GROUP-BASED HYPERSPECTRAL IMAGE DENOISING USING LOW RANK REPRESENTATION

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ABSTRACT
For the hyperspectral image (HSI) denoising problem, we propose a group-based low rank representation (GLRR) method. A corrupted HSI is divided into overlapping patches and the similar patches are combined into a group. The group is denoised as a whole using low rank representation (LRR). Our method can employ both the local similarity within the patch and the nonlocal similarity across the patches within a group simultaneously, while nonlocal similar patches within the group can bring extra structure information for the corrupted patch, which makes the noise more significant to be detected as outliers. Since the uncorrupted patches have an intrinsic low-rank structure, LRR is employed for the denoising of the patch group. Both simulated and real data are used in the experiments. The effectiveness of our method is proved both qualitatively and quantitatively.

Index Terms— Denoising, Hyperspectral image, Low rank representation, Nonlocal similarity.

1. INTRODUCTION
In recent years, hyperspectral images (HSIs) have been widely used in various fields, such as environment monitoring, agriculture, military and so on. However, corruption by various noises degrades the image quality greatly, leading to the low accuracy of classification, object segmentation, unmixing and subpixel mapping. Therefore, the denoising procedure is an essential preprocessing step before the following applications of HSIs.

Over the past several decades, many kinds of denoising algorithms have been proposed using different framework. Othman[1] reduces the noise using a wavelet-shrinkage filter in the hybrid spatial-spectral derivative domain. Zhang [2] proposed to employ a cubic total variation regularization and Yuan [3] extended the regularization item into a spectral-spatial adaptive one. In [4] and [5], tensor analysis is employed, with Tucker decomposition and rank-1 decomposition applied respectively. Zhang [6] introduced a denoising method based on low rank representation (LRR). The method received overwhelmingly good performance upon other methods as the uncorrupted HSI complies with the assumption of low rank structure highly. However, within the corrupted area, the noisy patches have a high percentage of different kinds of noises, which makes the specific noise is not significant for detection.

In this paper, we propose a novel algorithm for HSI denoising using group-based low rank representation (GLRR), in which local and nonlocal similarity of the HSI can be simultaneously considered under the unified framework of LRR, while the traditional LRR method just employs local similarity. In GLRR, similar patches are combined into a group, while the similarity across the patches within a group we denote as nonlocal similarity. The reconstructed unit with LRR is the group consisting of similar patches, instead of the individual patches. Within the group, the nonlocal similar patches bring auxiliary structure information for the corrupted patch, which makes the noise more significant to be detected.

The remainder of this paper is organized as follows. Section 2 describes the proposed method in detail. Experimental results and discussion are shown in Section 3. Section 4 draws the conclusion.

2. PROPOSED METHOD

2.1. Low Rank Representation

Low rank representation is originally proposed in [7] by Wright et al. The model is under the assumption that the uncorrupted data has a low-rank structure. And the low-rank property of HSIs has been demonstrated in [8, 9] from the perspective of a linear spectral unmixing model.

A corrupted matrix \( \mathbf{X} \in \mathbb{R}^{m \times n} \) is modelled as combina-
tion of a clean matrix and the additive noises, described as
\[
X = D + E + N
\]  
(1)
where \(D \in \mathbb{R}^{m \times n}\) means the uncorrupted data, with low-rank structure; \(E \in \mathbb{R}^{m \times n}\) is a sparse matrix, modeling the outliers, such as stripe noise; \(N \in \mathbb{R}^{m \times n}\) models the random noise, for example, Gaussian noise. From Eq.1, we can see that the denoising problem is modelled as the reconstruction of matrix \(D\) from the corrupted matrix \(X\). The optimization is organized as following
\[
\min_{D,E} \text{rank}(D) + \lambda \|E\|_0 \quad \text{s.t.} \|X - D - E\|_F^2 < \sigma^2
\]  
(2)
where \(\lambda\) is the balance parameter between the rank item and the sparse item. However, the optimization is a nonconvex problem and is hard to find an efficient solution. But it has been shown that the problem in Eq.2 can be solved by relaxing with replacement of rank with nuclear norm and \(l_0\)-norm with \(l_1\)-norm
\[
\min_{D,E} \|D\|_* + \lambda \|E\|_1 \quad \text{s.t.} \|X - D - E\|_F^2 < \sigma^2
\]  
(3)
Lin et al. [10] proposed an inexact augmented Lagrange multiplier method for the recovery of corrupted low-rank matrix in Eq.3. However, the method is quite time-consuming. In this paper, we use bilateral random projection [11] instead of singular value decomposition (SVD) for the approximation of low-rank structure, due to the high time-complexity of SVD. Within the bilateral random projection, two additional constraints are introduced, \(\text{rank}(D) \leq r\) and \(\text{card}(E) \leq k\), where \(r\) and \(k\) should be predefined.

2.2. Group Model

Traditional LRR-based HSI denoising method deals with the patches individually. However within the corrupted patch, the noise have a comparable energy with the data, as shown in Fig.1(b), leading to difficulty for the noise detection. Our proposed group-based model collects the similar patches as one group. Nonlocal similarity among the group patches is employed and extra information is brought by the nonlocal construction, as shown in Fig.1(c), the integration with patches B-D makes the noise in patch A more significant for detection. And the group as a whole still complies with the low-rank assumption, as it is derived from the perspective of spectral unmixing.

Euclidean distance is employed for the group criterion measurement in our method. For a series of matrices \(X_k \in \mathbb{R}^{m \times n}(k = 1, 2, \ldots, K)\), the Euclidean distance is defined as
\[
d_{ij} = d(X_i, X_j) = \left( \sum_{u=1}^{m} \sum_{v=1}^{n} |x_{uv}^{(i)} - x_{uv}^{(j)}|^2 \right)^{1/2}
\]  
(4)
where \(i\) and \(j\) are the index of the matrix. And the group is combined according to
\[
\mathcal{G}(X_k) = \{X_l : d_{kl} < \text{threshold}\}
\]  
(5)
where \(\mathcal{G}(X_k)\) denotes the corresponding group for \(X_k\).

2.3. HSI Denoising Based on GLRR

Here, a group-based LRR method is proposed for HSI denoising problem, which can be divided into the following three steps: data arrangement, LRR-based denoising and HSI reconstruction, as shown in Fig.2.

Firstly, HSI \(X \in \mathbb{R}^{M \times N \times B}\) is divided into \(K\) overlapping patches \(\{X_k \in \mathbb{R}^{q \times q \times B} , k = 1, \ldots, K\}\) with a fixed spatial size \(q \times q\) and stepsize \(s\). \(X_k \in \mathbb{R}^{q \times q \times B}\) is the reorganization 2D matrix of \(\{X_k\}\), as shown in Fig.2. For each patch \(X_k\), a group \(\mathcal{G}(X_k)\) is organized with Euclidean distance as the criterion, according to Eq.4 and 5. And the group is rearranged into a 2D matrix \(X_k^{(G)} \in \mathbb{R}^{m \times B}\), where \(m = |\mathcal{G}(X_k)| \times q^2\), \(|\mathcal{G}(X_k)|\) is the number of patches in \(\mathcal{G}(X_k)\).

Secondly, low-rank structure \(D_k^{(G)}\) of \(X_k^{(G)}\) is reconstructed according to the improved inexact augmented Lagrange method described in section 2.1. And the corresponding low rank matrix \(\mathcal{G}(D_k)\) of each item in \(\mathcal{G}(X_k)\) can be extracted from \(D_k^{(G)}\).
Finally, the denoised HSI is generated by a two-stage average of the reconstructed patches. At the first stage, each patch is reconstructed by weighted average. For patch $X_k$, it may be included in different groups $\{G(X_{k_1}), \ldots, G(X_{k_C})\}$, here $C$ is the number of groups including $X_k$. Therefore there will be multiple reconstructed patches for patch $X_k$, which we denote as $\{D^{(k_1)}_k, \ldots, D^{(k_C)}_k\}$, here $k_i (i = 1, \ldots, C)$ is the group index. The reconstructed low rank matrix $D$ for the $k$th patch is estimated by the following weighted average

$$D_k = \frac{\sum_{i=1}^{C} \omega_i D^{(k_i)}_k}{\sum_{i=1}^{C} \omega_i}$$

(6)

$$\omega_i = 1/(1 + \tau d_{kk_i})$$

(7)

where $\tau$ is an inverse parameter from distance measurement to similarity measurement and $d_{kk_i}$ is the distance between patch $X_k$ and $X_{k_i}$. And then, 2D matrix $D_k$ is reshaped to a 3D data cube $D \in \mathbb{R}^q \times n \times B$. As the patches are overlapped, thus at the second stage, the total denoised HSI image $D \in \mathbb{R}^{M \times N \times B}$ is generated by average of $D_k$.

3. EXPERIMENTAL RESULTS AND DISCUSSION

The experiments are conducted on both simulated data and real data. To demonstrate the effectiveness of our proposed method, both quality and quantity evaluation will be shown in this section.

3.1. Experiments on simulated data

The reflective optics system imaging spectrometer (ROSIS) image of the Pavia University, Italy is used in the simulated experiment. The data contains 103 bands with a spatial size of $610 \times 340$.

Various kinds of noises are added for the data simulation. All bands are added with Gaussian noise with $\sigma = 5\%$, and in addition, 10 bands are added with salt & pepper noise with a percentage of 20%, 10 bands are added with stripe noise with 10 random lines each band and 2 bands are added with all three kinds of noises. For example, Fig.3(b) is band 83 of the simulated data which is corrupted by all three kinds of noise, with Fig.3(a) showing the original clean band.

In our experiments, the parameters are set as: block size $q = 30$, stepsize $s = 10$, rank $r = 20$ and sparsity $k = 12000$. For comparison, BM4D [12] and LRR [6] are used as benchmark methods. BM3D receives state-of-art performance in natural image denoising to our knowledge, and BM4D is an extension of BM3D to volumetric data, while HSIs are typical volumetric data. And LRR uses the same framework of low rank representation, but just considers the local similarity of HSIs. In our experiments, we use the most appropriate parameters for LRR, with rank $r = 7$, sparsity $k = 6000$, the same block size and stepsize with our method.

3.2. Experiments on real data

The hyperspectral digital collection experiment (HYDICE) data, Urban of Copperas Cove, Texas, is used in our experiments as real data. The data consists of 210 spectral bands with a spatial size of $307 \times 307$. Polluted by atmosphere and water absorption, bands 104-108, 139-151 and 207-210 are removed from the data and a $307 \times 307 \times 188$ HSI is used in our experiments. Within the data, several bands are polluted by heavy Gaussian noise and also stripe noise because of the detector-to-detector difference, as shown in Fig.5(a).

In our experiments, the parameters are set as block size $q = 8$, stepsize $s = 5$, both for LRR and our method. Rank $r$ and sparsity $k$ are 7 and 2000 for LRR, while 8 and 4000 for our method. Experimental results are shown in Fig.3 (a)-(e), with (f)-(j) as the closeup of (a)-(e), respectively. From Fig.3(c) and (h), we can see that the BM4D cannot remove salt & pepper noise and stripe noise thoroughly, and also, it oversmooths the denoised area. LRR performs much better than BM4D, however, the method cannot detect all the noises, leading to a loss of some fine features, as shown by the red arrow in Fig.3(i). And in Fig.3(e) and (j), we can see that our proposed method outperforms BM4D and LRR subjective quality, with removing all the noises and retaining fine structures.

To further demonstrate the effectiveness of our method, peak signal-to-noise ratio (PSNR) index and structural similarity (SSIM) index [13] are employed for quantitative evaluation. The indexes are both calculated band-by-band between the reconstructed HSI and the original bands, as shown in Fig.4. It can be observed that BM4D shows very poor performance on some bands, because of its ineffectiveness on salt & pepper noise and stripe noise, while LRR performs unstable on band 75-103. PSNR of our method achieves an improvement of about 5.81 dB and 3.77 dB upon BM4D and LRR respectively. As for SSIM, although our method just achieves a small improvement upon LRR, it shows great stability, while that of LRR achieves a decline in band 75-103. It can be concluded that our method outperforms the other two methods both subjective qualitatively and quantitatively.

Fig. 4. Quantitative evaluation. Top: PSNR; Down: SSIM
Fig. 3. Experimental results of band 83 in the Pavia University data.

tal results are shown in Fig.5. It can be observed that BM4D oversmooths the data and fails to remove stripe noise totally. LRR is also unsatisfactory, as neither Gaussian noise nor stripe noise are removed thoroughly. From Fig.5(d), we can see that our method eliminates all kinds of noises and at the same time retains the spatial details.

4. CONCLUSION

In this paper, we proposed a group-based low rank representation(GLRR) denoising method for the reconstruction of corrupted HSIs. The group-based model takes the nonlocal similarity into consideration during the reconstruction, and introduces extra structure information for the reconstruction of corrupted patches. The low rank representation(LRR) framework employs the intrinsic low-rank structure of HSIs, which is derived from the linear spectral unmixing model. And a bilateral random projection-based inexact augmented Lagrange multiplier algorithm is employed for the efficiency solution of LRR. Both qualitative and quantitative assessment of the experiments confirm that our proposed method outperforms other method, such as BM4D and LRR, with detecting and removing the corrupting noise effectively, and retaining fine features of the HSIs simultaneously.

Fig. 5. Experimental results of band 188 in the urban data.
References


