Discrete Composite Control for Piezoelectric Actuator Systems

Xuehui Gao1,2, Xuemei Ren1, Changsheng Zhu2, Chengyuan Zhang2
1. School of Automation, Beijing Institute of Technology, Beijing, 10081, China
E-mail: xhao@163.com, xmren@bit.edu.cn
2. Department of Mechanical and Electrical Engineering, Shandong University of Science and Technology, Tai'An, 271019 China
E-mail: zhuchangsheng@gmail.com, c_y_zhang@sohu.com

Abstract: This paper considers the precise control of piezoelectric actuator systems with hysteresis nonlinearity. First, the piezoelectric nonlinearity model is described by a quasi-static hysteresis which is cascaded a non-hysteretic dynamics. Next, A discrete Preisach model represents the quasi-static hysteresis and a composite controller is proposed which consists of inverse model-based control(IMC) and discrete sliding mode control(DSMC) for piezoelectric actuator systems. In order to reduce the chattering, a hyperbolic tangent function replace signum function in DSMC. Moreover, a selector is proposed to select the appropriate input between the output of IMC and DSMC for the quasi-static hysteresis to promote the accuracy of the control strategy and the response speed. Finally, the stability of this piezoelectric system is guaranteed by a Lyapunov-based approach and the numerical simulation demonstrates the effectiveness of the proposed control strategy.

Key Words: composite, inverse, hysteresis, sliding mode control(SMC).

1 INTRODUCTION

Smart materials, e.g., piezoelectric actuators, shape memory alloys, and magnetostrictive sensors/actuators, play an important role in intelligent manufacturing systems (IMS). Among them, piezoelectric actuators are widely applied in microelectromechanical systems (MEMS), micropositioning systems and nanopositioning systems due to their rapid response, high stiffness and nonexistence of magnetic fields. However, the existence of hysteresis nonlinearity degrades their performance in practical application. Being a strict nonlinearity with memory, hysteresis can lead to inaccuracies, oscillations, or even instability in controlled systems. In order to promote the precision of the piezoelectric actuators, control strategies and models have been studied to characterize the hysteresis nonlinearity.

The common hysteresis models are Jiles-Atherton model [1], Preisach model [2, 3, 4, 5], Prandtl-Ishlinskii(PI) model [6, 7, 8] and Krasnosel’skii-Pokrovskii(KP) model [9], et. al. In these models, the Preisach model has been widely applied and investigated for many years. Song et. al ([10]) designed a cascaded PD/lead-lag feedback controller to improve the accuracy of Preisach model for piezoelectric patch actuator. For the hysteresis output was not directly measured, a radius basis functional neural networks adaptive controller was presented based on the Preisach model estimation in [11]. Xiao and Li ([3]) presented a new Preisach density function, which was based on the linearity property for piezoelectric actuators.

It is noted that these aforementioned models belong to the continuous domain. With the develop of the computer technology, controllers have been mostly discretized in practice. A new discrete model which combined a ferromagnetic material model with an ARMA model was presented in [12]. Tan and Baras ([5]) proposed a recursive identification of a Preisach operator with piecewise uniform density and addressed an adaptive inverse controller for magnetostrictive actuators in discrete domain.

Based on these continuous or discrete hysteresis models, many different inverse hysteretic models were designed to accurately control the hysteresis nonlinearities by feedback or feedforward element. Literature [6, 13] applied inverse Prandtl-Ishlinskii model as a feedforward compensator to compensate the hysteresis nonlinearities in smart material actuators. Other researchers were drawing considerable attentions to the Preisach model, the cascaded inverse Preisach model was introduced to compensate the hysteresis respectively in [3, 5, 10].

In contrast to the inverse hysteresis model, intelligent or adaptive controllers are also widely applied in recent researches. A recurrent fuzzy cerebellar model articulation controller(RFCMAC) was proposed for a dual-axis micro-motion stage powered by piezoelectric actuators that could deal with the effects due to modeling uncertainty, external disturbance in [14]. Shieh et. al ([15]) introduced an adaptive displacement tracking control of the parameterized hysteretic function for a piezo-actuated positioning mechanism.

In this paper, a composite control is designed for the piezoelectric actuator which consists of discrete sliding mode controller(DSMC) and estimated inverse model-based control(IMC). The model of piezoelectric system is described by a quasi-static hysteresis and a cascaded non-hysteretic dynamics (Fig.1). The quasi-static hysteresis is represented...
by Preisach model and its inverse model can be obtained by our prior work in literature [4]. In order to precise control this system, DSMC is adopted to improve the robustness for the control system. A selector selects the appropriate controlled input between the output of IMC and DSMC to stabilize the system and track the reference input.

This paper is organized as follows. Section 2 reviews the discrete Preisach model and the composite control is designed in Section 3. Section 4 provides some numerical simulations based on the piezoelectric actuator hysteresis system. Section 5 concludes this paper.

2 DISCRETE PREISACH MODEL

The Preisach model is reviewed in this section to fix the notation and provide the background for the remaining of the paper.

For a pair of thresholds $\{\alpha, \beta\}$, with $\alpha \geq \beta$, consider a delayed relay $\gamma_{\alpha, \beta}[\cdot, \cdot]$ and $u \in C([0, T])$, $\zeta \in [0, 1]$, then, the Preisach operator $\gamma_{\alpha, \beta}[u, \zeta]$ is defined as[4, 5]

$$\gamma_{\alpha, \beta}[u, \zeta] = \begin{cases} 1 & \text{if } u(t) < \beta, \\ 0 & \text{if } u(t) > \alpha, \\ \gamma_{\alpha, \beta}[u, \zeta](t^-) & \text{if } \beta \leq u(t) \leq \alpha. \end{cases}$$

(1)

where $t \in [0, T]$, $\lim_{\epsilon>0, t\rightarrow 0} \gamma_{\alpha, \beta}[u, \zeta](0^-) = \zeta$ and $t^- = \lim_{\epsilon>0, t\rightarrow 0} \gamma_{\alpha, \beta}[u, \zeta](\epsilon)$.

Therefore, the discrete Preisach model at time instant $k$, can be expressed as

$$y(k) = \sum_{i=1}^{k} \sum_{j=1}^{i} \omega_{ij}(k) \mu_{ij}(k)$$

(2)

where $x(k)$ is the state variable, $y(k)$ is the measured output, i.e., the displacement of piezoelectric actuator in this paper. $v(k)$ is the output of the hysteresis nonlinearity and $u(k)$ is the system input.

In (3), $v(k) = \Gamma(u(k))$ represents the quasi-static hysteresis of the nonlinear model in Fig.1. This quasi-static hysteresis can be represented by the Preisach model which is based on our prior work in literature [4]. The following Lemma expresses this quasi-static hysteresis:

**Lemma 1** For a monotonous section, if the input is $u = [u_1, u_2, \ldots, u_n]^T$, the output is $v = [v_1, v_2, \ldots, v_n]^T$, then, the output can be expressed as:

$$v = \bar{\mu} u$$

(4)

where $[\cdot]^T$ means transposed matrix and the Preisach density function $\bar{\mu}$ is defined as:

$$\bar{\mu} = \begin{bmatrix} \mu_{11} & 0 & \cdots & 0 \\ \mu_{21} & \mu_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{n1} & \mu_{n2} & \cdots & \mu_{nn} \end{bmatrix}.$$  

(5)

A composite controller is designed in this section based on the cascaded structure. It consists of an feedforward controller and a feedback controller, which is shown in Fig.2. The feedforward controller is comprised of inverse Preisach model and the inversion of the non-hysteretic dynamics and it can be called inverse model-based control(IMC). The feedback controller is made of discrete sliding model control(DSMC).

![Figure 2: The composite control structure](image)

3 THE HYBRID CONTROL FOR PIEZOELECTRIC ACTUATORS

Literature [16, 17] applied a similar model for smart material actuators where the structure was shown in Fig.1. The model of piezoelectric actuators is consisted of a quasi-static hysteresis and a cascaded non-hysteretic dynamics in this paper. The model can be defined as follows:

\[ x(k+1) = Ax(k) + Bu(k) \]
\[ y(k) = Cx(k) \]
\[ v(k) = \Gamma(u(k)) \]

where $x(k)$ is the state variable, $y(k)$ is the measured output, i.e., the displacement of piezoelectric actuator in this paper. $v(k)$ is the output of the hysteresis nonlinearity and $u(k)$ is the system input.

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![Figure 2: The composite control structure](image)

3.1 Design of the controller

In this section, the composite controller is discussed for the piezoelectric system. This composite controller is made of IMC and DSMC as shown in Fig.2. IMC consists of two parts: the estimation of the inverse non-hysteretic dynamics $G^{-1}$ and the estimation of the inverse quasi-static hysteresis nonlinearity $\Gamma^{-1}$. According to [4], we can get $\Gamma^{-1}$. Then, the estimated inverse dynamics $G^{-1}$ can be obtained form (3).

To guarantee the stability of the system, a discrete sliding mode controller is designed in this paper. Considering the system (3), we define the reference input as $y_r$, the first derivative as $dy_r$, the second derivative as $a y_r$, and so on. Let $R(k) = [y_r(k), dy_r(k), a y_r(k), \ldots]$, $y_r(k+1) = 2 \cdot y_r(k) - y_r(k-1), dy_r(k+1) = 2 \cdot dy_r(k) - dy_r(k-1...$
1. \( a_y(k+1) = 2 \cdot a_y(k) - a_y(k-1), \ldots \), then, the sliding manifold is designed as follows:

\[
s(k) = Z_eR(k) - Z_e x(k) \tag{6}
\]

where \( Z_e = [z^{n-1}, \ldots, z^2, z, 1], z > 0 \) denotes the parameter of sliding manifold.

The reaching law can be chosen as follows:

\[
s(k+1) = s(k) + T(-\varepsilon \tanh(s(k)) - ps(k)) \tag{7}
\]

where parameter \( p \) denotes the speed of dynamic transition, \( \varepsilon \) is the hyperbolic tangent function gain, \( T \) indicates the sampling period and \( p > 0, \varepsilon > 0 \).

According to the sliding manifold (6) and the reaching law (7), we can define the controller as follows:

\[
u(k) = \Gamma^{-1} \{ (Z_eB)^{-1}[Z_eR(k+1) - Z_eAx(k) - (1 - pT)s(k) + \varepsilon T \tanh(s(k))] \}
\]

\[
- \sum_{i=1}^{k-1} [\mu_{ki}u_i]/\mu_{kk}
\]

**Remark 1** In this controller, \( Z_e, p \) and \( \varepsilon \) are adjustable parameters for the sliding mode control. The velocity parameter \( p \) can influence the dynamic transition of switching function. We can improve the dynamic performance when we choose appropriate \( p \). \( Z_e \) affects the system’s regulation time and it guarantees the stability of system and the response speed. \( \varepsilon \) is the hyperbolic tangent function gain, which can weaken the perturbation and the chattering of the sliding mode control. In this paper, we choose the hyperbolic tangent function to replace the signum function for the continuity of the hyperbolic tangent function, which can better reduce the chattering in practice.

### 3.2 Stability analysis

**Theorem 1** For system (3), the sliding manifold is chosen as (6), the reaching law as (7) and the controller as (8), then, the control system is stable.

**Proof:** From (4) and (5), we can get the following equation:

\[
v(k) = \sum_{i=1}^{k} \mu_{ki}u_i. \tag{9}
\]

Then, the inverse hysteresis nonlinearity model can be expressed as follows:

\[
u(k) = \frac{v(k) - \sum_{i=1}^{k-1} \mu_{ki}u_i}{\mu_{kk}} \tag{10}
\]

According to (7), the reaching law can be rewritten as:

\[
s(k+1) - s(k) = -pT s(k) - \varepsilon T \tanh(s(k)). \tag{11}
\]

Thus, the non-hysteretic dynamics controller can be obtained as follows:

\[
v(k) = (Z_eB)^{-1}[Z_eR(k+1) - Z_eAx(k) - (1 - pT)s(k) + \varepsilon T \tanh(s(k))] \tag{12}
\]

Considering (10), we can get the controller:

\[
u(k) = \{(Z_eB)^{-1}[Z_eR(k+1) - Z_eAx(k) - (1 - pT)s(k) + \varepsilon T \tanh(s(k))] \}
\]

\[- \sum_{i=1}^{k-1} [\mu_{ki}u_i]/\mu_{kk}
\]

To evaluate the stability condition of the control system, a Lyapunov function candidate is chosen as

\[
V(k) = |s(k)|. \tag{14}
\]

We define \( \Delta V(k) \) as follows:

\[
\Delta V(k) = |s(k+1)| - |s(k)|. \tag{15}
\]

When \( s(k) > 0 \), based on (11), one can get \( s(k+1) - s(k) < 0 \), i.e., \( s(k+1) < s(k) \). When \( s(k) < 0 \), also based on (11), one can get \( s(k+1) - s(k) > 0 \). Therefore, the following equation can be obtained:

\[
|s(k+1)| < |s(k)| \tag{16}
\]

Thus, the condition for Lyapunov stability is satisfied. Then, the system is stable.

This end the proof.

For the model-based inversion feedforward control, the non-hysteretic dynamics \( G \) consists of electric and vibration dynamics. The whole transfer function can be written as follows from [16]:

\[
G = \frac{k}{\tau s + 1} \cdot \frac{\Pi_j^{-2}(s^2 + 2\epsilon_j \omega_j s + \omega_j^2)}{\Pi_i^0(s^2 + 2\epsilon_i \omega_i s + \omega_i^2)} \tag{17}
\]

where \( \tau \) is the electric dynamics time constant, \( \omega_i \) and \( \omega_j \) are the mode frequencies of vibration dynamics, \( \epsilon_i \) and \( \epsilon_j \) are the damping ratio, \( k \) is the gain of the non-hysteretic dynamics.

Liu et al. [16] discussed the identification of the estimated non-hysteretic dynamics \( \hat{G} \) and the hysteresis \( \hat{\Gamma} \). In this paper, the estimated hysteresis \( \hat{\Gamma} \) is identified by our prior work in [4] which is different from [16] and the identification of estimated non-hysteretic dynamics \( \hat{G} \) is based on [16].

The section of selector of Fig.2 is a special filter in this paper. There are many methods to select the appropriate quasi-static hysteresis input \( u \). To simplify the control strategy, this selector is designed as follows: Let \( e_u = |u_e - u_s| \), \( \gamma \) is a small positive constant, if \( e_u > \gamma \), then, \( u = u_e \), else \( u = u_s \).

### 4 NUMERICAL SIMULATION AND EXPERIMENT RESULTS

In this section, based on the literature [16], the non-hysteretic model of piezoelectric actuator is described by (17) and the estimated parameters([16]) are: \( \hat{k} = 0.8716, \hat{\tau} = 0.000474, \hat{\omega}_1 = 453.5, \hat{\omega}_2 = 792.7, \hat{\epsilon}_1 = 0.67 \) and \( \hat{\epsilon}_2 = 0.081 \), respectively. In this paper, the transfer function model is transformed into discrete state space model which is described as (3). \( u(k) = \Gamma^{-1}(v(k)) \) is identified
by the method which is proposed in literature [4]. The sliding mode controller is defined as (8). The parameters are taken to be: $Z_e = [1.65^2, 1.65, 1]$, $T = 0.4$, $p = 4$.

Fig.3 illustrates the tracking curve with the sinusoidal reference input and the error curve is shown in Fig.4 when only the DSMC work for this hysteresis system.

![Figure 3: The position tracking without the inverse model control](image1)

![Figure 4: The error of the position tracking without inverse model control](image2)

From Fig.3, we can clearly observe how the sliding mode reaches to the sliding surface. If we adjust $p$, the reaching velocity will be changed. In order to reduce the chattering and the perturbation and improve the velocity of the reaching sliding surface, the composite controller is applied as Fig.2. An IMC is adopted through the selector to control the system when the DSMC dynamic performance is not good at the adjusting stage.

Let $\gamma = 0.5$, the selector is worked as follows: if $e_u > \gamma$, $u = u_r$, else $u = u_s$. Therefore, when the DSMC doesn’t reached sliding surface, the input of quasi-static hysteresis is chosen $u_r$, which means IMC worked. But when the sliding mode reached the sliding surface, the quasi-static hysteresis input becomes $u_s$, which means DSMC worked. For IMC is a feedforward control, that has weak robustness but DSMC has strong robustness. Then, the proposed composite control not only has quickly response speed, but also has strong robustness. Fig.5 illustrates this situation and the error between the reference input and the system output is illustrated in Fig.6.

![Figure 5: The position tracking with composite control](image3)

![Figure 6: The error of the position tracking with composite control](image4)

Comparing Fig.4 and Fig.6, we conclude that the composite control strategy is more effective than the DSMC working alone for the hysteresis system. Adding to the IMC, the margin of error decreases clearly and the DSMC reaches sliding surface rapidly.

5 CONCLUSION

The hysteresis of piezoelectric actuator systems is represented by discrete Preisach model in this paper. Based on our prior work for the identification of discrete Preisach model[16], a composite control strategy which consisted of DSMC and IMC was proposed to precise control the piezoelectric actuator systems. DSMC could reduce the chattering by the hyperbolic tangent function instead of signum...
function and a selector was proposed to select the proper input between the output of IMC and DSMC to obtain the rapid response speed. In addition, the stability of system was discussed by Lyapunovon approach. Finally, the numerical simulations demonstrated the effectiveness of this composite controller.

REFERENCES