Combination of Analog Network Coding and Compressed Sensing in Clustered Chain-type Topology

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Abstract: Transmission in wireless sensor network (WSN) is energy intensive and time consuming. To solve this problem, combination of Analog Network Coding (ANC) and Compressed Sensing (CS) scheme is proposed to save transmission and time slots cost (TSC) in chain-type topology. In this paper, the WSN is divided into many rectangular areas. Then each rectangular area is divided into many clusters, and head nodes of these clusters form the chain-type topology which can take full advantage of the combination of ANC and CS scheme. Analysis and simulations show that with effective partition of clusters, our scheme can achieve perfect transmission and TSC performances with little loss, and it can save transmission and TSC by a factor of more than 10 when compared with other schemes in our simulation. Besides, our scheme is much more energy-saving and lifetime of our scheme outperforms the contrastive schemes.

Keywords: compressed sensing; CCT; transmission; TSC; lifetime

1 Introduction

Wireless Sensor Network (WSN) is a network made of micro sensors which have sensing, computing and communications capabilities with Ad hoc. WSNs can monitor the targets and collect information of the environment in time by the cooperation of large amount of sensors. Data acquisition and transmission are main tasks of the wireless sensor networks: data detected by the nodes are transmitted to the sink by wireless transmission, and the sink node would reconstruct and restore the original data detected.

In the traditional data transmission scheme, data of each sensor node are transmitted along the upstream of routing tree to the sink node hop by hop. And nodes close to the sink should not only transmit their own detected data, but also forward data of nodes far from the sink, resulting in high load of nodes close to the sink. As the traffic load is not balanced, the energy of sensor nodes near the sink will be used up firstly, and it will result in network failure. In addition, this data transmission scheme does not compress data with the spatial correlation, resulting in transmission inefficiencies of nodes and high energy consumption of wireless communication.

Taking the power limits of nodes, bandwidth limits and the correlation of data into account, many scholars propose that we could apply Compressed Sensing (CS) to the data transmission of WSNs. Proper transmission schemes may significantly reduce transmission and time slots cost (TSC) and improve the throughput. However, conventional data transmission schemes have many drawbacks. Compressive Data Gathering (CDG) [1], which employs distributed coding to compress data correlation, can realize the load balancing of the entire sensor network. However, the CDG scheme uses a uniform pattern in data transmission, where all nodes transmit the same amount of data regardless of their hop distances to the sink, making it inefficient in saving transmission costs in 2-D networks. Paper [2] points out CDG’s drawback and propose hybrid-CS scheme to save transmission, just as the clustered CS scheme [3], but both of them can’t reduce TSC. Paper [4] proposes a scheme which firstly uses CS operation and then delivers data using Analog Network Coding (ANC) transmission scheme, but both steps are independent, and it does not take full advantage of the two algorithms and together them at the same time to further improve the efficiency of signal processing.

In this paper, the WSN is divided into clustered chain-type topologies (CCTs) and apply the ANC-CS scheme we have proposed in paper [5] to CCTs. Besides, transmission and TSC performances of ANC-CS in CCT are compared with that of other schemes, and results show that beautiful solution can be got to obtain good transmission and TSC performances which are much better than these contractive schemes. Moreover, we discuss lifetime of these schemes when we adopt the solution, and the CCT uses our scheme can live much longer than CCTs use other schemes.

The rest of this paper is organized as follows. In Section 2, we briefly review the theory of CS and the proposed ANC-CS scheme. And we apply the ANC-CS scheme to CCT in Section 3. Section 4 presents and discusses the numerical results and analyzes the performance of our scheme. Finally, conclusion is drawn in Section 5.

2 Related works

Configured sensor nodes in WSN are used for monitoring a given region, and the detected values of different sensor nodes are related, for example, when detecting the temperature of a given region, the detected values of the adjacent nodes are similar, namely the detected data subject to the sparsity in a domain. Based on the above features of data in WSN, CS can be used to WSN to reduce transmission. Besides, the combination of ANC and CS transmission scheme can also be used in WSN to further improve performance.
2.1 Compressed sensing

Compressed sensing theory [6] figures out that as long as the signal is sparse or compressible, then it can be projected to low-dimensional space by a sensing matrix uncorrelated to the sparse base. And the original signal can be reconstructed correctly by solving the convex optimization problems.

Assuming that the original signal is \( X_{N \times 1} \), and there is an orthogonal basis under which the original signal is \( K \)-sparse (i.e. there are only \( K \) values aren’t 0 or the \( K \) values are much bigger than other values).

\[
X_{N \times 1} = \Psi_{N \times N} \times \theta_{N \times 1} \quad (\theta \text{ is a } K\text{-sparse vector}) \quad (1)
\]

Then it can be found that one observation matrix \( \Phi_{M \times N} \) \( (K < M \ll N) \) [6]) which is uncorrelated to \( \Psi \) to compress \( X \) to \( M \) measured values, i.e. \( Y = \Phi X \).

E.Candès et al. demonstrated that the original signal \( X \) can be reconstructed by solving the minimum 0-norm, and the underdetermined system of equations problem could be transferred to the minimum 0-norm problem, i.e.

\[
\hat{x} = \arg \min_x \|\xi\|_0 \quad \text{s.t. } y = \Phi X . \quad (2)
\]

Since the optimization problem of (2) is a nondeterministic polynomial-time hard problem and it can’t be solved. So D.Donoho proposed that 0-norm could be replaced by 1-norm, and then the convex optimization problem can be described as follows,

\[
\hat{x} = \arg \min_x \|\xi\|_1 \quad \text{s.t. } y = \Phi X . \quad (3)
\]

So the signal reconstruction problem can be solved by converting it into a linear programming problem.

As soon as CS is proposed, it attracts many scholars dedicated to it. Researches of compressed sensing mainly focus on the sparse representation of signals [7], design of transmission matrices [8] and reconstruction algorithms. Recently, researchers propose that we could apply CS to WSN, and they have designed the CS transmission schemes in WSN. In CS, since the sink node needs to receive the sum value of each node, then each node just need to deliver the sum value of its previous nodes’ data and its own data, instead of transmit all the data one by one. So it can avoid the high load of nodes close to sink and realize the load and energy consumption balance of nodes. The problem of transmission schemes based on CS is that since each node needs to send \( M \) signals, each sending and receiving need to occupy one time slot to avoid the wireless collision and interference. For the WSN with a large number of nodes, the number of required TSCs is proportional to the number of nodes, and it will result in a larger transmission delay and affect the real-time performance of the observing system.

2.2 ANC-CS transmission in chain-type topology

Network coding is a network data transmission method proposed by Ahlswede et al. in 2000. Nodes can only store and forward packets in the traditional routing-based networks. While in the network using network coding, nodes can even do many encoding operations (e.g. linear transformation). Katti et al. [9] have proposed an analog network coding scheme using interference and it could save TSC greatly. We have proposed the ANC-CS transmission scheme in chain-type topology, and analyzed its transmission and TSC performance in clustered WSN [5].

The ANC-CS scheme is mainly based on the CS transmission scheme in chain-type topology. In the chain-type topology with \( N \) nodes, assuming that node \( i (i = 1, 2, \ldots, N) \) needs to transmit \( x_i \). Based on the CS theory, if each node transmits \( \phi_i x_i \), in which \( \phi_i = \{\varphi_1, \varphi_2, \ldots, \varphi_M\}^T (M \ll N) \) is a column vector of \( \Phi_{M \times N} \) then the sink node can reconstruct the original information \( X \) with the received data \( \Phi X \) by the CS reconstruction algorithm, e.g. SL0, SL1 and OMP.

![Figure 1 CS and ANC-CS in chain-type topology](image)

As shown in Figure 1, different from the CS scheme which transmits packets hop by hop in WSN, in the ANC-CS scheme, sending or receiving of different nodes can be synchronous. When \( T = 2k \), nodes \( 2i (i = 1, 2, \ldots, k) \) transmit packets, and nodes \( 2i + 1 \) receive the sum value of packets from their adjacent nodes. It is similar when \( T = 2k + 1 \). It is remarkable that if the sensor node \( i \) has transmitted the \( m \) packets \( \phi_i x_i \), it does not need to transmit or receive packets until transmission for the next \( x_i \). Even though the transmission modes are different, the sink node of ANC-CS scheme can also receive the same data as that of CS scheme in chain-type topology while it can reduce TSC effectively. Considering the good performance of ANC-CS scheme in chain-type topology, with effective partition of WSN, we can take full advantage of ANC-CS scheme and get perfect performance in WSN.

3 ANC-CS scheme in CCT

Figure 2 shows a large scale sensor network where sensors are randomly deployed in the region of interest and monitor the environment regularly. In this paper, the WSN is divided into many rectangular areas to form chain-type topology. Based on the theory that Hybrid-CS could reduce the transmission effectively [2] and the sub-cell scheme of WSN in paper [3], each rectangular area is divided into clusters, i.e. the clustered chain-type topology. And transmission of cluster head nodes (CHNs) is just as transmission of a chain-type topology.
Assuming the rectangular area has $N$ sensor nodes (SNs), it is divided into $\beta$ clusters with $\alpha$ ($\alpha\beta = N$) SNs in each cluster. Even if clusters are not divided uniformly, data of CS will not change. For simplicity, we assume every cluster has the same number of SNs. We also assume each SN in cluster $j$ (for $j = 1, 2, \cdots$) in cluster $j$ will be transmitted to the CHN $j$ in different time slots which are allocated by the CHN $j$. So each SN needs to transmit only once to deliver packet to the CHN, and the CHN $j$ will get all the packets of SNs in cluster $j$, i.e. $X_j = [x_{j1}, x_{j2}, \cdots, x_{j\alpha}]$. Then the CHN $j$ can compress $X_j$ with sensing matrix $\Phi_j$, i.e.\[
\text{DATA}_j = \Phi_j X_j^T,\]where $\Phi_j$ is the $m \times \alpha$ CS compression matrix, and $\text{DATA}_j$ is the $m \times 1$ compressed data of the $\alpha \times 1$ signal $X_j$. Even if $m > \alpha$, to the whole CCT, packets are compressed at ratio of $m/N$ ($m \ll N$). Then each CHN transmit $\text{DATA}$ to the sink node as the ANC-CS scheme, and the sink node receives data\[
Y = \Phi_{\text{big}} X_{\text{big}} = [\Phi_1 \Phi_2 \cdots \Phi_{j}] \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{\beta} \end{bmatrix},\]where $\Phi_{\text{big}}$ is an $m \times N$ matrix, $X_{\text{big}}$ is the $N$ original packets needed to be transmitted and $Y$ is the compressed vector. Then at the sink node, the original packets $X_{\text{big}}$ can be reconstructed with CS reconstruction algorithm.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{clustered_chain_type_topology.png}
\caption{Clustered chain-type topology}
\end{figure}

\section{Performances analysis}

In this section, transmission, time slots cost and lifetime performances are test to verify the advantages of ANC-CS scheme in clustered chain-type topology.

\subsection{Transmission and TSC}

Assuming that there are $N$ SNs in the CCT which is divided into $\beta$ clusters, and there are $\alpha$ ($\alpha\beta = N$) nodes in each cluster. Figure 3 shows the transmissions relation of each node in clusters or between clusters of three schemes in CCT, and transmissions relation of ANC-CS scheme is the same as that of Hybrid-CS.

\begin{table}
\centering
\begin{tabular}{|c|c|c|}
\hline
Scheme & Transmission & TSC \\
\hline
No-CS & $N - \beta + \frac{(1+\beta)\alpha}{2}$ & $[\alpha-1]+\frac{(1+\beta)\alpha}{2}$ \\
Plain-CS & $m(N-\beta)+\frac{m\beta}{2}$ & $[m\beta]+[\beta+1]$ \\
Hybrid-CS & $N - \beta + \frac{m\beta}{2}$ & $[\alpha-1]+[\beta+2(m-1)]$ \\
ANC & $N - \beta + \frac{m\beta}{2}$ & $[\alpha-1]+[\beta+2(m-1)]$ \\
\hline
\end{tabular}
\caption{Transmission and time slots cost of 4 schemes}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{transmission_relationship_map.png}
\caption{Transmission relationship map}
\end{figure}

To highlight the transmission and TSC performances of ANC-CS in CCT, NTransmission and N/TSC are used as the performance criterion. In our simulation, SNs in the CCT is $N = 10000$, compressed dimension $m$ ranges from 40 to 200 to ensure the reconstruction performance and effective compression, and nodes in cluster ranges from 1 to 2000.

Figure 4 shows the results of N/Transmission and Figure 5 is the $\alpha$-transmission curves when $m = 40$. As shown in Figure 4, transmission of ANC-CS in CCT decreases with $\alpha$ and it increases with $m$ linearly, but when $\alpha$ is much bigger than $m$, the influence of $m$ can be neglected. In Figure 5, it can be seen that transmission of ANC-CS scheme decreases very fast only when $\alpha \leq 200$, while it decreases very slow when $\alpha > 200$. And when $\alpha$ is very big, the decrease ratio of transmission tend to be stable and the transmission tend to be $N$.

Similarly, Figure 6 and Figure 7 show the performances of TSC. The TSC performance of ANC-CS is much better than other schemes when $\alpha$ is smaller than 200, but it gets worse when $\alpha > 100 (\sqrt{N})$, and it even be worse than the Plain-CS scheme when $\alpha$ is bigger than $600 (\sqrt{(m-3)^2 + 4(m-1)N - (m-3)})/2$.
shown in Figure 6, TSC performance of ANC-CS in CCT increases with \( m \) linearly when \( \alpha \) is small, and the influence of \( m \) can also be neglected when \( \alpha \) is big.

In general, Plain-CS scheme in CCT needs much less transmission or TSC when \( \alpha \) is big while its transmission is much bigger than other schemes. But the ANC-CS scheme in CCT needs much less transmission or TSC when \( \alpha \) is small. Though there is no optimal solution for the ANC-CS scheme to get the best transmission and the best TSC performance in CCT, we can still get the second-best solution to bring the performance close to the best. When \( \alpha \) ranges from \( \sqrt{N} \) to \( 2\sqrt{N} \), the TSC is close to the minimum value meanwhile the transmission is also very small. So we can divide the CCT into \( N/\alpha \), \( \alpha \in [\sqrt{N}, 2\sqrt{N}] \) clusters to get the second second-best solution. Because \( m \ll N \) in CS, we can estimate that transmission of the ANC-CS in CCT is \( \Theta(N + m\sqrt{N}) \) while it is \( \Theta(mN) \) of Plain-CS in CCT. And TSC of ANC-CS in CCT is \( \Theta(\sqrt{N} + m) \) while it is \( \Theta(m\sqrt{N}) \) that of Plain-CS in CCT. E.g. when \( m = 40 \), if we set nodes in cluster \( \alpha \) to be 150, we can get the transmission ratio of the No-CS, Plain-CS, Hybrid-CS and ANC-CS which is 27.64:31.74:1:1 and the TSC ratio is 1152.47:9.21:9.59:1.

### 4.2 Analysis of lifetime in CCT

In this section, energy consumption of No-CS scheme and other three schemes which are based on CS in CCT are analyzed, and simulate lifetime of these schemes.

For simplicity, it is assumed that the energy consumption of each SN for receiving is equal to that for transmitting [10], i.e. 1. Since load balancing of CS, energy consumptions of CHNs in each cluster is the same in these CS schemes, but to the No-CS scheme, the last cluster consumes the most energy. It is defined that if there is one cluster can’t offer energy to receive and transmit packets, i.e. the CCT is dead. So lifetime of the No-CS scheme in CCT is lifetime of CCT’s last cluster. And to the three CS schemes, lifetime is any one cluster’s lifetime. Table II shows the cluster energy consumptions of each scheme for one complete transmission of one cluster in CCT. Energy consumption of each CHN consists of three parts: energy consumption for receiving packets from nodes in cluster (NIC), energy consumption for receiving packets from the prior CHN and energy consumption for transmitting its own packets. Meanwhile, to every NIC, they only need to transmit their own packets without receiving packets.

**Table II Cluster energy consumption of 4 schemes**

<table>
<thead>
<tr>
<th>Scheme</th>
<th>CHN</th>
<th>NIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-CS</td>
<td>([\alpha - 1] + [\alpha \times (\beta - 1)] + [\alpha \times \beta])</td>
<td>1</td>
</tr>
<tr>
<td>Plain-CS</td>
<td>([m] + [m] + [m])</td>
<td>(m)</td>
</tr>
<tr>
<td>Hybrid-CS</td>
<td>([\alpha - 1] + [m] + [m])</td>
<td>1</td>
</tr>
<tr>
<td>ANC</td>
<td>([\alpha - 1] + [m] + [m])</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 8 Lifetimes of Plain-CS and ANC-CS in CCT

As shown in Table II, cluster energy consumption of Hybrid-CS is the same as that of ANC-CS, and cluster energy consumption of No-CS is obviously bigger than that of three CS schemes. So we mainly analyze lifetimes Plain-CS and ANC-CS in CCT. In our simulation, selection of CHN refers to the selection of CHN in low energy adaptive clustering hierarchy,

\[ T(n) = \begin{cases} 
\frac{P}{1 - P \left( r \mod \frac{1}{P} \right)} & \text{if } n \in G \\
0 & \text{otherwise}
\end{cases} \]

where \( P \) denotes the ratio of CHNs, \( r \) denotes the index of round and \( G \) denotes the set in which there is no nodes becomes CHN during the latest given \( 1/P \) rounds. If \( T(n) > \text{threshold} \) and SN \( n \) has enough energy to receiving and transmitting packets as a CHN, SN \( n \) will become the next CHN.

Assuming that the initial energy of each SN is 5000 and lifetime is the time that there is no node in the cluster can afford enough energy to become a CHN, we can get lifetime of Plain-CS scheme and ANC-CS scheme as shown in Figure 8. ANC-1 and Plain-1 denote the first time when there are nodes can’t afford one transmitting in cluster. In some cases, it equals to the case that there is no nodes can become the CHN of the cluster. We can see that lifetime of ANC-CS is much higher than that of Plain-CS, and the gap increases with nodes in cluster \( \alpha \) quickly. Since energy consumption for each NIC’s transmitting of ANC-CS scheme is much smaller that of Plain-CS and the probability of each NIC becomes the CHN is the same, so to \( \alpha \) transmissions, the expectation energy consumption (EES) for every transmission is

\[ \text{EES} = \frac{E_{\text{CHN}} + E_{\text{NIC}} (\alpha - 1)}{\alpha}, \]

where \( E_{\text{CHN}} \) is energy consumption of CHN and \( E_{\text{NIC}} \) is energy consumption of NIC. EES of ANC-CS in CCT is \((2m - 2)/\alpha + 2, \alpha \in [1: N]\) which ranges from 2m to 2, so lifetime of ANC-CS increases with \( \alpha \) and decreases with \( m \) as shown in Figure 8. And to the Plain-CS in CCT, the EES is \( 2m/\alpha + m, \alpha \in [1: N]\) which ranges from 3m to m. Therefore, when nodes in cluster \( \alpha \in [\sqrt{N}, 2\sqrt{N}] \) as derived in Section 4.2, lifetime of ANC-CS in CCT is about \( m/2 \) times as much as that of Plain-CS in CCT. Because of that \( m \ll N \) in CS, EES of ANC-CS will be close to the lower bound 2 when \( \alpha \in [\sqrt{N}, 2\sqrt{N}] \), and it will not increase obviously with \( \alpha \) continues to increase.

5 Conclusions

Traditional CS scheme in WSN cannot reduce transmission effectively and its TSC is numerous. In this paper, we divide the WSN into CCTs, and apply the proposed ANC-CS transmission scheme to each CCT. Analysis and simulation show that when nodes in cluster \( \alpha \in [\sqrt{N}, 2\sqrt{N}] \), the ANC-CS scheme in CCT can obtain perfect transmission and TSC performances which are much better than that of other schemes. So our scheme can save energy and time effectively. Meanwhile, lifetime of CCT which uses ANC-CS scheme is much longer than CCTs which use other schemes.

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References


