A New Mesh Transformation Method for Hull Structure Shape Optimization Based on Parametric Technology

Yan-Yun Yu, Kai Li, Yan Lin, Ming Chen
State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology,
Ship CAD Engineering Center, Dalian University of Technology
Dalian, Liaoning Province, China

ABSTRACT
A new Parametric Mesh Transformation Method (PMTM), which is used for structure shape optimization of the hull, is proposed in this paper. The purpose of PMTM is to realize dimension-driven of the FEM model for hull structure, without calling mesh generator as the traditional parametric method doing. In this method, the plate is split into quadrilaterals with the concept of N-Sided region modeling technology. Then create a Coons surface that interpolates four edges for each quadrilateral. Get the dimensionless parameters for all nodes of the mesh within each Coons patch by surface reverse calculation method. A combined reverse calculation method, which takes into account of both efficiency and generality, is developed and used in calculation of dimensionless parameters. When the structure is changed, all the Coons surfaces are changed accordingly. Then substitute the dimensionless parameters of each node into the surface equations of the corresponding Coons surfaces by the new location of the node. Calculate new location for each node with the above procedure, and the new finite element model according to the new structure is obtained. This method is applied to the shape optimization of a 300 ft. jack-up rig compared with the traditional method. The result shows that PMTM is able to realize dimension-driven of FEM model. It is also proved that when the parameters of the parametric structure model changed gradually, the structure stresses change smoothly with PMTM. That is an important advantage which the tradition methods do not have, and it could improve the efficiency as well as quality of hull structure shape optimization.

KEY WORDS: hull structure; parametric; finite element method (FEM); mesh transformation; shape optimization.

INTRODUCTION
Structure optimization is one of the most efficient ways to improve the safety and economics for ships, platforms, and other ocean structures. Generally, structure optimization could be divided into two categories for the large structures constitute of stiffened plates, which are structure property optimization and structure shape optimization. Property optimization method, the purpose of which is to optimize the thickness of plate or the section dimensions of stiffeners, is relatively simple but effective in improving the structure strength as well as deducing the structure weight. For example, Yu et. al. (2010) proposed an efficient hull structure optimization method that could improve both yield strength and buckling strength on promise that the construction cost are properly taken into account. The purpose of structure shape optimization is to optimize the geometric shape of the plates. For example, Hougaz et. al. (2003) presented a shape optimization method for ship-line structure to reduce the structure weight by optimizing both shape variables and property variables. Mitsuru et. al. (2010) developed a shape optimization system to optimize the double bottom structure of hull with incorporating individual mesh subdivision and multi-point constraint. As to the platform, Yu et. al. (2010) introduced the parametric design method that based on geometric constraint solving into platform structure design, and proposed a structure shape optimization method based on the parametric model in 2012. Zheng et. al. (2012) proposed a shape optimization method for long span suspended platform based on the parametric modeling language of ANSYS software.

In the traditional hull structure shape optimization method mentioned above, Parametric Structure Model (PSM) is created and driven by parameters, and the finite element model is generated via meshing of PSM. When any parameter of PSM is changed, mesh generation should be performed in order to get the finite element model. A typical procedure proposed by Yu (2012) is shown in Fig. 1.

Fig. 1. Procedure of traditional structure shape optimization method
The above process is effective for structure shape optimization, but there are three main problems that hinder its further application. Firstly, mesh generation should be called each time when any of the parameters changes in the optimization process. As is known to all, mesh generation occupies most of the CPU time in the optimization process especially for the large scale hull structure problems, which seriously influences the efficiency of optimization. Secondly, mesh generation must be completely automatic when shape optimization starts, which means the designer could not control the mesh quality in the optimization process. Although there are quite a lot of advanced mesh generation methods, such as the ship structure mesh strategy proposed by Yu et al. (2009), manual intervention is still necessary to get high quality mesh in stress concentration region for the purpose of improving precision of FEM result. Finally, regeneration of mesh might cause change of mesh topology, which will result in discontinuity of structural mechanical response, and that brings great difficulties to optimization solution. As a result, although most of the structural mechanical response such as stress, deflection etc. satisfies the continuity condition, all the gradient-based optimization method and the sensibility analysis could not be performed for the traditional hull structure optimization.

To overcome the above problems, a new structure shape optimization procedure for hull structure is developed, as shown in Fig. 2. There are two significant differences between the new procedures compared with the existing one. First, the mesh generation is called only once at the beginning of shape optimization and manual interval is allowed if necessary. Second, in the optimization process, new mesh is obtained via mesh transformation method instead of calling the mesh generator each time. Obviously the key problem of this new optimization procedure is how to update the FEM model smoothly according to PSM, which is the rectangle with black dash line in Fig. 2. And that is the main issue to be solved in this paper.

**PRINCIPLE OF PARAMETRIC MESH TRANSFORMATION METHOD**

Assume $G$ is the initial contour of a plate, $G'$ is the contour changed by parameterization. $Es$ is the elements within $G$. The main purpose of PMTM is to map elements $Es$ from $G$ to $G'$, under the following three premises.

Firstly, the validity of elements should be guaranteed. In the finite element analysis, structured mesh without intersecting or self-intersecting is required. The initial mesh of shape optimization satisfies this requirement, and the mesh transformed by PMTM should not change this properties. That also means the transformation should not change the topology between nodes and mesh. Secondly, the coordination of mesh should be guaranteed. In the finite element model, the connection relationship of two connected structure is represented by common nodes on their common edges. The mesh transformation should satisfy this requirement. That requires the transformation should be uniqueness. It means that if a node on the common edge of two plates, it could be transformed along with both plates respectively and two new locations are obtained. Uniqueness requires that the two locations obtained by two ways should be exactly the same. Thirdly, the mesh transformation should be continuous. In Finite Element Analysis (FEA), the mesh shape is an important factor to the numerical result. So if the shape parameters of the structure change continuously, the mesh transformation method should ensure the continuity of mesh movement. If this requirement is not met, the structure mechanics response may be discontinuous, which will bring lots of troubles to structure shape optimization.

The geometric contour of the plate of hull structure could be quadrilateral, pentagon or other polygon with more than five edges. The plate contour consists of line segment, circle, arc and spline segment. The mesh transformation algorithm for quadrilateral contour plate is discussed in this section, and the one for polygon contour with more than five edges could be converted into the algorithm of quadrilateral contour problem.

**Transformation of point**

In PMTM, mesh transformation for quadrilateral is realized through a projection transformation algorithm. The transformation of mesh is transformation of point in essence. The point transformation principle is as follows.

Assume $Q$ is a quadrilateral. Firstly, establish $T$, a projection transformation function which is related to $Q$ and could convert each point within $Q$ including the points on the contour, into dimensionless coordinates. $T$ should be a two-way unique transformation, which means each geometric point corresponds to a dimensionless coordinate, and the reverse is also true. Assume $P$ is an arbitrary point on $Q$. Calculate $(u,w)$, the dimensionless coordinate of $P$.

When the contour of the quadrilateral changes from $Q$ to $Q'$, the dimensionless coordinate $u$ and $w$ remains unchanged, while the projection transformation is changed from $T$ to $Tn$, for it is related to the contour of the quadrilateral. Then calculate $Tn\ast$, the reverse transformation of $Tn$. By Applying $Tn\ast$ onto $(u,w)$, a new point $P'$ is obtained. $P'$ is the transformed point of $P$. Calculate the new position for each point on the mesh within $Q$, and then mesh transformation is complete.

**Transformation of mesh**

The transformation of mesh for quadrilateral could be explained by Fig. 3, where $ABCD$ is a quadrilateral plate, $E$ is an element with four nodes $I,J,K$ and $L$. Note: Issue to be solved in this paper.
Now the geometric contour is changed from $ABCD$ to $A'B'C'D'$. First, construct transformation function $T$ according to the contour of $ABCD$, apply $T$ to each node of element $E$, and the dimensionless coordinate of nodes are obtained, marked as $I_0$, $I_1$, $K_0$, $L_0$. Construct transformation function $Tn$ according to contour of $A'B'C'D'$, and then calculate $Tn^*$, the inverse function of $Tn$. Apply $Tn^*$ onto the dimensionless points $I_0$, $J_0$, $K_0$, $L_0$, and get $I'$, $J'$, $K'$, $L'$ four new nodes in $A'B'C'D'$. Then the objective element $E'$, which formed by $I'J'KL'$ is obtained.

**REPRESENTATION OF SURFACE PATCH AND INTERPOLATION METHOD**

Parametric surface is wildly used in geometric modeling. A point on the parametric surface could be represented by two parameter $u$ and $w$:

$$P(u, w) = [x(u, w), y(u, w), z(u, w)]$$

where $x(u, w)$, $y(u, w)$ and $z(u, w)$ are the three coordinate functions. In the parametric surface, each dimensionless coordinate $(u, w)$ corresponds to a point on the surface uniquely, and each point on the surface has only one dimensionless coordinate. Thus, the mesh transformation of PMTM could be realized based on parametric surface theory.

There are a lot of parametric surface theories, such as Ferguson surface, Coons surface, Bezier surface and NURBS surface etc. We need the surface interpolates four given edges, the type of which could be line, arc and spline etc. Among the parametric surface theories, Coons surface meets all the requirements. So Coons surface is used to complete the mesh transformation for PMTM.

The Coons surface has several forms for different boundary conditions. The one that interpolates four vertexes as well as the one interpolates four edges are used in PMTM. The Coons surface theory gives the expression of point on surface with dimensionless parameters. The reverse form, which is to get the dimensionless parameter for a given point is also needed in PMTM. For both of the two Coons surfaces, the reverse calculation methods are derived in this section.

**Coons surface interpolates four vertexes**

As to the quadrilateral with linear edges, the geometric contour could be expressed by four vertex $P(0,0)$, $P(1,0)$, $P(1,1)$ and $P(0,1)$. With Coons surface modeling technique, the parametric surface is given by

$$P(u, w) = P(0,0)(1-u)(1-w) + P(0,1)(1-u)w + P(1,0)u(1-w) + P(1,1)uw$$

(2)

From Eq. 2, we can get any geometric point on surface corresponding to parameter $(u, w)$ $(u, w \in [0, 1])$. Obviously, Eq. 2 satisfies the first requirement of mesh transformation, which is the transformation function from real coordinate to dimensionless coordinate, $Tn$. And also we need $Tn^*$, the reverse transformation function of $Tn$ to complete the mesh transformation for PMTM. $Tn^*$ is actually the inverse calculation problem of parametric surface, and the inverse calculation equation for Coons surface interpolates four vertexes is derived bellow.

Assume that $P(0,0)$, $P(1,0)$, $P(1,1)$ and $P(0,1)$ are the four vertexes for a quadrilateral with four linear edges. $(x_0, y_0)$, $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ are the locations for points $P(0,0)$, $P(1,0)$, $P(1,1)$ and $P(0,1)$ respectively. $(x, y)$ is an arbitrary point on quadrilateral surface. The objective of inverse calculation is to calculate $u$ and $w$, the dimensionless parameter of point $(x, y)$.

Substituting $x$ and $y$ into Eq. 2, we get a quadratic equation set

$$x = x_0(1-u)(1-w) + x_1(1-u)w + x_2u(1-w) + x_3uw$$

(3)

$$y = y_0(1-u)(1-w) + y_1(1-u)w + y_2u(1-w) + y_3uw$$

Eq. 3 could be also be written as

$$k_1uw + k_2w + k_3u + k_4 = 0 \quad (a)$$

$$s_1uw + s_2w + s_3u + s_4 = 0 \quad (b)$$

(4)

where

$$k_1 = x_0 - x_3 - x_1 + x_2 \quad s_1 = y_0 - y_3 - y_1 + y_2$$

$$k_2 = x_1 - x_0 \quad s_2 = y_1 - y_0$$

$$k_3 = x_1 - x_3 \quad s_3 = y_1 - y_3$$

$$k_4 = x_0 - x \quad s_4 = y_0 - y$$

Multiply Eq. 4(b) by $\frac{k_1}{s_1}$, and subtract it by Eq. 4(a), then the quadratic item is eliminated. $u$ could be denoted as the explicit expression of $w$

$$u = \frac{(k_1s_1 - k_2s_0)w + k_4s_0 - k_3s_1}{k_3s_2 - k_2s_1}$$

(6)

Let

$$k_1s_1 - k_2s_0 = k_4s_0 - k_3s_1 = 0$$

and $w_1 = w_2 = 0$. Then

$$u = \frac{k_4s_0 - k_3s_1}{k_3s_2 - k_2s_1}$$

$$w = \frac{k_3s_2 - k_2s_1}{k_3s_1 - k_2s_0}$$

(7)

Thus, we get the explicit expression of $u$ and $w$.
\[ \alpha = \frac{k_2s_2 - k_1s_1}{k_3s_3 - k_2s_2} \]
\[ \beta = \frac{k_3s_3 - k_1s_1}{k_3s_3 - k_2s_2} \]  
(7)

Substitute Eq. 7 into Eq. 6, \( u \) is obtained as
\[ u = \beta w + \alpha \]  
(8)

Substituting Eq. 7 and Eq. 6 into Eq. 4(a) to eliminate the unknown number \( u \) and a binary linear equation about \( w \) is obtained as follows
\[ k_1\beta w^2 + (k_1\alpha + k_2\beta + k_3)w + k_1\alpha + k_3 = 0 \]  
(9)

By solving Eq. (9) \( w \) is obtained as
\[ w = \frac{-k_1\alpha + k_2\beta + k_3 \pm \sqrt{(k_1\alpha + k_2\beta + k_3)^2 - 4k_1\beta(k_1\alpha + k_3)}}{2k_1\beta} \]  
(10)

Eq. 10 has two solutions, the one within \([0, 1]\) is the correct one.

**Coons surface interpolates four edges**

As to the quadrilateral with one or more non-linear edge such as arc or spline, assume the four edges are \( P(u,0), P(u,1), P(w,0) \) and \( P(w,1) \) respectively, as shown in Fig. 5. With the Coons modeling method, such quadrilateral could be denoted as
\[ P(u,w) = P(u,0) + (P(u,1) - P(u,0))w + (P(w,1) - P(u,0))w \]  
(11)

Being similar to the surface interpolates four vertexes discussed above, the difficulty for Coons surface interpolates four edges is the inverse calculation algorithm. It is very difficult to derive the analytical solution of this problem, especially for the surface with spline edge. So the iteration method is used to get the numerical solution below.

1. As shown in Fig. 5, assume that \((x, y)\) is an arbitrary point on the quadrilateral surface interpolates four curves. The object is to calculate the dimensionless parameter, \( u \) and \( w \) \((0 \leq u, w \leq 1)\) for point \((x, y)\).
2. \( u \) and \( w \) are two unknown values. Firstly, \( u \) is calculated with an iteration method based on dichotomy. Then \( w \) could be calculated based on \( u \), \( l \) and \( r \) are the two location parameters of the dichotomy method, firstly set \( l = 0 \), \( r = 1 \).
3. Let \( m = (l + r) / 2 \).
4. According to Eq. 11, calculate \( P(l,w) \), the curve corresponding to parameter \( l \) on the Coons surface. Then calculate \( P(m,w) \), the curve corresponding to parameter \( m \).
5. Check if point \((x,y)\) lies between curve \( P(l,w) \) and \( P(m,w) \). If it does, let \( r = m \); otherwise, let \( l = m \).
6. Calculate \( d \), the minimum distance of point \((x,y)\) to curve \( P(m,w) \). if \( d < \varepsilon \) then execute 7; otherwise execute 3.
7. The \( u \) direction parameter of point \((x,y)\) is \( m \). Calculate \( d_{1\alpha} \) the length of curve \( P(m,w) \) between point \((x,y)\) and \( P(m,0) \). Then calculate \( d_{1\beta} \), the length of curve \( P(m,w) \) between point \((x,y)\) and \( P(m,1) \). The \( w \) direction parameter of point \((x,y)\) is calculated by
\[ w = \frac{d_{1\alpha}}{d_{1\alpha} + d_{1\beta}} \]  
(12)

8. Reverse calculation complete.

The above reverse calculation method for Coons surface has good generality, and it is suitable for surface interpolates four arbitrary curves. Obviously, this method could deal with the problem of Coons surface interpolates four vertexes that discussed in the last subsection. However, the iteration processing is time consuming, while the reverse calculation should be called for each node when the plate contour is changed. For the hull structure, the contour of most of the plates is polygon with linear edge only. So in consideration of both efficiency and generality, Eq. 9 and Eq. 10 are used for plate with linear edge, and the iteration method described in this subsection is only applied for plate with non-linear edge in PMTM.

**PROCEDURE FOR PARAMETRIC MESH TRANSFORMATION METHOD**

**Algorithm for polygon plate**

With the N-Sided region concept, such as the method proposed by Piggin and Tiller (1999), the polygon plate of hull structure with \( n \) edge \((n \geq 5)\) could be divided into \( n \) quadrilateral pieces, and the mesh transformation problem for polygon with more than four edges could be down to that of quadrilateral plate. The algorithm is as follows.

Assume \( n \) is the number of polygon edge, \( V_i \) \((1 \leq \leq n)\) is the \( i \)th vertex of polygon, \( M_i \) is the middle point of the \( i \)th edge, \( O \) is the geometric center of the polygon, whose coordinate \((x_c,y_c,z_c)\) is
\[ O(x_c,y_c,z_c) = \left( \sum_{i=1}^{n} x_i, \sum_{i=1}^{n} y_i, \sum_{i=1}^{n} z_i \right) \]  
(13)

where \( x_i, y_i, z_i \) are the \( x, y \) and \( z \) coordinate of the \( i \)th vertex respectively. Separate the \( i \)th edge with \( j \)th middle point \( M_i \) and connect center point \( O \) with each middle point \( M_i \) \((1 \leq \leq n)\). The quadrilateral plate is divided into \( n \) pieces, which are \( VM_M M_i M_{j+i} \) \((1 \leq \leq n, M_{j+i} \) means \( M_{j+i} \)). Each piece could be represented by a Coons surface with the methods discussed in the last section.

With the above approach, a pentagon is divided into five quadrilaterals as shown in Fig. 6.

**Basic procedure for application of PMTM**

The application procedure of PMTM based on N-Sided region method is as follows.

First, create the sketch driven by dimensions, and then establish PSM...
according to the sketch. Split each plate into quadrilateral patches with N-Sided region concept, and create a Coons surface for each patch. Generate the initial finite element model, and modify the initial mesh for mesh quality manually if necessary. Establish the relationship between Coons patches and nodes of mesh, which means find the Coons patch that each node belongs to. Calculate the initial dimensionless parameter $u$ and $w$ for each node via reverse calculation algorithm of the Coons patch it belongs to.

When the parameters of the sketch are changed, the structure is changed automatically by geometry constraint solving, and the Coons patches are changed along with the structure simultaneously. For each node, find its new position according to the dimensionless parameters by the equation of Coons surface it belongs to. The mesh transformation is complete after all nodes are changed.

The above process could be explained more clearly by Fig. 7, where (a) is the initial parametric model of a plate, and the initial finite element model is shown in (b). When the parameters are changed, the plate contour is changed according to the parameterization, as shown in Fig. 7(c). Then transform the initial mesh according to the new contour, and new mesh is obtained and shown in Fig. 7(d), where crude dashed line denotes the initial contour of the plate.

The uniqueness property of PMTM

There are quite a lot of nodes lay on the common edge of Coons patches, some are on the common edge of plates, and the others are on the auxiliary lines to split polygon into quadrilaterals.

Assume $A$ and $B$ are two Coons patch with a common edge $E$, $P$ is an arbitrary point on edge $E$. When $A$ and $B$ are changed, $P$ could be transformed from $A$ or from $B$ respectively, assume that the new points are $P_A$ and $P_B$. From Eq. 2 and Eq. 11, it can be derived that, $P_A$ and $P_B$ are exactly the same for PMTM. That is because the parameter of point $P$ on curve $E$ only depends on the ratio of curve length between $P$ to each ends of $E$, and it is independent to the other three edges of the Coons surface. This uniqueness property is very important for the finite element model, which requires that nodes distribution on the common edge of two adjacent plates should be exactly the same.

APPLICATION OF PMTM IN ENGINEERING PRACTICE

It is noteworthy that PMTM is mainly used for structure shape optimization of the hull. However, the theme of this paper is not the shape optimization method itself. So in this section, a 300ft jack-up rig is taken as instance to discuss the advantage of PMTM compared to the traditional methods.

The principal dimensions of the 300 ft. jack-up rig are shown in Table 1, and the general arrangement drawing is shown in Fig. 8.

<table>
<thead>
<tr>
<th>Length</th>
<th>Breadth</th>
<th>Depth</th>
<th>Drilling Depth</th>
<th>Spudcan Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.0 m</td>
<td>58.0 m</td>
<td>7.50 m</td>
<td>7000 m</td>
<td>14.0 m</td>
</tr>
</tbody>
</table>

Fig. 8. General arrangement drawing of the 300 ft. jack-up rig

Create PSM and initial finite element model

Firstly, the sketch of hull structure based on geometric constraint solving is created are shown in Fig. 10. Then create 3D PSM according to the sketch. PSM consists of main deck plate, outer bottom plate, inner bottom plate, outer shell plate, longitudinal bulkhead, transverse bulkhead, cantilever, three legs and living quarter structure etc. Create the initial finite element model with automatic mesh generation method combined with manually control at stress concentrated region. The initial finite element model is shown in Fig. 10.

Fig. 10. Structure finite element model of the 300 ft. jack-up rig
Take a typical drilling condition as example to calculate the structure stress. The load taken into account includes hull gravity, variable deck load, hook load, wind load, current load and wave load etc.

**Parameters in PSM to be changed**

Applying PMTM into PSM, the finite element model could be driven by any parameter in the sketch. The traditional parametric method could also realize the above object, with different principle. To observe the difference of PMTM compared to the traditional method, two typical parameters, which are the leg web diameter and transverse leg distance, are selected as variables, which are shown in Fig. 9 marked as C1 and C2. The purpose of this example is to check the difference of the structure stress response when C1 or C2 change with PMTM and the traditional methods.

**Variables for mechanical response to be observed**

Structure stress is one of the most important indexes for shape optimization of hull. So structure equivalent stress at critical region is chosen as the observed variable in this example, as shown in Table 2 and Fig. 11.

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Stress on bottom plate adjacent to leg well</td>
</tr>
<tr>
<td>S2</td>
<td>Stress on main deck plate adjacent to leg well</td>
</tr>
<tr>
<td>S3</td>
<td>Stress on cantilever plate adjacent to transom plate</td>
</tr>
<tr>
<td>S4</td>
<td>Stress on leg well around fore leg</td>
</tr>
<tr>
<td>S5</td>
<td>Stress on leg well around aft left leg</td>
</tr>
<tr>
<td>S6</td>
<td>Stress on leg well around aft right leg</td>
</tr>
</tbody>
</table>

The above phenomenon is obviously inconsistent with this general knowledge.

** Differences in stress response for the two methods**

Change parameter C1 from 6.8 m to 7.2 m, increased by 0.01 m. Get the finite element model from the traditional method and PMTM respectively. Calculate stress variables S1–S6 for each scheme. The stress-C1 curve is shown in Fig. 12 for the traditional method, and the one for PMTM is shown in Fig. 13.

For each C1, the only difference between the two schemes is that the finite element mesh is different, one from automatic mesh generator and the other from mesh transformation. From the result we can see that the general trend of stress change for the two methods is similar, but small differences exist. For the traditional method, when C1 changes from 6.8 m to 7.2 m, the stress curves for S1, S2, S3, S4, S5, S6 have a sudden drop at 6.86 m, and the one for S5 also drops suddenly at 6.98 m. As is known to all, the stress response of hull structure should be continuous when the geometry shape dimensions change continuously.

As to PMTM, things are different. From Fig. 13 we can see that all the six variables change smoothly when C1 increase gradually. Compared with the result of the traditional method, there are no sudden drop or arise in all the curves, and the fairness of all the six curves are also improved significantly.

Change parameter C2 from 19.8 m to 20.2 m, increased by 0.01 m. The stress-C2 curve is shown in Fig. 14 for the traditional method, and Fig. 15 for PMTM. The conclusion of changing C2 is similar with that of changing C1. For the traditional method, there is sudden change in
some of the curves. For PMTM, all the curves are smooth and no fluctuation exists.

CONCLUSION

In this paper, a parametric mesh transformation method named PMTM is presented to realize dimension-driven of FEM model for hull structure. Being different from the traditional method, PMTM generate the finite element model via mesh transformation from the initial mesh based on Coons surface and N-Sided region modeling technique. Compared to the traditional method, it has three main advantages when applied in shape optimization of hull structure.

Firstly, PMTM is high efficient. For traditional method, the mesh generator should be called each time when the parameters are changed in the optimization process. As the mesh generation is a seriously time consuming procedure, the traditional method is low in efficiency. As to PMTM, the time complexity for mesh transformation of plates with linear contour is $O(n)$, where $n$ is number of node in FEM model. Although time complexity for plate with non-linear edge may be worse, but those plates occupy only a small proportion in the whole structure. Although time complexity for plate with non-linear edge may be worse, but those plates occupy only a small proportion in the whole structure. However, PMTM could greatly improve the efficiency of hull structure shape optimization.

Secondly, PMTM has good continuity. In shape optimization based on PMTM, the topology of the mesh keeps unchanged in the whole optimization process, which avoids the discontinuous of stress when shape dimensions are changed gradually. Besides, from the expression of Coons surface, it can be found that when the shape of edges is changed continuously, the location of the point corresponding to a fixed dimensionless parameter will also change continuously. As a result, the mesh transformation of PMTM meets the continuity requirements, and highly efficient optimization methods, such as the gradient-based algorithms, are applicable in the shape optimization of hull structure.

Finally, PMTM provides high quality mesh. In the traditional method, the mesh is generated fully automatically, which might result in poor element quality, especially in stress concentrated region. Low quality mesh may deduce the precision of the FEA result, and ulteriorly influence the accuracy of shape optimization. As to PMTM, the mesh is generated only once at the beginning of optimization, and manually interval is allowed if necessary to improve the mesh quality. So it can be assumed that the quality of the initial mesh is high. Generally, structure shape optimization optimizes the dimensions of the hull within a small range, which means there should not be great differences for the shape of the plate before and after optimization. Meanwhile, from the expression of Coons surface, it can be seen that the coordinate of the internal point is the linear function of the contour curves. Obviously, if the contour of plate changes in small amount, the mesh quality will not change too much. That is to say, if the initial mesh is high-quality, the finite element model in each optimization step will also be high quality.

There are two preconditions that should be satisfied for the application of PMTM. First, the parameters of PSM should not be changed too much, otherwise, the mesh quality could not be guaranteed, which will reduce the precision of FEA. Second, the algorithm is only suitable for planar plates, and it could not deal with the plates with non-planar surface, such as the hull surface of the traditional transport ship. If those non-planar plates exist in the hull structure, their shape should not be changed when using PMTM. Despite of these limitations, PMTM could resolve the principal problems of hull structure shape optimization, and it might be an effective way to improve efficiency and quality of structure design for ships, platforms and other ocean structures.

ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China (Grant No. 51409042).

REFERENCE


