Review of some important beam physics issues in electron–positron collider designs

Jie Gao
Institute of High Energy Physics, Yuquan Road 19, Beijing 100049, P. R. China
gaoj@ihep.ac.cn

Received 11 December 2014
Accepted 15 January 2015
Published 25 March 2015

In this paper, we will give a brief review of some important beam physics in circular and linear electron–positron collider designs, covering beam–beam tune limits, longitudinal and transverse single bunch collective effects, electron cloud and space charge effects, dynamic aperture estimations, etc. The main feature of this review is that the corresponding beam physics treatments are coming from author’s previous research works which are scattered in different scientific publications both for circular and linear colliders. With the progresses of future linear colliders, such as ILC, and future circular electron–positron colliders, such as CEPC, it is high time to review the key beam physics issues in the optimization designs of these two kinds of machines.

Keywords: Circular colliders; linear colliders; electron–positron; ILC; CEPC.
PACS Nos.: 29.20.db, 29.20.Dh, 29.20.Ej, 29.27.Bd

1. Introduction

Particle physics is not only the most fundamental research area in physics, but in Science as well. It contains both genes of ancient Greek natural Philosophy and Science, i.e. hypothesis, a priori, and the experimental proof of scientific theories with instruments. Nowadays, the ever existing largest scientific instruments in the world are particle accelerators, such as LEP and LHC at CERN, one for electron–positron collision and another for proton–proton collision. It is on LHC that the Higgs boson particle, predicted in the Standard Model Theory, was announced to be found on July 4, 2012. The discovered Higgs boson, about 125 GeV in the mass region, provides a very important subject of studies on its detailed properties by electron–positron colliders, both linear and circular ones. In the era of Higgs boson physics, supersymmetry and beyond, parallel to a hadron collider which continues to discover new territories in energy frontier, electron–positron colliders assert their unique roles in precision measurements.
As far as particle physics experiments are concerned, there are two important issues,\(^3\) the first one is the reaction energy in the center of momentum frame, which determines the happening of the event to be studied, and the second one is the reaction luminosity, i.e. the reaction rate per reaction transverse surface, with the specific event production rate being the multiplication of luminosity and the cross-section of a specific physics reaction process. For social aspects of these Big-instruments, there are other two important issues as well, i.e. construction cost and AC power consumption, which become super-key factors for political decisions in addition to their scientific goal and social impacts. For a good development of particle physics based on accelerators, an excellent balance of the four issues mentioned above demands a good choice of accelerators based on both excellent accelerator physics designs and advanced technologies.

For high energy physics experiments, compared to fix target scheme, apparently, the colliding beam experimental scheme is the most efficient way to reach the maximum reaction energy, however, the luminosity of colliding beams are much more difficult to increase compared with fixed target scheme, which is the key issue for all types of colliders in terms of accelerator beam physics and technologies.

In this paper, we will make a brief review of some important beam dynamics issues in circular and linear electron–positron colliders, which fundamentally influence the machine parameter choices and their performances, and these advanced beam dynamics knowledge are not included in the basic lectures both for circular colliders\(^4\) and linear colliders.\(^5\)

2. Circular Electron–Positron Colliders
To design an electron–positron storage ring collider, the basic knowledge has been provided in Sand’s lecture,\(^4\) however, advanced knowledge on beam–beam effects, bunch lengthening and bunch energy spread increasing, single bunch transverse emittance growth and instability threshold, dynamic aperture due to nonlinear multipoles, etc. should be addressed in much more detail, since they are the key factors to limit the machine luminosity.

2.1. Luminosity
The luminosity of an electron–positron circular collider can be expressed as\(^4\)

\[
L = \frac{I_{\text{beam}} \gamma \xi_y}{2 r_e \beta_y^*} \left( 1 + \frac{\sigma_y^*}{\sigma_x^*} \right) F_h, \tag{1}
\]

where \(r_e\) is the electron radius (2.818 \(\times\) 10\(^{-15}\) m), \(\beta_y^*\) is the beta function value at the interaction point (IP), \(\gamma\) is the normalized particle energy, \(\sigma_x^*\) and \(\sigma_y^*\) are the bunch transverse dimensions at the IP, respectively, \(I_{\text{beam}}\) is the circulating current of one beam, \(F_h\) is Hour glass reduction factor,

\[
F_h = \frac{\beta_y^*}{\sqrt{\pi} \sigma_z} \exp \left( \frac{\beta_y^*^2}{2 \sigma_z^2} \right) K_0 \left( \frac{\beta_y^*^2}{2 \sigma_z^2} \right), \tag{2}
\]
where $\sigma_z$ is the bunch length and $K_0$ is the zeroth order modified Bessel function of the second kind, and

$$\xi_y = \frac{N_e r_e \beta_y^*}{2\pi \gamma \sigma_y^* (\sigma_z^* + \sigma_x^*)} \quad (3)$$

is the vertical beam–beam tune shift, $N_e$ is the particle population inside the bunch. Practically, Eq. (1) could be expressed as follows

$$L = 2.17 \times 10^{34} (1 + r) \xi_y \frac{E_0(\text{GeV}) N_b I_b (A) F_h}{\beta_y^*(\text{cm})} [\text{cm}^{-2} \text{s}^{-1}], \quad (4)$$

where $r = \sigma_y^*/\sigma_x^*$, $N_b$ is the number of bunches inside a beam, $I_b$ is the average current of a bunch, and $I_{\text{beam}} = N_b I_b$.

In fact, since ACO,\(^6\) it is found that for all circular colliders, $\xi_y$ is not a free parameter, and for a given collider, there is a maximum $\xi_y$ value, or $\xi_y,_{\text{max}}$, which could not be surpassed with working point optimization, and beyond $\xi_y,_{\text{max}}$, the colliding bunch transverse dimensions blow-up and bunch lifetime drops drastically (exponentially in fact). These beam–beam interaction induced phenomena are called beam–beam effects. Understanding the beam–beam effects and finding out the beam–beam tune limit, $\xi_y,_{\text{max}}$, are the key subjects for accelerator physicists.

Before beam–beam effects were understood, for a long time, in a collider design, $\xi_y,_{\text{max}}$ was thought to be a constant independent of specific machine parameters, i.e. regardless whether $\xi_y,_{\text{max}}$ was a function of the machine energy, damping time, number of IPs and particle revolution period, etc. In fact, as we know from Ref. 7, for flat colliding beams, from transverse emittance blow-up mechanism, $\xi_y,_{\text{max}}$ of an $e^+e^-$ circular collider can be expressed as

$$\xi_y,_{\text{max}} = \frac{H_0}{2\pi} \sqrt{\frac{T_0}{\tau_y \gamma N_{\text{IP}}}}, \quad (5)$$

where $H_0 = 2845$, $\tau_y$ is the transverse damping time, $T_0$ is the revolution time, and $N_{\text{IP}}$ is the number of IPs. Or, for isomagnetic case, one has

$$\xi_y,_{\text{max}} = H_0 \gamma \sqrt{\frac{r_e}{6\pi R N_{\text{IP}}}}, \quad (6)$$

where $R$ is the local dipole bending radius.

Knowing the analytical expression of $\xi_y,_{\text{max}}$, one could have luminosity expressed as

$$L_{\text{max}}[\text{cm}^{-2} \text{s}^{-1}] = 2.17 \times 10^{34} (1 + r) \xi_y,_{\text{max}} \frac{E_0(\text{GeV}) N_b I_b (A) F_h}{\beta_y^*(\text{cm})}, \quad (7)$$

or in other different forms

$$L_{\text{max}}[\text{cm}^{-2} \text{s}^{-1}] = 0.158 \times 10^{34} (1 + r) \frac{I_b [\text{mA}]}{\beta_y^* [\text{mm}]} \sqrt{\frac{U_0[\text{GeV}]}{N_{\text{IP}} F_h}}, \quad (8)$$
J. Gao

where \( U_0 \) is the energy loss due to synchrotron radiation per turn, or

\[
L_{\text{max}}[^{\text{cm}}^{-2}s^{-1}] = \frac{0.158 \times 10^{34}(1 + r)}{\beta_y^{y}[\text{mm}]} \sqrt{\frac{J_b[\text{mA}]P_b[\text{MV}]}{N_{\text{IP}}}} F_h ,
\]

where \( P_b \) is the synchrotron radiation power of one colliding beam. It is obvious that to have high luminosity, one has to make the machine radiate beam energy as high as possible.

If the collider has \( N_{\text{IP}} \) IPs, and the total luminosity of the collider is denoted as \( L_{\text{total}} \), it is clear that \( L_{\text{total}} = N_{\text{IP}}L_{\text{max}} \propto \sqrt{N_{\text{IP}}} \). Thanks to the analytical expression of \( \xi_{y,\text{max}} \) expressed in Eq. (5), CEPC parameter could be optimized in a consistent way.\(^8\)

Before going on further with discussions on other subjects, let us remark an important thing, i.e. the difference between a lepton and a hadron collider in terms of beam–beam effects. According to Ref. 9, one has the general analytical beam–beam tune shift, \( \xi_{b,y,\text{max}} \), for a hadron circular collider, expressed as follows:

\[
\xi_{b,y,\text{max}} = H_0 \gamma f(x) \sqrt{\frac{r_h}{6\pi R N_{\text{IP}}}}
\]

or

\[
\xi_{b,y,\text{max}} = \frac{H_0}{2\pi f(x)} \sqrt{\frac{T_0}{\tau_y \gamma N_{\text{IP}}}} ,
\]

where

\[
f(x) = 1 - \frac{2}{\sqrt{2\pi}} \int_0^x \exp\left(-\frac{t^2}{2}\right) dt ,
\]

\[
x^2 = \frac{4f(x)}{\pi \xi_{y,\text{max}} N_{\text{IP}}}
\]

and \( x_0 \) in Eq. (11) could be solved by the following equation:

\[
x_0^2 = \frac{4f(x_0)^2}{H_0 \gamma} \sqrt{\frac{6\pi R}{r_h N_{\text{IP}}}} .
\]

If a proton–proton circular collider is concerned, one has to replace \( r_h \) by the classical radius of proton, \( r_p \), in Eqs. (10) and (14). In Super proton–proton Collider (SppC)\(^2\) design, Eqs. (11)–(14) have been used to estimate the maximum beam–beam tune shift.

2.2. Beam–beam tune shift limit coming from beam–beam induced beam lifetime reduction

As mentioned above, in addition to beam blow-up effect due to beam–beam interaction, beam–beam effects reduce beam lifetime, and in fact, this can be understood
from beam–beam effect induced dynamic aperture reduction. For two head-on colliding bunches, the incoherent kick felt by each particle can be calculated as:

$$\delta y' + i \delta x' = - \frac{N_e r_e}{\gamma_*} f(x, y, \sigma_x, \sigma_y),$$  \hspace{1cm} (15)

where $x'$ and $y'$ are the horizontal and vertical slopes, $\sigma_x$ and $\sigma_y$ are the standard deviations of the transverse charge density distribution of the counter-rotating bunch at IP, $\gamma_*$ is the normalized particle’s energy, and $*$ denotes the test particle and the bunch to which the test particle belongs. When the bunch is Gaussian, $f(x, y, \sigma_x, \sigma_y)$ can be expressed by Basseti–Erskine formula:

$$f(x, y, \sigma_x, \sigma_y) = \sqrt{\frac{2\pi}{\sigma_x^2 - \sigma_y^2}} \left( w\left( \frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) - \exp \left( - \frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) \left( w\left( \frac{\alpha_{xy} x + i \alpha_{xy} y}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right) \right),$$  \hspace{1cm} (16)

where $w$ is the complex error function expressed as

$$w(z) = \exp(-z^2)(1 - \text{erf}(-iz)).$$  \hspace{1cm} (17)

It is clearly seen that the transverse kick forces are strongly nonlinear, and according to Ref. 12, when flat beams collide, the beam–beam effect induced dynamic aperture could be expressed as

$$A_{\text{dyna,bb},y} = \frac{3}{\sqrt{2\pi \xi_y}}.$$  \hspace{1cm} (18)

From Eq. (18), the reason why $\xi_y$ is an important parameter to describe the beam–beam effect is clearly seen.

In Ref. 12, we have derived beam–beam effect limited beam lifetimes for a rigid flat beam

$$\tau_{bb,y,\text{flat}} = \frac{\tau_y}{2} \left( \frac{3}{\sqrt{2\pi \xi_y N_{IP}}} \right)^{-1} \exp \left( \frac{3}{\sqrt{2\pi \xi_y N_{IP}}} \right),$$  \hspace{1cm} (19)

$$\tau_{bb,x,\text{flat}} = \frac{\tau_x}{2} \left( \frac{3}{\pi \xi_x N_{IP}} \right)^{-1} \exp \left( \frac{3}{\pi \xi_x N_{IP}} \right)$$  \hspace{1cm} (20)

and a rigid round beam

$$\tau_{bb,y,\text{round}} = \frac{\tau_y}{2} \left( \frac{4}{\pi \xi_x N_{IP}} \right)^{-1} \exp \left( \frac{4}{\pi \xi_x N_{IP}} \right).$$  \hspace{1cm} (21)
From Eqs. (19) and (21), one finds that for the same $\tau_{y, bb, flat}/\tau_y$, $\tau_{x, bb, flat}/\tau_x$ and $\tau_{y, bb, round}/\tau_y$, one has $\xi_{x, flat} = 2\xi_y, flat$ and $\xi_{y, round} = \frac{2}{3} \xi_y, flat = 1.89 \xi_y, flat$.

Now taking into account the emittance blow-up effect due to beam–beam interactions, in a heuristic way, one gets\(^7\)

$$\tau_{bb, y, flat} = \frac{\tau_y}{2} \left( \frac{3 \xi_{y, max, em, flat}}{\sqrt{2\pi}\xi_{y, max, 0} N_{IP}} \right)^{-1} \exp \left( \frac{3 \xi_{y, max, em, flat}}{\sqrt{2\pi}\xi_{y, max, 0} N_{IP}} \right)$$

and

$$\tau_{bb, y, round} = \frac{\tau_y}{2} \left( \frac{3 \xi_{y, max, em, round}}{\sqrt{2\pi}\xi_{y, max, 0} N_{IP}} \right)^{-1} \exp \left( \frac{3 \xi_{y, max, em, round}}{\sqrt{2\pi}\xi_{y, max, 0} N_{IP}} \right),$$

with

$$\xi_{y, max, em, round} = 1.89 \xi_{y, max, em, flat},$$

where $\xi_{y, max, em, flat}$ is $\xi_{y, max}$ expressed in Eq. (5), and $\xi_{y, max, 0}$ is rigid beam case limiting value. Taking $\xi_{y, max, 0} = 0.0447$ means that we quantify the term “beam–beam limit” for the beam–beam limited beam lifetime being one hour at $\tau_y = 30$ ms with $N_{IP} = 1$.

Finally, as far as beam–beam effects are concerned, if beams collide with definite crossing angle or meet each other in parasitic crossing points, the relevant detailed discussion could be found in Refs. 13 and 14.

### 2.3. Longitudinal short range wakefield of a storage ring

To study single bunch longitudinal collective effect, it is very important to know the short range wakefield of the whole ring. A storage ring vacuum beam passage is a complicated metallic boundary system, it consists of rf cavities, vacuum chamber, bellows, collimators, etc., the development of an analytical formula to describe the short range wake potential of storage ring whole vacuum system is a challenging task. Here, according to Ref. 15, we provide one empirical analytical wakefield expression as follows

$$\mathcal{W}_z(z)(V/C) = -ak(\sigma_z) \exp \left( -\frac{2z^2}{\sigma_z^2} \right) \cos \left( 1 + \frac{2}{\pi} \arctan \left( \frac{Z_i}{2Z_r} \right) \right) \frac{z}{\sqrt{3\sigma_z}} \left( \frac{Z_i}{2Z_r} \right),$$

where $\sigma_z$ is bunch length, $k(\sigma_z)$ is the total loss factor of the ring, $L(\sigma_z)$ is the total inductance, $a = 2.23$, $Z_i = 2\pi L(\sigma_z)/T_0$, $Z_r = k(\sigma_z) L(\sigma_z)/T_0$, $T_0 = 2\pi R_{av}/c$, $T_k = 3\sigma_z/c$,
Review of some important beam physics

$R_{av}$ is the average radius of the ring, $c$ is the velocity of light, and $z = 0$ corresponds to the center of the bunch, and all variables are in MKS units.

### 2.4. Bunch lengthening and bunch energy spread increasing effects

In a collider, the bunch length of the colliding beam is always one of the top concerns, since it influences the luminosity through Hourglass effect and Piwinski angle, if beam collides with definite angle. To estimate the bunch length with respect to bunch current is of great importance. The bunch lengthening phenomenon in an electron storage ring was first observed in ACO\textsuperscript{6} and later in other machines, where accompanying bunch lengthening, one finds an increase in the single bunch energy spread with a more or less appreciable threshold current. The first empirical bunch lengthening formula\textsuperscript{6} found in ACO is expressed as:

$$
\sigma_{\tau}^2(\text{ns}) = \sigma_{\tau 0}^2(\text{ns}) \left(1 + 2 \times 10^{-3}\frac{I_b (mA)}{E^4(\text{GeV})}\sigma_{\tau}(\text{ns})\right),
$$

where $\sigma_{\tau}(\text{ns})$ is the bunch rms duration measured in nano-second, $\sigma_{\tau 0}(\text{ns})$ corresponds to $\sigma_{\tau}(\text{ns})$ at zero bunch current, $I_b$ is the bunch current, and $E$(GeV) is the particle energy. Since then, understanding these single bunch longitudinal collective phenomena has become one of the main battle fields for accelerator physicists.

According to Ref. 16, bunch lengthening, $R_z = \sigma_z/\sigma_{z0}$, and bunch energy spread increasing, $R_\varepsilon = \sigma_\varepsilon/\sigma_{\varepsilon0}$, with respect to the bunch current could be estimated as follows:

$$
R_z^2 = 1 + \frac{\sqrt{2C R_{av} R K_{\text{tot}}^\parallel I_b}}{\gamma^{\frac{3}{2}}(R_z)^{\frac{1}{2}}},
$$

where

$$
C = \frac{576\pi^2\epsilon_0}{55\sqrt{3}hc^3},
$$

$\epsilon_0$ is the permittivity in vacuum, $h$ is Planck constant, $c$ is the velocity of light, $K_{\text{tot}}^\parallel$ is the total longitudinal loss factor of the whole ring at the natural bunch length, $\sigma_{z0}$, $I_b = eN_e c/(2\pi R_{av})$, and $R_{av}$ is the average radius of the ring. If SPEAR scaling law\textsuperscript{7} is used (for example), $\zeta \approx 1.21$ (in fact each machine has its own $\zeta$), Eq. (27) can be written as

$$
R_z^2 = 1 + \frac{\sqrt{2C R_{av} R K_{\text{tot}}^\parallel I_b}}{\gamma^{\frac{3}{2}}(R_z)^{\frac{1}{2}}},
$$

$$
= + \frac{C(R_{av} R I_b K_{\text{tot}}^\parallel)^2}{\gamma^{\frac{7}{2}}(R_z)^{\frac{4}{2}}},
$$

The procedure to get the information about the bunch lengthening and the energy spread increasing is first to find $R_z(I_b)$ by solving bunch lengthening equation, i.e. Eq. (27), and then calculate energy spread increasing, $R_\varepsilon(I_b)$ ($R_\varepsilon = \sigma_\varepsilon/\sigma_{\varepsilon0}$), by putting $R_z(I_b)$ into Eq. (30):

$$
R_\varepsilon^2 = 1 + \frac{C(R_{av} R I_b K_{\text{tot}}^\parallel)^2}{\gamma^{\frac{7}{2}}(R_z)^{\frac{4}{2}}},
$$

1530006-7
J. Gao

2.5. Fast single bunch transverse instability threshold

For a given storage ring, it has been observed that when the bunch current surpasses certain value, the transverse dimension of the bunch will blow-up, and this threshold current depends on chromaticity. In fact, once $R_{\varepsilon}(|\xi_c|)$ is found, one can use the following formulae to calculate the fast single bunch transverse instability threshold current: \[ I_{th\text{, \, gao}} = \frac{F' f_s E_0}{\epsilon (\beta_{y,c}) K_{\perp}^{tot}(\sigma_z)} \] with \[ F' = 4R_{\varepsilon}\frac{\nu_y \sigma_{e0}}{\nu_x E_0} \] where $\nu_x$ and $\nu_y$ are synchrotron and vertical betatron oscillation tunes, respectively, $\langle \beta_{y,c} \rangle$ is the average beta function in the rf cavity region, $\xi_{c,y}$ is the chromaticity in the vertical plane (usually positive to control the head-tail instability), $K_{\perp}^{tot}(\sigma_z)$ is the total transverse loss factor over one turn, $\sigma_{e0}$ is the natural energy spread, and $E_0$ is the particle energy. The magnitude of $F'$ can be easily estimated as around 4 for $|\xi_{c,y}| = 1$ and $R_{\varepsilon} = 1$. In practice, it is useful to express $K_{\perp}^{tot}(\sigma_z)$ as $K_{\perp}^{tot}(\sigma_z) = K_{\perp}^{tot,0}/R_{\varepsilon}^{\Theta}$, where $K_{\perp}^{tot,0}$ is the value at the natural bunch length, and $\Theta$ is a constant depending on the machine concerned. By using SPEAR scaling law, $\Theta$ can be taken as $2/3$.$^{17}$ Equation (31) is therefore expressed as: \[ I_{th\text{, \, gao}} = \frac{F' f_s E_0 R_{\varepsilon}^{2/3}}{\epsilon (\beta_{y,c}) K_{\perp}^{tot,0}}. \] The notation $I_{th\text{, \, gao}}$ is used with the aim of distinguishing it from the formula given by Zotter.$^{18,19}$

2.6. Bunch transverse emittance growth due to vacuum chamber impedance

In an electron storage ring, in addition to fast single bunch instability mentioned above, it is observed that with the increasing bunch current not only a bunch suffers from bunch lengthening, increase in energy spread, but also transverse emittance growth. The usual explanation to the transverse emittance grow is based on the intrabeam scattering theory$^{20-22}$ which has its origin from H. Bruck’s idea.$^{23}$ Comparison of the emittance growth between experimental results and those from intrabeam scattering theory shows, however, that in the vertical plane the agreement is not satisfactory.$^{24,25}$ In this paper, we will draw attention to another important physical cause for the transverse emittance grows in addition to the intrabeam scattering, i.e. the short range transverse wakefield of the machine.$^{26}$ It is not difficult to imagine that if the closed orbit is distorted and (or) the vacuum chambers are misaligned from the ideal geometric center, the particles in a bunch will suffer from transverse deflections due to single bunch short range wakefield which result
in its emittance growth similar to what happens in a linac when the accelerating structures’ axes do not coincide with the trajectory of the passing bunch.\(^{27}\)

If we distinguish now the horizontal plane denoted by the subscript \(x\) and the vertical plane denoted by the subscript \(y\), one gets two emittance equations

\[
\mathcal{R}_{e,x} = \frac{\epsilon_{\text{total},x}}{\epsilon_{0,x}} = 1 + \frac{\sigma_x^2 \tau_x \langle \beta_x(s) \rangle}{4T_0 \epsilon_{0,x} \mathcal{R}_z^3} \left( \frac{e^2 N_e k_{1,x}(\sigma_{z,0})}{m_0 c^2 \gamma \mathcal{R}_z^3} \right)^2, \tag{34}
\]

\[
\mathcal{R}_{e,y} = \frac{\epsilon_{\text{total},y}}{\epsilon_{0,y}} = 1 + \frac{\sigma_y^2 \tau_y \langle \beta_y(s) \rangle}{4T_0 \epsilon_{0,y} \mathcal{R}_z^3} \left( \frac{e^2 N_e k_{1,y}(\sigma_{z,0})}{m_0 c^2 \gamma \mathcal{R}_z^3} \right)^2, \tag{35}
\]

where \(\sigma_{z,0}\) is the bunch length of zero current, \(\sigma_x\) and \(\sigma_y\) are standard deviations of the beam orbit with respect to the geometric center of vacuum chamber, \(\langle \beta_x(s) \rangle\) and \(\langle \beta_y(s) \rangle\) are the average of the beta functions over the ring, \(\tau_x\) and \(\tau_y\) are the damping times, \(\epsilon_{0,x}\) and \(\epsilon_{0,y}\) are the natural emittances, and \(T_0\) is the revolution period, \(\mathcal{R}_z = \sigma_z/\sigma_{z,0}\), and \(\Theta = 0.7\), which corresponds to SPEAR scaling for transverse loss factor.\(^{28}\) Since \(\mathcal{R}_z\) is also a function of \(N_e\), it is obvious that one can start to solve Eqs. (34) and (35) only when \(\mathcal{R}_z(N_e)\) has been solved from the bunch lengthening equation.\(^{16}\) Equations (34) and (35) set the requirements of the orbit alignment tolerances with respect to the center of vacuum chamber.

### 2.7. Electron cloud effect

For a positron storage ring, the positron beam attracts the residual electrons inside vacuum chamber both from discharged residual gas and the secondary emission electrons from the vacuum wall towards the center of the beam orbit, and the attracted electron density depends on positron beam current, bunch space structure, residual gas density, vacuum wall property, and synchrotron radiation, etc. Different from beam–beam interaction, the electron cloud effect results from a fast moving positron beam colliding with quasi stationary electron cloud on the beam orbit.

The differential positron linear tune shift (i.e. per meter) due to electron cloud is expressed as:\(^{14}\)

\[
\xi_{e,c} = \frac{r_e N_{ec} \beta_{+y}(s_0)}{2\pi \gamma_{+y}(s_0)(\sigma_{+x}(s_0) + \sigma_{+y}(s_0))} \left( \frac{1}{2L_0} \right), \tag{36}
\]

where \(L_0\) is the storage ring length, \(\sigma_{+x}\) and \(\sigma_{+y}\) are the transverse rms dimensions of the electron clouds and positron beam (both transverse dimensions are assumed same), \(\beta_{+y}\) is the vertical beta function for positrons, \(\gamma_{+}\) is the normalized positrons’ energy, and finally \(N_{ec}\) is total electron cloud charge numbers around the ring within a transverse cross-section of \(2\pi \sigma_{+x} \sigma_{+y}\). According to Ref. 14, one has the normalized dynamic aperture due to electron cloud expressed as

\[
\frac{A_{ec,y}}{\sigma_{+y}} = \sqrt{\frac{3\sqrt{2\gamma_{+}}}{\pi r_e \beta_{av,y} \rho_{ec} L_0}}, \tag{37}
\]
where $\beta_{av,y}$ is the average vertical beta function around the ring, $\gamma_+$ is normalized positron’s energy, and $\rho_{ec}$ is the average electron cloud density inside the vacuum chamber which is defined as follows:

$$\rho_{ec} = \frac{N_{ec}}{2\pi \sigma_{av,+x,} \sigma_{av,+y} L_0},$$

(38)

where $\sigma_{av,+x}$ and $\sigma_{av,+y}$ are the average beam transverse dimensions around the ring.

From Eq. (37), one could estimate the limit to which the electron cloud density should be suppressed either by using solenoid magnet field along the vacuum pipe or suppressing the electron production sources, such as better vacuum inner surface coating.

2.8. Space charge effect

Space charge effect matters not only in a hadron machine, but also in electron storage rings if the machine parameters happen to be the case, such as in TESLA dog-bone damping ring.

Considering an electron storage ring, we start with the linear incoherent space charge tune shift of the machine at the center of the bunch over the whole ring:

$$\xi_{sc} = \frac{r_c N_e \beta_{av,y}}{2\pi \gamma \sigma_{y,av} (\sigma_{x,av} + \sigma_{y,av})} \left( \frac{L_0}{\sqrt{2\pi \beta^2 \gamma^2 \sigma_z}} \right),$$

(39)

where $\sigma_z$ is the bunch length, $\beta_{av,y}$ is the average over the ring and $\beta_{av,x}$ is assumed to be equal to $\beta_{av,y}$, $\sigma_{x,av}$ and $\sigma_{y,av}$ are the average bunch’s transverse dimensions in the ring, $\beta$ and $\gamma$ are the normalized electron’s velocity and the energy, respectively. The normalized nonlinear space charge limited dynamic aperture could be expressed as

$$\left( \frac{A_{total,sc,y}(s)}{\sigma_y(s)} \right) = \sqrt{\frac{3}{2\pi \xi_{sc}}},$$

(40)

From Eq. (40), one could estimate the limit to which the space charge effect matters.

2.9. Dynamic aperture induced by multipoles

For the circular accelerator designers, one of the preoccupations is to estimate the influence of nonlinear forces on the single particle’s motion. These nonlinear forces manifest themselves as the systematic and random errors from optical elements, the voluntarily introduced functional ones, such as the sextupoles used for the chromaticity corrections, the octupoles used for stabilizing the particles’ collective motion, wiggler used to enhance synchrotron radiation, or from nonlinear beam–beam interaction forces. Even though the nonlinear forces mentioned above compared with the linear forces are usually very small, what is observed in reality,
however, is that when the amplitudes of the transverse oscillation of a particle are large enough, the transverse motion might become unstable and the particle itself will finally be lost on the vacuum chamber. Apparently, the above implied maximum oscillation amplitudes, $A_{x,y}$, corresponding to the stable motions are functions of the specific longitudinal position, $s$, along the machine, and these functions $A_{x,y}(s)$ are the so-called dynamic apertures of the machine. In this subsection, we will give the general expression of the dynamic aperture due to a multipole and the dynamic aperture due to many multipoles. As a special case of the dynamic aperture due to many multipoles, we give the dynamic aperture from a wiggler also. If we express the magnetic field in the direction perpendicular to the orbit surface

$$B_z = B_0(1 + x b_1 + x^2 b_2 + x^3 b_3 + \cdots + x^{m-1} b_{m-1} + \cdots).$$

(41)

One has from Ref. 30 the general expression of the dynamic aperture in the horizontal plane ($z = 0$) of a single $2m$ ($m \geq 3$) pole component:

$$A_{\text{dy}n\alpha,2m} = \sqrt{2\beta_x(s)} \left( \frac{1}{m \beta_x^m(s\{2m\})} \right)^{1/2} \rho \left( \frac{\rho}{b_{m-1}} \right)^{1/(m-2)} L,$$

(42)

where $s\{2m\}$ is the location of this multipole, $\rho$ is the radius of curvature, and $L$ is the length of the multipole. The question that follows immediately is: If there are more than one nonlinear components how can one estimate their collective effect? Obviously, this is a difficult question to answer in a general way. Fortunately, one can distinguish two cases:

(1) If the components are independent, i.e. there are no special phase and amplitude relations between them, the total dynamic aperture can be calculated as:

$$A_{\text{dy}n\alpha,\text{total}} = \sqrt{\sum_i \frac{1}{A_{\text{dy}n\alpha,\text{ext},i}} + \sum_j \frac{1}{A_{\text{dy}n\alpha,\text{oct},j}} + \sum_k \frac{1}{A_{\text{dy}n\alpha,\text{deca},k}} + \cdots}.$$

(43)

(2) If the nonlinear components are dependent, i.e. there are special phase and amplitude relations between them (for example, in reality, one use some additional sextupoles to cancel the nonlinear effects of the sextupoles used to make chromaticity corrections), there is no general formula as Eq. (43) to apply.

As a direct application of Eqs. (42) and (43), one gets the dynamic aperture of an ideal wiggler:31

$$A_{N_w,y}(s) = \frac{3\beta(s)}{\beta_{y,m}^2} \frac{1}{\beta_y \sqrt{L_w}},$$

(44)

where $N_w$ is the wiggler period number, $\lambda_w$ is the wiggler period length, the wiggler length $L_w = N_w \lambda_w$, $\rho_w$ is the radius of curvature of the wiggler peak magnetic field $B_0$, and $\rho_w = E_0/e c B_0$ with $E_0$ being the electron energy, and $\beta_{y,m}$ is the beta function value in the middle of the wiggler.
3. Linear Colliders

3.1. Introduction

Since the discovery of Higgs boson on LHC at CERN in 2012, the scientific goal for the future linear collider is more clear than before. It is required that the center-of-mass energy ($E_{cm}$) should start from 250 GeV, upgradable to 1 TeV (ILC), even above to several TeV, with different accelerator technologies (CLIC), and the luminosity should be in the order of $10^{34}$ cm$^{-2}$s$^{-1}$ with the luminosity proportional to the square of the energy to keep constant event rates. To make an optimization design of linear colliders and to make fair comparisons between different linear colliders with different technologies, one needs good understanding of machine design philosophy based on background limitations from detectors.\(^{32}\)

In a linear collider, the luminosity of two Gaussian head-on colliding beams is given by

$$L_0 = \frac{f_{\text{rep}}N_bN_e^2}{4\pi\sigma_x^*\sigma_y^*}H_{D_x}H_{D_y}, \quad (45)$$

where $f_{\text{rep}}$ is the repetition rate of the bunch train, $N_b$ is the number of bunches in the train, $N_e$ is the number of particles per bunch, $\sigma_x^* = (\epsilon_x^*\beta_x^*)^{1/2}$, $\sigma_y^* = (\epsilon_y^*\beta_y^*)^{1/2}$, $\beta_x^*$ and $\epsilon_x^*$ are the values of the beta functions and the emittance at the IP, respectively, and $H_{D_x,y}$ are the pinch enhancement factors which are functions of the so-called disruption parameters $D_{x,y}$ of a bunch. In Sec. 3.2, we demonstrate a general procedure to determine the beam parameters from the constraints at the IP. In Secs. 3.3 and 3.4, we discuss the efficiencies of transferring the rf power to the beam, and the constraints on the accelerating structure misalignments to limit the luminosity reduction.

3.2. Constraints from IP

When two head-on colliding electron and positron beams penetrate each other, every particle in each beam will feel the electromagnetic field of the other beam and will be deflected. This deflection process has several effects. First, the deflected particle will lose part of its energy due to the synchrotron radiation, as “beamstrahlung”, which will increase the energy spread of the colliding beams, and hence increase the uncertainty of the experiments. Secondly, after the collision the particles will change their flying directions with respect to the axis by an angle $\theta_x,y$ (assuming the particle is parallel to the axis before the collision). If this angle is large enough, the particles after the collision will interfere with the detection of small-angle events. Finally, the deflected particles will emit photons, hadrons, etc. which will increase the noise background level in the detector. In the following, we discuss these effects in detail.

The beamstrahlung fractional energy spread $\delta_B$ is expressed as,\(^{33}\)

$$\delta_B = \frac{2r_e^2N_e^2}{3\sigma_x^*\sigma_y^*\sigma_z}F(R), \quad (46)$$

1530006-12
where \( r_e = 2.82 \times 10^{-15} \) m is the classical electron radius, \( R = \sigma_x^*/\sigma_y^* \) is the aspect ratio of the bunch, \( \gamma \) is the ratio of electron energy to its rest energy, \( F(R = 1) = 0.325 \), and \( F(R \gg 1) \approx 1.3/R \). It is obvious that in order to keep \( \delta_B \) small it is better to make the beam flat. In the following discussion, we assume always \( R \gg 1 \).

If \( \delta_B^* \) is the maximum tolerable beamstrahlung energy spread, one has

\[
\frac{N_e}{\sigma_x^*} = \left( \frac{3\delta_B^* \sigma_z}{2.6r_e^3 \gamma} \right)^{1/2}.
\]  

(47)

During the collision the collective fields of one beam will deflect the other beam. The effect is equivalent to that of a thin lens with the focal lengths in both the horizontal and vertical planes expressed as \( f_{x,y} = \sigma_z/D_{x,y} \), where \( D_{x,y} \) are called disruption parameters.\(^{34}\)

\[
D_{x,y} = \frac{2r_e N_e \sigma_z}{\gamma (\sigma_x^* + \sigma_y^*)}.
\]  

(48)

If the colliding particles are parallel to the axis before collision, the disruption angles \( \theta_{x,y} \) are

\[
\theta_x = \theta_y = \frac{2r_e N_e}{\gamma (\sigma_x^* + \sigma_y^*)} = \frac{2r_e N_e}{\gamma \sigma_x^*}.
\]  

(49)

If \( \theta^* \) is the maximum tolerable disruption angle, one has

\[
\frac{N_e}{\sigma_x^*} = \frac{\theta^* \gamma}{2r_e}.
\]  

(50)

The mean number of beamstrahlung photons per incident particle is,\(^{35,36}\)

\[
n_\gamma \approx \frac{5\alpha^2 \sigma_z^2}{2r_e \gamma} \Upsilon_0,
\]  

(51)

\[
\Upsilon_0 = \frac{5\gamma^2 \gamma N_e}{6\alpha \sigma_z (\sigma_x^* + \sigma_y^*)},
\]  

(52)

where \( \alpha \) is the fine structure constant. If \( \Upsilon_0 \ll 1 \) and the beam is very flat \( (R \gg 1) \),\(^{36}\)

\[
n_\gamma \approx \frac{2\alpha r_e N_e}{\sigma_x^*}.
\]  

(53)

If the number of the maximum tolerable beamstrahlung photon is \( n_{\gamma}^* \), one has,

\[
\frac{N_e}{\sigma_x^*} = \frac{n_{\gamma}^*}{2\alpha r_e}.
\]  

(54)

The number of hadronic events per crossing, \( N_{\text{Had}} \) is\(^{37,38}\)

\[
N_{\text{Had}} = \frac{1}{4\pi} \left( \frac{N_e}{\sigma_x^*} \right)^2 \frac{\sigma_x^*}{\sigma_y^*} H_{\gamma \gamma} n_{\gamma}^* \sigma_{\gamma \gamma} \rightarrow \text{Had},
\]  

(55)

1530006-13
where $\sigma_{\gamma \gamma \rightarrow \text{had}}$ is the $\gamma \gamma \rightarrow$ hadron total cross-section. If the maximum tolerable $N_{\text{Had}}$ is denoted as $N^*_{\text{Had}}$, one gets the maximum aspect ratio $R^*$ ($H_{D_y}$ is almost a constant, about 1.5 within the range of $D_y$ which will be discussed later),

$$R^* H_{D_y} = \frac{\sigma^*_y}{\sigma^*_y H_{D_y}} = \frac{1}{(n^*_x)^2(\frac{N_e}{\sigma^*_z})^2} \left( \frac{4\pi N^*_{\text{Had}}}{\sigma_{\gamma \gamma \rightarrow \text{had}}} \right).$$  \hspace{1cm} (56)

From constraints expressed in Eqs. (50) and (54), one can find out the maximum $N_e/\sigma^*_x$ value depending on which constraint is the more stringent one. Assuming that one has to take $n^*_x$ more seriously which is of order unity, one can find the minimum bunch length $\sigma^*_z$ in a linear collider from Eqs. (47) and (54),

$$\sigma^*_z = \frac{1.3(n^*_x)^2 \gamma r_e}{6 \delta^*_p \alpha^2}.$$ \hspace{1cm} (57)

In order to fix the beam parameters from $n^*_x$ and $N_{\text{Had}}$, we will go back to Eq. (48) to see the constraint on the maximum disruption $D_y$ (since $D_x \ll D_y$). It is well known that when the disruption is too large ($D_y > 20$) the pinch enhancement factor $H_{D_y}$ is very sensitive to the misalignment of the colliding beams due to the kink instability. It is found, however, that when $D_y$ is about 9, the sensitivity to the offsets is reduced by a maximum factor of 2. If the constraint, $D_y = 9$, is included in Eq. (48), one gets the relation between $\sigma^*_y$ and $\sigma^*_z$.

$$\sigma^*_y = \frac{2\gamma r_e}{9 \gamma} \left( \frac{N_e}{\sigma^*_z} \right) \sigma^*_z.$$ \hspace{1cm} (58)

One sets as usual

$$\beta^*_y \approx \sigma^*_z,$$ \hspace{1cm} (59)

due to the “hourglass effect”, and assumes the beta ratio at IP to be equal to the emittance ratio,

$$\frac{\beta^*_x}{\beta^*_y} = \frac{\epsilon^*_x}{\epsilon^*_y},$$ \hspace{1cm} (60)

which corresponds to equal horizontal and vertical beam envelope divergence angles. Finally, to determine the value of $f_{\text{rep}} N_b$, one can express Eq. (45) in the following way

$$L_0 = f_{\text{rep}} N_b \left( \frac{N_{\text{Had}}}{(n^*_x)^2 \sigma_{\gamma \gamma \rightarrow \text{had}}} \right).$$ \hspace{1cm} (61)

It is obvious that for the same luminosity, the same backgrounds, $f_{\text{rep}} N_b$ is a constant for the different machine designs. Finally, we arrive at the stage that once $\gamma$, $n^*_x$, $N_{\text{Had}}$ and $L_0$ are given, one can determine the values of $\sigma^*_x$, $\sigma^*_y$, $R^*$, $\sigma^*_z$, $N_e$, $\beta^*_y$, $\epsilon^*_y$, $R^*$ and $f_{\text{rep}} N_b$. The difference between different machine designs (different frequency choices and normal or superconducting rf structures) is interpreted by the different combination of $f_{\text{rep}}$ and $N_b$ while keeping their product constant.
3.3. Alternating current power to beam power transfer efficiency

Knowing the beam parameters, one should make a choice among a conventional and a nonconventional linear accelerator system (Two Beam Accelerator (TBA), for example), a normal conducting and a superconducting system, and estimate the ac power to run these linear colliders.

3.3.1. Normal conducting linear accelerators

In this section, we consider a linear collider using the discrete modulator-klystron powering system. The efficiency of converting the ac power to the rf power by means of modulators and klystrons is denoted as $\eta_{rf_{ac}}$, which is in the order of 50%, and the efficiency of transferring the rf power to the beam should be estimated accordingly. Since this efficiency is accelerating structure dependent, we will start with constant gradient travelling wave structures.

The average rf power needed to create an accelerating field $E_0$ along the total active beam path $L$ is

$$\langle P \rangle_{rf} = \tau_{rf} f_{rep} \frac{E_0^2}{R_{sh}(1 - e^{-2\tau})} L,$$

where $\tau_{rf}$ is the rf pulse duration, $R_{sh}$ is the shunt impedance per unit length, $E_0$ is the average accelerating field strength without beam loading, $\tau$ is the attenuation of the structure, and $L$ is the machine length (2 linacs). In the following we distinguish two cases. First, if the duration of the bunch train ($\tau_e$) is larger than the filling time of the accelerating structure, the total machine length $L$ can be calculated as

$$L = \frac{W_{cm}}{eE_{eff}} \frac{W_{cm}}{eE_0(1 - ekN_eN_b\tau_{fill}(1 + e^{-\tau})/2E_0\tau_e)},$$

where $W_{cm}$ is the center of mass energy, $k$ is the accelerating mode loss factor, $e$ is the charge of the electron, $\tau_{fill}$ is the filling time of the accelerating structure with $\tau_{rf} = \tau_{fill} + \tau_e$, and $E_{eff}$ is the effective accelerating field strength including the beam loading. The rf to the beam power transfer efficiency $\eta_{bf_{rf}}$ is

$$\eta_{bf_{rf}} = \frac{eR_{sh}N_bN_e(1 - e^{-2\tau})(1 - e(2 - \tau_e/\tau_{fill}))kN_eN_b(1 + e^{-\tau})/2E_0)}{\tau_{rf}E_0}.$$ 

Second, if $\tau_e \leq \tau_{fill}$

$$L = \frac{W_{cm}}{eE_{eff}} = e(2 - \tau_e/\tau_{fill})E_0(1 - ekN_eN_b(1 + e^{-\tau})/2E_0),$$

$$\eta_{bf_{rf}} = \frac{eR_{sh}N_bN_e(1 - e^{-2\tau})(1 - e(2 - \tau_e/\tau_{fill}))kN_eN_b(1 + e^{-\tau})/2E_0)}{\tau_{rf}E_0}.$$

The total ac power is

$$P_{ac} = \frac{\langle P \rangle_{bf}}{\eta_{bf_{rf}}},$$

where $\langle P \rangle_{bf} = eW_{cm}f_{rep}N_0N_e$ is the total average beam power.
3.3.2. Superconducting linear accelerators

As far as superconducting standing wave structures are concerned, as in the case of ILC, the rf power to the beam power transfer efficiency is simply

$$\eta_{bf} = \frac{\tau_e}{\tau_{rf}},$$

(68)

where $\tau_{rf} = Q_L L \omega \ln 4 + \tau_e$ is the rf pulse duration, $Q_L$ and $\omega$ are the loaded quality factor and the angular resonance frequency of the accelerating cavity, respectively. It is clear that when $Q_L \ln 4/\omega \ll \tau_e$ one can get a high rf to beam power transfer efficiency.

3.3.3. Two beam accelerator scheme

For a TBA scheme, the power transformation efficiency from the ac power to the rf power to be injected into the main linac is very difficult to estimate in a general way, since it depends very much on the detailed process of the transformation. Taking the CLIC scheme for example, one has to transform, first, the ac power to an rf power of 350 MHz by using klystrons in cw mode, secondly, to transfer the rf power to the drive beam power, and finally, to convert the drive beam power to the 30 GHz rf power required by the main linac, by means of transfer structures. The total transfer efficiency will be the product of the efficiency of all the three processes. In this paper, some handy formulae are given to find out the drive beam parameters starting from the main linac power requirement. If $f_{rf}$, $P_{rf}$, $\tau_{rf}$, and $f_{rep}$ are the rf frequency, the peak rf power, the rf pulse duration, and the rf pulse repetition rate of the main linac, the drive beam has to be composed of $N_d$ trains with $\tau_{rf} = \tau_{d\text{fill}} N_d$, where $\tau_{d\text{fill}}$ is the filling time of the transfer structure. The repetition rate of these $N_d$ trains is $f_{rep}$. The total electric charge in each train is $Q_d$ which can be determined as

$$Q_d = \left(\frac{2\tau_{d\text{fill}} P_{rf}}{k_d n_0 k_d v_d (1 - e^{-2\tau_d})}\right)^{1/2},$$

(69)

$$\tau_d = \frac{\pi f_{\text{rf}} \tau_{d\text{fill}}}{Q_0^d},$$

(70)

where $Q_0^d$ is the quality factor of the transfer structure, $n_0$ is the number of the main beam accelerating structures powered by one transfer structure, $\tau_d$ and $k_d$ are the attenuation and the working mode loss factor of the transfer structure. When the charge $Q_d$ is the sum of $n_d$ bunchlets, the electric charge in each bunch should be $q_d = Q_d / n_d$. The average power of one drive beam is

$$(P)_{bf}^d = W_d q_d n_d N_d f_{rep} / e,$$

(71)

where $W_d$ is the drive beam’s energy which has to be large enough to keep the longitudinal and transversal stabilities. Assuming that $\delta_w$ is the maximum tolerable...
energy spread within a bunch train of the drive beam, the minimum drive beam injection energy can be determined as

\[ W_d = \frac{eq_d k_d L_d^T}{2} \left(1 + \frac{n_d}{\delta_w} \left(1 - e^{-\tau_d/\tau_e}\right) F\right), \tag{72} \]

where \( L_d^T \) is the total active length of the transfer structures, \( \tau_d^d \) is the time duration of the \( n_d \) bunchlets. If \( n_d \gg 1 \), the average power of one drive beam can be expressed as

\[ \langle P \rangle^d_b = \frac{F}{2\delta_w} f_{\text{rep}} P_{\text{rf}} \tau_{\text{rf}} N_{m}^T, \tag{74} \]

where \( N_{m}^T \) is the total number of the main linac accelerating structures. It is clear that \( \langle P_{\text{rf}} \rangle_m = f_{\text{rep}} P_{\text{rf}} \tau_{\text{rf}} N_{m}^T \) is the total average main linac rf power, and

\[ \eta_{\text{rf},m}^d = \delta_w/F, \tag{75} \]

is the transfer efficiency from the total drive beam power to that of the rf power of the main linac and it is connected with the drive beam linac. A good drive beam linac design corresponds to a maximum \( \eta_{\text{rf},m}^d \) value. If superconducting structures are used to accelerate the drive beam with very high rf to beam transfer efficiency, the total ac power will be approximately (the ac power for the refrigerators is not included)

\[ P_{\text{ac}} = 2 \frac{\langle P_{\text{rf}} \rangle_m}{\eta_{\text{ac}} \eta_{\text{rf},m}^d}, \tag{76} \]

where \( \eta_{\text{ac}} \) is the transfer efficiency from the ac power to the average rf power of the drive beam linac. Taking the CLIC transfer structure for example, \[ ^{42,43} \] one has \( f_{\text{rf}} = 30 \text{ GHz}, P_{\text{rf}} = 40 \text{ MW}, \tau_{\text{rf}} = 11.2 \text{ ns}, N_d = 4, n_0 = 2, \tau_{\text{fill}}^d = 2.8 \text{ ns}, e_v^d/c = 0.595, Q_0^d = 3808, k_d = 0.156 \text{ V/pC/m}, \] one finds from Eq. (69) that \( Q_d = 1.765 \mu\text{C} \). If \( n_d = 43, \eta_d = 0.04 \mu\text{C} \) (or \( 2.5 \times 10^{11} \) electrons). Assuming \( \delta_w = 50\% \) and \( L = 6.4 \text{ km} \), one gets from Eqs. (72) and (75) that \( W_d = 3.2 \text{ GeV}, \langle P \rangle^d_b = 36.5 \text{ MW}, \) and \( \eta_{\text{rf},m}^d = 33\% \), respectively.

### 3.4. Emittance growth and the tolerances

With the properly designed damping rings, the required single bunch emittances \( \epsilon_x, \epsilon_y \) can be obtained at the entrances of the main linacs (a different machine has a different damping ring, since \( f_{\text{rep}} \) and \( N_b \) are different from one machine to another). Due to the misalignments of the accelerating structures, the quadrupoles, the beam position monitors (BPMs), the injection offsets, and the ground motions, etc. the trajectories of the particles are not always on the axis of the accelerating structures, and as a consequence, the bunch train will undergo the single bunch
J. Gao

and the multibunch emittance growths due to the short range and the long range wakefields.\textsuperscript{27} To keep these emittance growths within a certain range one has to use BNS damping to compensate the short range wakefields due to the coherent betatron oscillations caused by injection errors, to detune and damp the higher order modes excited inside the accelerating structures to reduce the long range wakefields, and to use different emittance correction techniques to release the alignment tolerances. Assuming that the long range wakefield induced multibunch emittance growth is made less than the single bunch emittance growth by means of damping and detuning, or by means of increasing the bunch separation, one knows that the luminosity reduction is\textsuperscript{44,45}

$$\frac{\Delta L_0}{L_0} \approx \frac{1}{2} \left( \frac{\Delta \epsilon^*_y}{\epsilon^*_y} \right) = -\frac{1}{2} \frac{N_s^2 f_{rf}^6 (\lambda_{rf}/a_{iris})^6 \sigma_z (\Delta y_c)^2 \beta_0}{E_{eff}^2 \epsilon^*_y},$$

(77)

where $a_{iris}$ is the average iris radius of a constant gradient structure, $\Delta y_c$ is the rms misalignment error of the accelerating structures, and $\beta_0$ is the maximum beta function value at the entrance of the main linac. The constant $C$ in the formula can be determined by numerical simulations, and it is found that $C \approx 1 \times 10^{-70} \ V^2 s^6 m^{-5}$. Equation (77) assumes that the same beta function scaling law has been applied to different machines.

4. Conclusions

In this paper, we have made a brief review on the key accelerator beam physics issues in order to be helpful in making optimization designs of both $e^+e^-$ circular and linear colliders, especially in the era of Higgs boson studies and beyond by future colliders, such as CEPC(SppC) and ILC.

Acknowledgments

The author thanks Dr. Y. W. Wang for editing this paper. This work was supported by NSFC 11175192.

References

Review of some important beam physics

18. B. Zotter, Mode-coupling or transverse turbulence of electron or positron bunches in the SPS and LEP, LEP note 363 (1982).
29. J. Gao, Theoretical analysis on the limitation from the nonlinear space charge forces to TESLA damping ring, TESLA Note 2003-12.
33. H. Wiedemann, 1981 Summer School on Particle Physics, SLAC-PUB-2849.
37. P. Chen, T. Barklow and M. Peskin, Hadron production in gg collisions as a background for e⁺e⁻ linear colliders, SLAC-PUB-5873.
43. I. Wilson, CLIC, the basic scheme, critical issues and status of test facilities, CLIC
44. R. Brinkmann, Beam dynamics in linear colliders, what are the choices?, Internal Report, DESY M-95-10 (1995).