Nonlinear response prediction of cracked rotor based on EMD

Yongfeng Yang*, Hu Chen, Tingdong Jiang

Northwestern Polytechnical University, Xi'an 710072, PR China

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Abstract

The empirical mode decomposition (EMD) method is introduced, to improve the prediction accuracy of cracked rotor's nonlinear response during a long-term period. The EMD method was applied to decompose the nonlinear response into series of intrinsic mode functions (IMF). Consequently, the prediction results of IMF were obtained, based on the maximal local Lyapunov exponent (LLE). By adding all the prediction results of IMF, the nonlinear response of cracked rotor can be predicted, called the IMF prediction method. Compared with the response predicted directly by the maximal local Lyapunov exponent, when the forecasting step is less than the maximal prediction time which is calculated by the multiplicative inverse of maximal Lyapunov exponent, the IMF method has the same prediction accuracy. When the forecasting step is greater than maximal prediction time, the IMF prediction method is more advantageous than the Lyapunov prediction method. Bently RK4 rotor test is used to validate the IMF prediction method's advantage.

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1. Introduction

Rotating machines represent the maximal and most important class of machinery used for fluid media transportation, for metal working and forming, for energy generation, for providing aircraft propulsion, and for other purposes. Dynamic behavior of a high speed rotor system has been investigated extensively by linear analysis procedures. Most of the components that
comprise a rotor dynamic system can be quite accurately modeled as linear. Fatigue cracking of the rotor shaft observed in the rotating machinery should be avoided, which may lead to catastrophic failure [20,29]. In this situation, there are multi-periodic, quasi-periodic and chaos responses. These responses are nonlinear and non-stationary vibration. However, the strongly nonlinear can make the system possess characteristics that are substantially different from those of a linear system, such as self-excited oscillations and jump discontinuities. The inherently nonlinear characteristics cannot be predicted by linear models [22]. Detailed investigation into the nonlinear dynamic response prediction of a rotor with crack in the shaft is very important for diagnosing and preventing rotor cracks [11,12,16].

The influence of a transverse crack on the vibration of a rotating shaft has been the focus of many researchers. Extensive reviews of the dynamic response of cracked rotor systems were published by Wauer [23] and Dimarogonas [3]. Patel and Darpe [15] investigated the influence of the crack breathing models (switching crack model and response-dependent breathing crack model) on the nonlinear vibration characteristics of the cracked rotors. Switching crack modeling reveals chaotic, quasi-periodic, and subharmonic vibration response for deeper cracks and more realistic breathing crack model reveals no evidence of them. Ishida and Inoue [7] used harmonic excitation force to investigate the nonlinear response of cracked rotor. The occurrence of various types of nonlinear resonances due to crack is clarified, and types of these resonances, their resonance points, and dominant frequency component of these resonances are clarified numerically and experimentally. Rubio and Fernandez-Saez [18] propose a new procedure to analyze the nonlinear dynamic of cracked rotors using an iterative technique that transforms the full nonlinear problem in a succession of time-dependent linear ones. The calculations using the proposed method are over a 100 times faster than the corresponding to integrate the full nonlinear problem, being very helpful in on-line crack identification procedures.

Several methods have been applied to nonlinear and non-stationary signal analysis, such as short-time Fourier analysis, wavelet analysis and Wigner–Ville distribution [1]. However, all these methods are based on the traditional Fourier transformation. The Fourier transformation is a mathematical tool to expand signals into a spectrum of sinusoidal components. It is applied to facilitate signal representation and the analysis of system performance. In certain applications the Fourier transformation is capable to decompose the input signal into uncorrelated components. Hence, signal processing can be more effectively implemented on the individual spectral components. However, the Fourier transform cannot get the resolution in the time domain. For the non-stationary signals, Fourier transformation, a linear sum of harmonics, cannot obtain the fundamental transient characteristics [6].

In 1998, Hilbert–Huang Transformation was first proposed by Huang et al. [6]. It is a new time-frequency analytical method for non-linear and non-stationary signals, which includes the empirical mode decomposition (EMD) and Hilbert transformation. EMD can be applied to, the adaptive decomposition of signals, either stationary signals or non-stationary signals. Panagopoulos [14] used numerical Morlet wavelet transform and empirical mode decomposition to study the damped dynamics of an elastic rod with an essentially nonlinear end attachment by superimposing the wavelet transform spectra and the instantaneous frequencies of the intrinsic mode functions to the frequency–energy plot of the underlying Hamiltonian system, one is able to clearly identify the multi-scaled transitions that occur in the transient damped dynamics, and to interpret them as ‘jumps’ between different branches of periodic orbits of the underlying Hamiltonian system. Georgiades [5] used empirical mode decomposition in combination with wavelet transforms to study targeted energy transfers and nonlinear modal interactions attachments occurring in the dynamics of a thin cantilever plate on an elastic foundation. Faiz [4] used IMF as a competent method for broken bars fault identification in line-start and inverter-fed industrial induction motors. An [2] used grey forecasting...
model and largest Lyapunov exponent prediction method to predict the IMF in order to getting the short-term prediction of wind farm power. Nowadays, the EMD method has been widely applied in many engineering fields, such as machinery, transportation, marine, medicine, electricity, etc. [8,13,21,27]. The basic idea of EMD decomposition is that any signal is composed of series basic model components, with their amplitudes and phases varying with time. These model components are defined as the intrinsic mode function (IMF). The IMF components can be extracted from the multi-component signals by the EMD method. Based on the EMD method, a nonlinear response prediction method, named IMF prediction, is proposed in the current work. The EMD is used to decompose the nonlinear response of a rotor. Then the prediction results of the IMF are obtained on the basis of the maximal local Lyapunov exponent. By adding all the prediction results of IMF, the nonlinear response of a cracked rotor can be predicted.

2. Chaos identification and prediction methods

A variety of techniques have been proposed to identify chaos, which include the EMD method, the Lyapunov exponent method, the Lyapunov prediction method, etc. In this study, the EMD method was applied to decompose nonlinear response into a series of IMF. The Lyapunov exponent was employed to analyze the chaotic nature of the time series. The Lyapunov prediction was conducted as an alternative method for prediction.

2.1. EMD method

EMD method was developed based on the assumption that any signal consists of different simple intrinsic modes of oscillations. Each linear or non-linear mode has the same number of extrema and zero-crossings. There is only one extremum between successive zero-crossings. Each mode is independent of the others. In this way, each signal can be decomposed into a number of IMFs, which must satisfy the following definition [6]:

(1) In the whole data set, the number of extrema and the number of zero-crossings must either equal to or differ at most by one.
(2) At any point, the mean value of the envelope defined by local maxima and the envelope defined by the local minima is zero.

An IMF represents a simple oscillatory mode, compared with the simple harmonic function. By this definition, any signal \( x(t) \) can be decomposed as follows:

(1) Identify all the local extrema, and then connect all the local maxima by a cubic spline line as the upper envelope.
(2) Repeat the procedure for the local minima to produce the lower envelope. The upper and lower envelopes should cover the whole data set.
(3) The mean value of the upper and low envelopes is denoted as \( m_1(t) \), and the difference between \( x(t) \) and \( m_1(t) \) is the first component of IMF, noted as \( h_1(t) \). That is

\[
x(t) - m_1(t) = h_1(t)
\]

In the ideal case, if \( h_1(t) \) is an IMF, then \( h_1(t) \) is the first component of \( x(t) \).
(4) If $h_1(t)$ is not an IMF, $h_1(t)$ is treated as the original signal. Repeating steps (1)–(3), we can get

$$h_1(t) - m_{11}(t) = h_{11}(t)$$

(2)

where $m_{11}(t)$ is the mean value of upper and low envelopes of $h_1(t)$. After repeated sifting, i.e. up to $k$ times, $h_{1k}(t)$ becomes an IMF, which is

$$h_{1(k-1)}(t) - m_{1k}(t) = h_{1k}(t)$$

(3)

Then it is designated as

$$c_1(t) = h_{1k}(t)$$

(4)

The first IMF component obtained from the original data $c_1(t)$ should contain the finest scale or the shortest period component of the signal.

(5) Separate $c_1(t)$ from $x(t)$ and we can get

$$r_1(t) = x(t) - c_1(t)$$

(5)

where $r_1(t)$ is treated as the original data. Repeat the above processes. Hence, the second IMF component $c_2(t)$ of $x(t)$ can be obtained. Repeat the above process for $n$ times, the $n$-IMFs of signal $x(t)$ can be calculated. Then

$$\begin{align*}
  r_1(t) - c_2(t) &= r_2(t) \\ 
  \vdots \\ 
  r_{n-1}(t) - c_n(t) &= r_n(t)
\end{align*}$$

(6)

The decomposition process continues until $r_n(t)$ becomes a monotonic function, from which no more IMF can be extracted. By summing up Eqs. (5) and (6), the signal can be written as

$$x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t)$$

(7)

Hence, the signal can be decomposed into $n$-empirical modes, together with a residue $r_n(t)$ which is the mean trend of $x(t)$. The IMFs $c_1(t), c_2(t), \ldots, c_n(t)$ include different frequency bands ranging from high frequency to low one. The frequency components contained in each frequency band are different and change with the variation of signal $x(t)$. In this sense, EMD is a self-adaptive signal decomposition method.

2.2. Lyapunov exponent method

The most significant feature of a chaos system is the unpredictability due to the sensitive dependence on initial conditions. Even very small deviations in initial conditions of all the trajectories can lead a chaos system blown up after a few time steps. For a chaotic system, divergence will be exponentially quick [25]. Lyapunov exponent gives the averaged information of this divergence and thus the unpredictability of the system. It characterizes the rate of separation of infinitesimally close trajectories. Let $s_{t_1}$ and $s_{t_2}$ be two points in two trajectories in the state space such that the distance between them satisfies $\|s_{t_1} - s_{t_2}\| = \delta_0 < 1$. After $\Delta t$ time
steps, the distance between these two trajectories will be $\partial_{\Delta t} \simeq \|s_{t1+\Delta t} - s_{t2+\Delta t}\|$, $\partial_{\Delta t} < 1$, $\Delta t > > 1$. Hence, trajectories with initial separation $\partial_0$ diverge in the form of an exponential function, $\partial_{\Delta t} \simeq e^{\lambda \Delta t} \partial_0$, where $\lambda$ is the Lyapunov exponent [24]. Since the rate of separation is different for various orientations of initial separation vector, the total number of Lyapunov exponents is equal to the number of dimensions in the phase space defined, i.e., a spectrum of exponents will be available. Among them, only the maximal (global) Lyapunov exponent needs be considered, because it determines the total predictability of the system.

A positive $\lambda$ indicates an exponential divergence of the nearby trajectories, and thus chaos. The orbit is unstable and chaotic. Negative Lyapunov exponents are characteristic of dissipative or nonconservative systems. Their orbits attract to a stable fixed point or periodic orbit. The stability is directly proportional to the negativeness of the exponent. Conservative systems exhibit a zero Lyapunov exponent. The orbit is a neutral fixed point.

Since a positive Lyapunov exponent is a strong feature of chaos, many algorithms have been developed to calculate the maximal Lyapunov exponent. Wolf's algorithm is one of the first kinds developed for this, although it requires much care because the algorithm does not allow testing the presence of exponential divergence and it can lead to wrong results. However, exponential divergence can be examined using algorithms introduced by Rosenstein et al. [17] and Kantz and Schreiber [9].

To calculate the maximal Lyapunov exponent, one has to compute [24]

$$S(\Delta t) = \frac{1}{N} \sum_{i=1}^{N} \ln \left( \frac{1}{|U(s_{i0})|} \sum_{s \in U(s_{i0})} \left| s_{i0+\Delta t} - s + \Delta t \right| \right)$$

(8)

where $s_{i0}$ are reference points or embedding vectors, $U(s_{i0})$ is the neighborhood of $s_{i0}$ with diameter $\xi$. For a reasonable range of $\xi$ and for all embedding dimensions $m$ which is larger than some minimum dimension $m_0$, if $S(\Delta t)$ exhibits a linear increase, then its slope can be treated as an estimation of the maximal Lyapunov exponent $\lambda$.

2.3. Lyapunov prediction method

According to chaos theory, Lyapunov exponents measure the rate of divergence of neighboring orbits in the phase space. The inverse of the maximal Lyapunov exponent is frequently called the Lyapunov time [19]. The Lyapunov time reflects the limits of the predictability of a dynamic system. A larger maximal Lyapunov exponent indicates shorter Lyapunov time. When the forecasting time is less than the Lyapunov time, the prediction error would increase as the forecasting step increases. But the error still increases linearly. When the forecasting time is greater than the Lyapunov time, the prediction error grows with time exponentially and predictive ability would rapidly fall.

By definition, the maximal Lyapunov exponent represents the dispersion rate of the system at the initial point. It tells us precisely by how much the distance between the current state and its nearest neighbor will expand (or contract) over time, so that the distance between the nearest-neighbor predictor can be easily obtained (i.e., the neighbor's successor) and the future we are trying to predict (tomorrow's state). Thus, we know exactly by how much to correct the prediction of the nearest-neighbor predictor. So we can use the maximal Lyapunov exponent to predict the nonlinear responses. This method can be described as following [28,31].

Consider a one-dimensional series of $T$ observations from a chaotic system, $(x_1, \ldots, x_T)$, whose future values are to be forecasted. Recall that a chaotic system is characterized by the existence of an attractor in a $d$-dimensional phase space, where $d > 1$ is the embedding dimension. A
possible embedding method involves building a \(d\)-dimensional orbit, \((X_t)\), with 
\[ X_t = (x_t, x_{t-\tau}, ..., x_{t-(d-1)\tau}). \]
For the sake of exposition, recall that \(\tau = 1\).

Consider an orbit \((X_1, ..., X_T)\) whose one-step-ahead future, \(X_{T+1}\), is to be predicted. The nearest-neighbor predictor returns \(\hat{X}_{T+1} = X_{i+1}\), where \(X_i\) is the element of the orbit with minimal distance to \(X_T\). Because the dynamic system is periodic (or else, forecasting would not be an issue), the nearest-neighbor predictor is inevitably biased. Indeed, because \(|X_T - X_i| > 0\), it must also be the case that
\[ |\hat{X}_{T+1} - X_{T+1}| \approx |X_T - X_i| e^\lambda > 0 \]  \(\text{(9)}\)
where \(\lambda\) is the maximal Lyapunov exponent.

It follows from Eq. (9) that knowing the distance between the predictor and the nearest neighbor as well as the LLE at the nearest neighbor allows us to predict the distance of the predictor’s image to the neighbor’s image. Note that this is true regardless of the sign of \(\lambda\); i.e., regardless of whether the system is locally chaotic or locally stable. Moreover, because the orbit considered results from the embedding of a one-dimensional series, we also know all but the first coordinate of \(X_{T+1} = (x_{T+1}, x_T, ..., x_{T-d+2})\). Hence, \(X_{T+1}\) lies in the intersection of the sphere of radius \(|X_T - X_i| e^\lambda|\) centered on \(X_T\) and the line defined by \(\{z, x_T, ..., x_{T-d+2}\} \in \mathbb{R}\) which, in the Euclidean space, amounts to solving the following polynomial for \(z\in \mathbb{R}\):
\[ (z-x_{i+1})^2 + (x_T-x_i)^2 + ... + (x_{T-d+2}-x_{d+2})^2 - |X_T - X_i| e^\lambda = 0 \]

Typically, two candidates emerge, \(\hat{x}_{T+1}^-\) and \(\hat{x}_{T+1}^+\), respectively, underestimating and overestimating the true value of observation \(x_{T+1}\). To choose the \(\hat{x}_{T+1}^-\) and \(\hat{x}_{T+1}^+\), one can use the following method.

For two vector \(P = [x_1, x_2, ..., x_n]\) and \(Q = [y_1, y_2, ..., y_n]\), the angle is defined as \([30]\)
\[ \theta = \arccos \left(\frac{P \cdot Q}{\|P\| \cdot \|Q\|}\right) = \arccos \left(\frac{\sum_{i=1}^{m} x_i y_i}{\sqrt{\sum_{i=1}^{m} x_i^2} \sqrt{\sum_{i=1}^{m} y_i^2}}\right) \]

The smaller \(\theta\) represent that two vectors are closer. Use \(\hat{x}_{T+1}^-\) and \(\hat{x}_{T+1}^+\) for building two vectors \(\hat{X}_{T+1}^-\) and \(\hat{X}_{T+1}^+\). Calculate \(\theta\) with their nearest-neighbor and choose the smaller one as the prediction result.

3. Cracked rotor model

In order to verify the proposed methods, a Jeffcott rotor model is considered consisting of a rigid disk and a shaft with crack supported by two rigid bearings, as shown in Fig. 1, using fixed co-ordinate system \((OXY)\) and a rotating co-ordinate system \((O'\xi\eta)\). The crack is at mid-span of the shaft and at the left side of the disk.

![Fig. 1. Cracked rotor and the cross-section coordinate.](image-url)
Although the actual crack on the shaft opens and closes gradually while rotating, the typical ‘on–off’ model can still reflect the characteristics of a crack, and it is applied in most cases [20]. A step function is used to represent the opening and closing of the crack.

The motion equations of the system can be written as

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \dd x + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \dd y + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} f \Delta k \begin{bmatrix} \cos^2 t & \sin t \cos t \\ \sin t \cos t & \sin^2 t \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} mg \\ 0 \end{bmatrix} + \begin{bmatrix} me \omega^2 \cos (\beta + \omega t) \\ me \omega^2 \sin (\beta + \omega t) \end{bmatrix}$$

where $m$ is the mass of the disk, $c$ is the external damping coefficient, $e$ is the eccentricity of the disk unbalance, $k$ is the bending stiffness of the uncracked rotor and $f \Delta k$ is the stiffness changing of the rotor with periodic opening and closing of the crack. $\beta$ and $\omega$ are the original phase angle and the rotation speed, respectively.

Supposing $\delta$ is a static deflection of the disk, dividing both sides of Eq. (10) by $m \delta \omega$ and introducing the non-dimensional form of Eq. (10) can be obtained

$$\begin{bmatrix} X'' \\ Y'' \end{bmatrix} + \begin{bmatrix} \frac{1}{4} (1 - f \cdot \Delta K \cos^2 \tau) & -\frac{1}{4} f \cdot \Delta K \cos \tau \sin \tau \\ -\frac{1}{4} f \cdot \Delta K \cos \tau \sin \tau & \frac{1}{4} (1 - f \cdot \Delta K \sin^2 \tau) \end{bmatrix} \begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \frac{2D}{\Omega^2} & 0 \\ 0 & \frac{2D}{\Omega^2} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

where

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + U \cos (\tau + \beta) \\ U \sin (\tau + \beta) \end{bmatrix}$$

where

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{d}{d\tau} \end{bmatrix}, \quad D = \frac{c}{2m \omega_n}, \quad \Omega = \frac{\omega}{\omega_n}, \quad \Delta K = \frac{\Delta k}{k}, \quad \tau = \omega t, \quad U = \frac{e}{\delta}, \quad \omega_n = \sqrt{\frac{k}{m}},$$

where $f$ is a switch function, whose value is dependent of $\tau$. It can be written as

$$f = \begin{cases} 1 & x \cos \tau + y \sin \tau \leq 0 \\ 0 & x \cos \tau + y \sin \tau > 0 \end{cases}.$$

4. Numerical prediction of nonlinear response

In this section, two methods are applied to predict the nonlinear response of cracked rotor. The results are compared with the simulation one. The first method is the Lyapunov prediction method, directly forecasting the nonlinear response. The other one is the IMF prediction method. It first employs Lyapunov prediction method to forecast the response of IMF, and then adds all the prediction results of IMF. Consequently, the nonlinear response of cracked rotor can be predicted.

It is well known that the end effect (or boundary effect) is an important open problem related to the EMD method [10]. For this problem, we used the largest Lyapunov prediction method to solve the end issue of empirical mode decomposition for the nonlinear data [26]. The simulation results showed that the effect from the end issue is limited.
In this case, the prediction based on the end point of an IMF is highly questionable. It is helpful to the readers if the authors can clearly explain how the end effect affects the final prediction and how they treat the end effect. Will the final prediction depend on the method used to deal with the end effect?

Eq. (11) is solved by the Newmark-β method with the integration step of \( \pi/100 \). From the numerical simulation results, it is found that the response before the first several periods is generally not real due to the influence of free decreasing vibration, the exact period value depends on the external damping. So the following results are all obtained after the first 1000\( T \) (here \( T \) is the period of simulating force, it is equal to the time of a revolution of the rotor).

4.1. Numerical simulation: test case 1

When \( D = 0.01, U = 0.1, \Omega = 0.6, \Delta k = 0.58, \) and \( \beta = 0 \), the response of the cracked rotor is chaos. Fig. 2 shows the Poincaré map. The maximal Lyapunov exponent is 1.9157, Lyapunov time is 0.522 and the maximal prediction step \( n \) is 104. Fig. 3 shows the result of EMD. The maximal Lyapunov exponent of IMF1, IMF2 and IMF3 is 1.3773, 1.1991 and 0.3930 respectively. The maximal Lyapunov exponent for higher IMF is zero or negative. The calculation parameters of these Lyapunov exponents are shown in Table 1.

Fig. 4 shows the comparison of simulation results between IMF prediction method and Lyapunov prediction method. The simulation data have 90,000 points. EMD method is used to decompose all these data. In order to deal with the end effect, 10,000–79,600 points are used as known point to calculate the maximum Lyapunov Exponent. 79,601–80,000 are used as checksum values. In the figure, \( n \) is prediction point number. When the forecasting step is less than the maximal prediction step, these two methods almost predict the same results. When the forecasting step is greater than the maximal prediction step, it can be seen that the prediction error of Lyapunov prediction method increases fast and the results by the IMF prediction method are better than the Lyapunov prediction method.

4.2. Numerical simulation: test case 2

When \( D = 0.008, U = 0.1, \Omega = 0.617, \Delta k = 0.61, \) and \( \beta = 0 \), the response of the cracked rotor is chaos. Fig. 5 shows the Poincaré map. The maximal Lyapunov exponent is 1.4045, Lyapunov time is 0.714 and the maximal prediction step is 143. Fig. 6 shows the result of EMD. The maximal Lyapunov exponent of IMF1 and IMF2 is 0.9337 and 0.8360 respectively. The

Fig. 2. Poincaré map of test case 1.
maximal Lyapunov exponent for higher IMF is zero or negative. The calculation parameters of these Lyapunov exponents are shown in Table 2.

Fig. 7 shows the comparison of the results between IMF prediction method and the Lyapunov prediction method in the test case 2. It can be seen that the forecasting error of IMF prediction method is less than the Lyapunov prediction method when the forecasting step is larger than the maximal prediction step.

5. Experiment

In order to validate the IMF prediction method, a vibration test of a rotor system is conducted. The experimental model is shown in Fig. 8. The crack on the shaft is cut by a line cutting machine (width <0.2 mm). Four eddy current sensors are used to measure the key phase signal, the horizontal displacement, the vertical displacement, and the disk swing. Sensor 5 is used for speed feedback control. The measured position of the horizontal and vertical displacements is near the disk. However, the measurement position for the swing signal is close to disk edge.

The experimental equipment includes three parts: rotor, sensors, sampling systems. Bently RK4 rotor is used in this experiment (Fig. 9). The geometric parameters are as follows: effective span between two support $l=430$ mm, shaft diameter $d=10$ mm, disk weight $w=800$ g, diameter
D = 76 mm, and thickness t = 23 mm. The data test system is YE6261B which is made by China Jiangsu Lianneng Electronic Technology Company Ltd. The sampling frequency is 10 kHz.

Three shafts are tested in the experiment. One shaft is normal without crack; the crack depths for the other two are 2 mm and 4 mm. For the normal one, we have done a dynamic balance when the rotating speed is 1500 rpm/min and the first critical speed is 2100 rpm/min which can be obtained by Bode diagram. In the experiment, the crack will produce plastic deformation. After several times, the shaft must be scrapped. Because of the bending and deep crack, the shaft cannot bear the high-speed load. Therefore, our experiment speeds are all below the first critical speed.
After several experiments, when the crack depth is 4 mm and the rotating speed is 1410 rpm/min (near 2/3 of the first critical speed), the rotor response appears obvious nonlinear. The trajectory (Fig. 10) is a class pentagonal shape. The amplitude of the response was more significant compared with the one of the other speed. Clipping phenomenon appeared in the response. 1/2 (12.6 Hz), 3/2 (35.9 Hz), 5/2 (59.8 Hz) and 3 (72.1 Hz) octave can also be observed in the power spectrum. The maximal Lyapunov exponent of the response in the horizontal direction is 0.5624.

Fig. 11 shows the result of EMD. The maximal Lyapunov exponents of IMF1 and IMF2 are 0.2952 and 0.0852 respectively. The maximal Lyapunov exponent for a higher IMF is zero or negative. The calculation parameters of these Lyapunov exponents are presented in Table 3. Fig. 12 indicates the comparison of simulation results between the IMF prediction method and the Lyapunov prediction method. In order to show the result clearly, the local prediction points between 18,000 (1.75 s) and 26,000 (2.54 s) are shown in Fig. 12. According to the maximal Lyapunov exponent, the Lyapunov time is 1.78 s, which means 18,207 points. When the forecasting step is less than the maximal prediction step, the results of these two methods are almost the same. When the forecasting step is greater than the maximal prediction step, the results by the IMF prediction method are better than the ones by the Lyapunov prediction method.

Table 2
Maximal Lyapunov exponent calculation parameters of the test case 2.

<table>
<thead>
<tr>
<th></th>
<th>Delay time</th>
<th>Embedding dimension</th>
<th>Average period (No. of point)</th>
<th>Maximal Lyapunov exponent</th>
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<tr>
<td>Cracked rotor response</td>
<td>39</td>
<td>3</td>
<td>14</td>
<td>1.4045</td>
</tr>
<tr>
<td>IMF1</td>
<td>24</td>
<td>3</td>
<td>14</td>
<td>0.9337</td>
</tr>
<tr>
<td>IMF2</td>
<td>50</td>
<td>3</td>
<td>25</td>
<td>0.8360</td>
</tr>
</tbody>
</table>
6. Conclusion

The EMD method was applied to decompose the nonlinear response of a rotor and to get the forecasting results of the IMF, based on the maximal local Lyapunov prediction method. From the above simulation and experimental results, the following conclusions are made.

Usually, IMFs have a smaller maximal Lyapunov exponent than the original response. This is a significant advantage to increase the Lyapunov time. Compared with the Lyapunov prediction method.
Fig. 9. Picture of Bently RK4 rotor system.

Fig. 10. Experimental response with the crack depth of 4 mm and the rotating speed of 1410 rpm/min (a) trajectory diagram and (b) power spectrum.

Empirical Mode Decomposition

Fig. 11. EMD of the experimental response in the horizontal direction.
method, the IMF prediction method is more applicable when the forecasting step exceeds the maximal prediction step.

For the nonlinear response, the Lyapunov prediction method is better than the other methods which are based on the linear theory. Although the errors of these two methods are quite large, there is still a better way for nonlinear response. Especially, for the rotor system we show in this paper, when the forecasting time is less than the Lyapunov time, the Lyapunov prediction method is good enough for response prediction. When the forecasting time is larger than the Lyapunov time, the IMF prediction method needs to be employed to predict the long-term period response.

Table 3
Maximal Lyapunov exponent calculation parameters for the experimental response.

<table>
<thead>
<tr>
<th></th>
<th>Delay time</th>
<th>Embedding dimension</th>
<th>Average period (No. of point)</th>
<th>Maximal Lyapunov exponent</th>
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<td>Experimental response</td>
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<td>397</td>
<td>0.5624</td>
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<tr>
<td>IMF1</td>
<td>74</td>
<td>3</td>
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<td>0.2952</td>
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<tr>
<td>IMF2</td>
<td>41</td>
<td>2</td>
<td>753</td>
<td>0.0852</td>
</tr>
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</table>

Fig. 12. Prediction of the experimental response (a) prediction results and (b) prediction error.
Acknowledgments

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