analytical formulae in the mathematic model.

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