Dynamic output feedback robust model predictive control based on ellipsoidal estimation error bound for quasi-LPV systems

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Abstract: This paper presents a dynamic output feedback robust model predictive control (DOFRMPC) for the quasi-LPV (quasi-linear parameter varying) systems with both polytopic uncertainty and bounded disturbance. Our previous studies have shown that the refreshment of the estimation error set (EES) is a key knob for improving the control performance. In our previous work, for calculating the EES, the recursion of polyhedral set is utilized, which also involves the polyhedral outer approximation of the ellipsoid and the outer ellipsoidal approximation of polyhedral set. By applying S-procedure, a new method for refreshing EES is proposed in this paper which directly utilizes the recursion of ellipsoidal EES. A numerical example is given to illustrate the effectiveness of the approach.

Key Words: Dynamic output feedback, model predictive control, quasi-linear parameter varying systems, estimation error set

1 INTRODUCTION

The model predictive control (MPC), also known as the receding horizon control, is a class of optimization based control method, which explicitly utilizes the process model and measurements to optimize a performance function. At each time, the optimization is carried out, and the control input sequence generated. However, only the first element in this sequence is implemented. Due to its ability to handle the systems with hard constraints explicitly, it has attracted a lot of attentions, from both academic community and industrial society (see e.g., [1–4]). However, in the real processes, the precise model parameters are seldomly available. Hence, the robust MPC (RMPC) is more practical for the real applications.

The linear parameter varying (LPV) systems have their dynamics depending on the time-varying parameters, but take their real-time model parameters in the pre-specified sets. When the time-varying parameters of the linear system can be exactly known at the current time, the systems are called quasi-LPV (quasi-linear parameter varying) systems — it means that the current model parameters are exactly known at each time, but their future evolutions are uncertain but contained in the prescribed bounded sets. The parametric uncertainty can be dealt with within the frame of LPV systems. In RMPC, at each sampling time, usually a min-max optimization is utilized to enhance the performance of the LPV system, which considers all the possible realization of model parametric uncertainty (see e.g., [5]). The authors in [6] extended the approach in [5] to the quasi-LPV system with quasi-min-max optimization. The authors in [7] considered the quasi-LPV system with parameter-dependent control law. In [8–11], the controller design considers the time-varying parameters of the quasi-LPV system having bounds on their rates of variations.

In the real processes, the true states are usually unmeasurable, and only outputs with disturbance are available. Under this situation, the output feedback RMPC (OFRMPC) is more practical than the state feedback one for the real applications. For the OFRMPC with bounded disturbance without model parametric uncertainty, see [12]. For on-line OFRMPC with both polytopic uncertainty and bounded disturbance, see [13–19]. In [14], the authors considered the OFRMPC with model parametric uncertainty represented by Takagi-Sugeno (T-S) fuzzy model, which is the same as the quasi-LPV model. For the off-line OFRMPC with polytopic model parametric uncertainty, see [20–22]. For the system with norm-bounded model parametric uncertainty, [23] combines the off-line control law with the on-line optimization.

For the OFRMPC, the calculation of the bounds on the estimation error set (EES) is an important issue, which has to be considered for robust stability and physical constraints. The works [19, 21, 22] consider the fixed estimation error constraints in the off-line optimization problems. Hence, the on-line searched controller parameters can guarantee the satisfaction of estimation error constraints in the on-line searching stage. Both the off-line and the on-line optimization problems in [23] consider the fixed estimation error constraints. In [13–15, 17], the estimation error constraints are not considered in the main optimization problem, which are advantageous to enhance feasibility and optimality. Instead, the EESs can be refreshed in the separate on-line auxiliary optimizations. In [18], the bounds of true state, which plays the same role as the EESs, are calculated, and...
the method of refreshing the bounds of true state is investigated. The auxiliary optimizations in [13, 18] can determine whether or not the main optimization problem at next sampling time is to be solved. Different from [13, 18], the optimization problems proposed in [14, 15, 17, 19] can guarantee the recursive feasibility of main optimization problem.

In Algorithm 3 of [14], a polyhedral set is indirectly utilized to calculate the ellipsoidal EES, which is less conservative than the ellipsoidal EES directly obtained from the invariance condition of the closed-loop system. In the process of calculating one-step ahead EES by estimation error equation, the recursion of the polyhedral set is utilized, which also involves the polyhedral outer approximation of the ellipsoid and the outer ellipsoidal approximation of polyhedral set. Different from [14], by applying the S-procedure [24], the present paper directly utilizes the recursion of ellipsoidal EES.

Notations: For any vector $x$ and positive-definite matrix $W$, $||x||_W^2 := x^T W x$. $x(i)$ is the value of $x$ at time $k+i$, predicted at time $k$. For brevity, denote the current value $x(0)$ as $x(k)$. $I$ is the identity matrix with appropriate dimension. $\varepsilon_M := \{ \xi | T \xi \leq 1 \}$ denotes the ellipsoid associated with the symmetric positive-definite matrix $M$. All vector inequalities are interpreted in an element-wise sense. An element belonging to $\text{Co}\{\cdot\}$ means that it is a convex combination of the elements in $\{\cdot\}$, with the scalar combing coefficients nonnegative and their sum equal to 1. The symbol ‘*$’ induces a symmetric structure in the matrix inequalities. The superscript ‘$T$’ represents the matrix transposition. A value with superscript ‘*$’ means that it is the optimal solution of the optimization. The time-dependence of the MPC decision variable is often omitted for brevity.

2 PROBLEM STATEMENT

Consider the discrete-time uncertain LPV system

$$
\begin{align*}
x(k+1) &= A(k)x(k) + B(k)u(k) + D(k)w(k), \\
y(k) &= C(k)x(k) + E(k)w(k),
\end{align*}
$$

(2.1)

where $u \in \mathbb{R}^{n_u}$, $x \in \mathbb{R}^{n_x}$, $y \in \mathbb{R}^{n_y}$ and $w \in \mathbb{R}^{n_w}$ are the input, state, output and disturbance, respectively. The disturbance is persistent, unknown, bounded and satisfies $w(k) \in \varepsilon_{p_w}$. The input and output are required to satisfy

$$
-\bar{u} \leq u(k) \leq \bar{u}, \quad -\bar{\psi} \leq y(k+1) \leq \bar{\psi},
$$

(2.2)

where $\bar{u} = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_{n_u}]^T$, $\bar{u}_j > 0$, $j \in \{1, \ldots, n_u\}$; $\bar{\psi} = [\bar{\psi}_1, \bar{\psi}_2, \ldots, \bar{\psi}_{n_y}]^T$, $\bar{\psi}_j > 0$, $j \in \{1, \ldots, q\}$; $\Psi \in \mathbb{R}^{q \times n_y}$. Assume that $[A|B|C|D](k) \in \Omega$, i.e., there exist nonnegative coefficients $\lambda_i(k)$, $i \in \{1, \ldots, L\}$, such that $\sum_{i=1}^{L} \lambda_i(k) = 1$ and $[A|B|C|D](k) = \sum_{i=1}^{L} \lambda_i(k)[A_i|B_i|C_i|D_i|E_i]$. For the above system (2.1)-(2.2), the output feedback controller in the predictive control is of the following form ([13]):

$$
\begin{align*}
x_c(i+1|k) &= A_c(i) x_c(i|k) + B_c(i) u(i|k) + D_c(i) y(i|k), \\
u(i|k) &= C_c(i) x_c(i|k) + D_c(i) y(i|k),
\end{align*}
$$

(2.3)

(2.4)

where $x_c \in \mathbb{R}^{n_x}$ is the controller state; $\{A_c, B_c, C_c, D_c\}(i|k)$ are the controller parametric matrices. For this paper, we consider the special case where $\lambda_i(k)$’s are exactly known at the current time $k$ (see [13, 18]; otherwise, see [16, 17]). The controller parameters take the following parameter-dependent form

$$
\begin{align*}
A_c(i|k) &= \sum_{l=1}^{L} \sum_{j=1}^{L} \lambda_l(k+i) \lambda_j(k+i) A_{ij}^t, \\
B_c(i|k) &= \sum_{l=1}^{L} \lambda_l(k+i) B_{ij}, \\
C_c(i|k) &= \sum_{j=1}^{L} \lambda_j(k+i) C_{ij}^t,
\end{align*}
$$

(2.5)

Hence, the augmented closed-loop system, based on (2.3) and the predictions made by (2.1), is

$$
\dot{x}(i+1|k) = \Phi(i, k) \dot{x}(i|k) + \Gamma(i, k) w(k+i), \\
\forall i \geq 0, \quad \dot{x}(0|k) = \bar{x}(k),
$$

(2.6)

where

$$
\bar{x} = [\bar{x}^T, \bar{x}^T] = \Phi(i, k) \bar{x}(i|k) + \Gamma(i, k) w(k+i),
$$

and satisfies $e(k) = x(k) - x_c(k)$

(2.7)

and satisfies $e(k) \in \varepsilon_{M_e}(k)$.

3 Synthesis approach of output feedback robust MPC

3.1 The optimization problem

In the DOFRMPC, at each time $k$, the following optimization problem is solved (see [13, 14]):

$$
\begin{align*}
\min_{\gamma, Q, A_{ij}^t, B_{ij}, C_{ij}^t, D_{ij}} \max_{\lambda_i(k+i) \in \Omega, w(k+i) \in \varepsilon_{p_w}} & \sum_{l=1}^{L} \lambda_l(k+i) \sum_{j=1}^{L} \lambda_j(k+i) \gamma_{ij}^{gb} \\
\text{s.t.} & \sum_{j=1}^{L} \lambda_j(k+i) \sum_{j=1}^{L} \lambda_j(k+i) \gamma_{ij}^{gb} \geq 0,
\end{align*}
$$

(3.1)

$$
\begin{align*}
\hat{\gamma}_{ij}^{gb} &\geq 0, \quad Z_{ss} \leq \frac{1}{2} \gamma_{ss}^2, \quad j = 1, \ldots, L, \quad s = 1, \ldots, n_{w}, \\
\sum_{s=1}^{L} \lambda_s(k+i) \sum_{j=1}^{L} \lambda_s(k+i) \gamma_{ss}^{gb} &\geq 0,
\end{align*}
$$

(3.2)

$$
\begin{align*}
\Xi_{ss} &\leq \bar{\phi}_{ss}^2, \quad h = 1, \ldots, L, \quad s = 1, \ldots, q, \\
x_T^k(k)(M_1 - M_1) x_c(k) &\leq 1 - \phi,
\end{align*}
$$

(3.3)

(3.4)

$$
\begin{align*}
M_1 &\leq g M_e(k),
\end{align*}
$$

(3.5)

$$
\left[ \begin{array}{cc}
- M_1 & - M_1 \\
- M_1 & - M_1
\end{array} \right] \geq 0, \quad \left[ \begin{array}{cc}
M_1 - M_1 & I \\
I & Q_1
\end{array} \right] \geq 0,
$$

(3.6)

(3.7)

where (3.2) is the quadratic boundedness condition; (3.3) and (3.4) are the input and output constraints, respectively; moreover,
\[ \begin{align*}
\mathbf{Q}^2_{BH} &= \begin{bmatrix} *_{11} & *_{12} & * \\ *_{21} & *_{22} & * \\ *_{31} & *_{32} & * \end{bmatrix},
\mathbf{Q}_{22} &= \begin{bmatrix} Q_1 & * \\ I & M_1 \end{bmatrix}.
\end{align*} \]

\[ \begin{align*}
\mathbf{\gamma}_{11} &= \begin{bmatrix} (1-\alpha)M_1 & * \\ 0 & \gamma I \end{bmatrix},
\mathbf{\gamma}_{21} &= \begin{bmatrix} (1-\alpha)I & (1-\alpha)Q_1 \\ 0 & 0 \end{bmatrix},
\mathbf{\gamma}_{31} &= \begin{bmatrix} A_t + B_t\mathbf{D}_tC_j & A_tQ_1 + B_t\mathbf{C}_t \\ M_1A_t + B_t\mathbf{D}_tC_j & M_1A_t + B_t\mathbf{D}_tE_j + D_t \end{bmatrix}.
\end{align*} \]

\[ \begin{align*}
\mathbf{\gamma} = \begin{bmatrix} M_1 & * & * \\ I & Q_1 & * \\ 0 & 0 & P_w \end{bmatrix},
\mathbf{\gamma}^T = \begin{bmatrix} \sqrt{\mathbf{D}_tC_j} & \sqrt{\mathbf{C}_t} & \sqrt{\mathbf{D}_tE_j} & Z \end{bmatrix}.
\end{align*} \]

\[ \begin{align*}
\mathbf{\gamma}_{31} &= \sqrt{\Psi}C_h \begin{bmatrix} A_t + B_t\mathbf{D}_tC_j \\ \mathbf{A}_tQ_1 + B_t\mathbf{C}_t \end{bmatrix},
\mathbf{\gamma}_{52} &= \sqrt{\Psi}C_h \begin{bmatrix} A_t + B_t\mathbf{D}_t \end{bmatrix},
\mathbf{\gamma}_{53} &= \sqrt{\Psi}C_h \begin{bmatrix} B_t\mathbf{D}_tE_j + D_t \end{bmatrix}.
\end{align*} \]

\[ \begin{align*}
Q^{-1} &= \begin{bmatrix} M_1 & M_2^T \\ M_2 & M_1 \end{bmatrix},
Q &= \begin{bmatrix} Q_1 & Q_2^T \\ Q_3 & Q_3 \end{bmatrix}.
\end{align*} \]

The new algorithm is as follows:

**Algorithm 1.** Initially, choose \( x_c(0) \) and \( \varepsilon_{M_c(0)} \). At each time \( k \geq 0 \), perform the following steps:

1. Solve the optimization problem (3.1)-(3.7) to obtain \( (\alpha, \gamma, g, A_t^j, B_t^j, C_t^j, D_t, Q_1, M_1, M_2, M_3, M_4, Z, \Xi) \).

2. Calculate \( [A_t^j, B_t^j, C_t^j, D_t]^\ast \) by (3.8), then obtain the control parameters \( [A_c, B_c, C_c, D_c]^\ast (k) \).

**4 On-line refreshment of EES to improve the control performance**

4.1 A new method of refreshing EES

When the step (iv) of Algorithm 1 is performed, based on the invariance constraint of the close-loop system (see [14]), the one-step ahead EES denoted by \( \varepsilon_{M_c(k)} \) is obtained. The following Lemma 1 is used to calculate the one-step ahead EES, which is based on the estimation error equation.

**Lemma 1.** For the quasi-LPV system (2.1), the problem (3.1)-(3.7) is solved at each time \( k \). If \( x(k) \) belongs to an elliptoidal EES, i.e.,

\[ \mathcal{E}(k) = \{ x(k) \in \mathcal{R}^n \mid (x(k) - x_c(k))^TQ_c^{-1}(k) \times (x(k) - x_c(k)) \leq 1, Q_c(k) \geq 0 \}, \]

then the one-step ahead EES is

\[ \mathcal{E}(k') = \{ x(k') \in \mathcal{R}^n \mid (x(k') - x_c(k'))^T\bar{Q}_c^{-1}(k') \times (x(k') - x_c(k')) \leq 1, \bar{Q}_c(k') \geq 0 \}, \]

and can be solved by the following optimization problem:

\[ \min \quad \text{trace}(\bar{Q}_c(k')), \]

s.t. \[ \begin{bmatrix} \Delta_5 \quad \Pi \\ \Pi \quad \bar{Q}_c(k') \end{bmatrix} \geq 0, \]

\[ \Delta_5 = \text{diag}(1 - \phi_1 - \phi_2, \phi_1 I, \phi_2 P_w), \]

\[ \Pi = [A(k)x_c(k) - x_c(k') + B(k)u(k), A(k)E_c(k), D(k)]. \]

**Proof.** According to (4.1),

\[ x(k) = x_c(k) + E_c(k)z, \]

where \( E_c(k) \) is the Cholesky factorization of \( Q_c(k) = M_c^{-1}(k) \), satisfying \( Q_c(k) = E_c(k)E_c^T(k) \) and \( \| z \| \leq 1 \). Define \( \zeta = [1, z, w]^T \in \mathcal{R}^n \), \( n_\zeta = 1 + n_x + n_w \). According to (2.1)(2.3)(4.4)(4.5), for the current system parameters, based on the estimation error equation, the one-step ahead EES is

\[ e(k') = x(k') - x_c(k') = A(k)x_c(k) - x_c(k') + B(k)u(k) + A(k)E_c(k)z + D(k)w(k) = \Pi \zeta. \]

Hence, \( x(k') \in \mathcal{E}(k') \) is guaranteed by

\[ \zeta^T \text{diag} \{1, 0, 0\} \zeta - \zeta^T \Pi^T \bar{Q}_c^{-1}(k') \Pi \zeta \geq 0. \]

\[ \| z \| \leq 1 \text{ and } w(k)^T P_w w(k) \leq 1 \text{ can be rewritten as, respectively,} \]

\[ \zeta^T \text{diag} \{1, -I, 0\} \zeta \geq 0, \]

\[ \zeta^T \text{diag} \{1, 0, -P_w\} \zeta \geq 0. \]
According to the S-procedure, the sufficient condition for “(4.8)-(4.9) $\implies$ (4.7)” to hold true is that there exist non-negative scalars $\phi_1$, $\phi_2$ such that
\[
\zeta^T \text{diag}\{1, 0, 0\} \zeta = \zeta^T \Pi^T \hat{Q}_e^{-1}(k') \Pi \zeta - \phi_1 \zeta^T \times \\
\text{diag}\{1, -1, 0\} \zeta - \phi_2 \zeta^T \text{diag}\{1, 0, -P_w\} \zeta \geq 0 .
\] (4.10)

For all $\zeta$, a necessary and sufficient condition for (4.10) is
\[
\text{diag}\{1, 0, 0\} - \Pi^T \hat{Q}_e^{-1}(k') \Pi - \phi_1 \text{diag}\{1, -1, 0\} - \phi_2 \text{diag}\{1, 0, -P_w\} \geq 0 ,
\] (4.11)
i.e.,
\[
\text{diag}\{1 - \phi_1 - \phi_2, \phi_1 I, \phi_2 P_w\} - \Pi^T \hat{Q}_e^{-1}(k') \Pi \geq 0 .
\] (4.12)

By applying the Schur complement, it is shown that (4.12) is equivalent to (4.3).

**Remark 1.** To calculate one-step ahead EES in [13], and the bounds of true state at the next time in [18], an outer polyhedral approximation of $\varepsilon_{P_w}$ is used. The present paper removes this possible conservatism.

### 4.2 The overall solutions

At each time $k$, $x_\varepsilon(k)$ (dashed line) and $E(k')$ (solid line) satisfy one of the relationships shown in Figure 4.1. Combining Algorithm 1 and the method of refreshing EES, the following Algorithm 2 is proposed:

**Algorithm 2. (On-line DOFRMPC with ellipsoidal EES)**

Initially, choose $x_\varepsilon(0)$ and the EES $\varepsilon_{M_e(0), M_e(0)} = \hat{Q}_e^{-1}(0)$. At each time $k \geq 0$,

(a) perform steps 1-4 of Algorithm 1;

(b) solve the optimization problem (4.3) to obtain $\hat{Q}_e(k')$.

If $\hat{Q}_e^{-1}(k') \geq M_e(k')$, then $M_e(k') = \hat{Q}_e^{-1}(k')$.

The proof of recursive feasibility of Algorithm 2 can be referred to as in section D of [14]. When the main optimization problem (3.1)-(3.7) is solved at time $k$, the solutions $\{\alpha, \gamma, 30, \hat{A}^1, \hat{B}^1, \hat{C}^1, \hat{D}, \hat{Q}_4, \hat{M}1, \hat{M}4, Z, \Xi\}^*(k)$ are obtained.

Choose
\[
M_4(k') = M_4^*(k) - Q_4^{-1}(k)^{-1}, \\
M_3(k') = M_3^*(k)[M_3^*(k)^{-1} - Q_4^{-1}(k)^{-1}]M_4^*(k)^T ,
\]
\[
\varrho(k') = [1 - x_\varepsilon^T(k')(M_3^*(k) - M_4^*(k))(k)x_\varepsilon(k')]^{-1} ,
\]
\[
M_e(k') = M_e^*(k)\varrho(k'), Q_4(k') = Q_4^*(k) .
\] (4.13)

then (3.5)-(3.7) are satisfied as follows:
\[
x_\varepsilon^T(k')(\hat{M}_3(k') - M_4(k'))x_\varepsilon(k') = 1 - \varrho(k') ,
\] (4.14)
\[
M_1(k') = \varrho(k')M_e(k') ,
\] (4.15)
\[
\begin{bmatrix}
\hat{M}_3(k') - M_4(k') \\
\hat{M}_1(k') & \hat{M}_4(k')
\end{bmatrix} \geq 0 ,
\] (4.16)
\[
\begin{bmatrix}
M_1(k') - M_4(k') & I \\
I & Q_4(k')
\end{bmatrix} \geq 0.
\] (4.19)

Furthermore, when $\hat{Q}_e^{-1}(k') \geq M_e(k')$, let $M_e(k') = \hat{Q}_e^{-1}(k')$. Then, (3.5)-(3.7) are satisfied as follows:
\[
x_\varepsilon^T(k')(\hat{M}_3(k') - M_4(k'))x_\varepsilon(k') = 1 - \varrho(k') ,
\] (4.17)
\[
M_1(k') \leq \varrho(k')\hat{Q}_e^{-1}(k') ,
\] (4.18)
\[
\begin{bmatrix}
\hat{M}_3(k') - M_4(k') \\
-\hat{M}_1(k') & \hat{M}_4(k')
\end{bmatrix} \geq 0 ,
\] (4.19)
\[
\begin{bmatrix}
M_1(k') - M_4(k') & I \\
I & Q_4(k')
\end{bmatrix} \geq 0.
\] (4.19)

In the optimization (3.1)-(3.7), only (3.5)-(3.7) are related with the process state information. Therefore, at time $k'$, (3.1)-(3.7) is feasible by further choosing $\{\alpha, \gamma, 30, \hat{A}^1, \hat{B}^1, \hat{C}^1, \hat{D}, \hat{Q}_4, \hat{M}1, \hat{M}4, Z, \Xi\}^*(k') = \{\alpha, \gamma, 30, \hat{A}^1, \hat{B}^1, \hat{C}^1, \hat{D}, \hat{Q}_4, \hat{M}1, \hat{M}4, Z, \Xi\}^*(k)$.

**Theorem 1.** For the system (2.1), DOFRMPC is implemented according to Algorithm 2. Suppose $w(k) \in \varepsilon_{P_w}$ for all $t \geq 0$, and $e(0) \in \varepsilon_{M_e(0)}$. If (3.1)-(3.7) is feasible at time $k = 0$, then $x$ will converge to a neighborhood of $\tilde{x} = 0$, and constraints (2.2) is satisfied for all $k \geq 0$.

**Proof.** The recursive feasibility of Algorithm 2 means that, the optimal solutions at time $k$ will be feasible solutions at time $k'$. Suppose at time $k$, the performance index is $\gamma(k')$, then at time $k'$, $\gamma(k') = \gamma(k)$ is the upper bound of the performance index. By solving the optimization problem (3.1)-(3.7) at time $k'$, it will satisfy $\gamma(k') \leq \gamma(k)$. Considering all time $k \geq 0$, $\gamma(k)$ will not increase with time $k$. Hence, the augmented state $\tilde{x}$ will converge to the neighborhood of $\tilde{x} = 0$ and stay in the neighborhood of $\tilde{x} = 0$ thereafter. Satisfication of constraints (2.2) is due to (3.3)-(3.4).

### 5 Numerical example

Consider a system with $L = 2$, $A(k) = \begin{bmatrix}
0.385 & 0.33 \\
0.21 & 0.59
\end{bmatrix}$, $B(k) = \begin{bmatrix}
1 & 0
\end{bmatrix}^T$,
\[
C(k) = \begin{bmatrix}
0 & 1
\end{bmatrix}, D(k) = \begin{bmatrix}
0.3 & 0.3
\end{bmatrix}^T , E(k) = 1,
\]
where $\mu(k)$ is an uncertain parameter satisfying $|\mu(k)| \leq 0.11$, $\lambda_1(k) = \frac{1 + \sin(k)}{2}, \lambda_2(k) = \frac{1 - \sin(k)}{2}$.

The input and output constraints are $|u| \leq 2$, $|y| \leq 24$, respectively.

For Algorithm 1 (Alg1), Algorithm 2 (Alg2) and Algorithm 3 in [14] (TFS11Alg3), choose $P_w = 25$, $\mathcal{Q} = 25$, $\mathcal{R} = 1$.

For Algorithm 1 (Alg1), the resultant state trajectories of augmented closed-loop system and the corresponding control input signals are shown in Figures 5.1 and 5.2, respectively. It shows that, as compared with Alg1 and TFS11Alg3, the control performance in Alg2 is improved. The control input signals shown in Figure 5.2 illustrate that the input constraints are satisfied. Figure 5.3 shows the responses of the true state and estimated state, with the corresponding evolution of EES for three algorithms.
Figure 4.1: The relationship between $E(k')$ and $\varepsilon_M(k')$ 
(up, $E(k') \supset \varepsilon_M(k')$; middle, $E(k') \subseteq \varepsilon_M(k')$; down, the boundaries of $E(k')$ and $\varepsilon_M(k')$ intersects)

Figure 5.1: The state trajectories of the closed-loop system

Figure 5.2: The control input signals

Figure 5.3: The responses of the true state and estimated state, with the corresponding evolution of the estimation error set, for Alg1, TFS11Alg3 and Alg2
LMI Toolbox of Matlab 7.6 (Pentium 4 CPU 2.40GHz, 1G Memory) is utilized for the simulation. Let $J = \sum_{i=0}^{25} \left[ \mathcal{L} y(i)^2 + \mathcal{R} u(i)^2 \right]$. For Algorithm 1 (Algorithm 2, TFS11Alg3), $J = 13167(J = 12898, J = 13059)$ is obtained and it takes $32.37s (41.37s, 38.93s)$ within $k \leq 25$.

6 Conclusions

For the systems with both polytopic uncertainty and bounded disturbance, a new method of refreshing ellipsoidal EES is investigated. By applying S-procedure to refresh EES, less conservative EES can be obtained. The augmented state can converge to a neighborhood of $\ddot{x} = 0$ and stay in it thereafter.

REFERENCES