Experimental violation of the local realism for four-qubit Dicke state

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Abstract: Dicke state is an widely used type of multi-particle entangled state in quantum information. However, very few works have been done on its nonlocality. Here we prepare a four-photon symmetric Dicke state, whose fidelity is as high as 0.904 ± 0.004, and devise a simple Bell-type inequality to demonstrate that it violates the local realism with 12 standard deviation.

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References and links
1. Introduction

Multi-particle entanglement states provide an important resource in quantum information and computing. Therefore, efficient preparation of these entangled states, e.g., Greenberger-Horne-Zeilinger (GHZ) states, cluster states and Dicke states, is an important task in experiment. The violations of local hidden variable theory for multipartite entangled states become the criteria of some Device-Independent (DI) quantum information processing tasks, such as the security of quantum key distribution (QKD) [1, 2], decreasing the computation complexity [3, 4] and the validity of random number generation [5], etc.

Observations of violations of local realism in the case of multipartite correlations are a challenging task because of the lack of handy Bell inequalities for multipartite systems.

For two-qubit system, the Clauser-Horne-Shimony-Holt (CHSH) inequality [6] has a simple structure and is easy to observe the violation of local hidden variable theory experimentally. For arbitrary many observers, each choosing between two local dichotomic observables, the complete set of tight CHSH-type Bell inequalities has been obtained [7–9]. Such inequalities have been pointed out to possess a common structure [10]. However, the explicit expressions of Bell inequalities are too complicate to be used to verify the violation of local realism. The multipartite Mermin-Ardehali-Belinskii-Klyshko (MABK) inequalities have the explicit expressions, however, the multipartite MABK inequality has $2^N$ terms [11–13] for $N$-partite system, it is not convenient to use MABK inequalities to check the nonlocality of a multipartite entangled states if the number of the particles is large. The multipartite Bell inequalities involving only two-body correlations still reveal the Bell- nonlocality in many-body system. Those Bell inequalities open a way to verify multipartite Bell-nonlocality experimentally [14, 15].

The less the number of terms of a Bell inequality is, the less difficulty of the implementation of nonlocality for multipartite entangled states is. Based on the geometrical properties of correlation polytopes [16], the structural features of CHSH-type Bell inequalities are found [17]. Furthermore, the multipartite CHSH-type Bell inequalities with four terms are proposed. Therefore, it is possible to propose the experimental scheme to observe the violation of local realism for multipartite entangled states.

As a kind of important multipartite states which naturally arise in symmetric systems [18],
Dicke states are highly persistent against photon loss and projective measurements [19]. These states possess high level multipartite entanglement. Recent researches also demonstrate that they are optimal for quantum metrology [20, 21] and easy to quantify the entanglement depth for many-body systems [22]. For these reasons, many works have been done on preparing the high quality Dicke states. In the optical system, the four-photon symmetric Dicke state with a fidelity of 0.844 ± 0.008 was prepared in experiment by N. Kiesel et al in 2007 [19], and the six-photon case was simultaneously realized by W. Wieczorek et al. [23] and R. Prevedel et al. [24]. However, the general method used to detect the entanglement of Dicke states cannot reveal the violation of local realism [19, 20, 23–25].

In this work, we report an experimental demonstration of the local realism of the four-photon Dicke state $D_4^{(2)}$. Moreover, the Bell-type inequality we put forward has only four terms. It is more simple and easier to be realized in experiment than the one in [26].

2. Experiment

![Experimental setup for Dicke state preparation](image)

In our experiment, the frequency-doubled ultraviolet (UV) pulses (390 nm, 76 MHz repetition rate, 400 mW average power) from a mode-locked Ti:sapphire laser are used to pump a type-I beta-barium-borate (BBO) crystal ($6 \times 6 \times 2$ mm$^3, \theta = 29.9^\circ$). This result in four horizontal polarization photons in a single pulse,

$$|\Psi\rangle = |H\rangle_a \otimes |H\rangle_a \otimes |H\rangle_b \otimes |H\rangle_b.$$  

(1)

After passing through the 45° rotated half-wave plate1 (HWP1), the polarization of photons in mode 'a' is changed from horizontal orientation to vertical orientation. By adjusting a good
Hong-Ou-Mandel-type interference [27] (the visibility of the interference is about 0.97), we removed the path information of photons in both modes (mode 'a' and mode 'b') and obtained four indistinguishable photons. As Fig. 1 shown, three Beam Splitters distribute them equally onto four spatial modes ('c', 'd', 'e', 'f'). At last, only the four photon coincidence counts are recorded. Then we can get the two excitations four-photon Dicke state $D_4^{(2)}$,

$$|D_4^{(2)}\rangle = \frac{1}{\sqrt{6}}(|HHVV\rangle + |HVVH\rangle + |VHHV\rangle + |HVHV\rangle + |VHVH\rangle + |VVHH\rangle),$$  

(2)

the four entries in the state vector indicate the horizontal (H) or vertical (V) polarizations of the photons in one mode. For example, $|HHVV\rangle = |H\rangle_c \otimes |H\rangle_d \otimes |V\rangle_e \otimes |V\rangle_f$ denotes that the photons of mode 'c', mode 'd' are horizontal polarization and the photons mode 'e', mode 'f' are vertical polarization. For the purpose of obtaining high interference visibility, all the photons are coupled into single mode fibers in our experiment. Even though, nearly 23 four-fold coincidence counts can collect in one minute and this is enough to perform the full tomography.

By performing the standard tomography, we get the fidelity of state is $0.904 \pm 0.004$, which is better than the previous published results (details in Fig. 2). This is mainly due to the photon source we used. For our experiment setup, the photons are generated from the type-I nonlinear SPDC, then a HOM type interference with high visibility guarantees a good identity in their spatio-temporal mode structure.

![Fig. 2. State tomography. Histograms of the density matrix for the ideal state and the real part of the density matrix derived from the measurement data.](image)

3. Local realism

The Bell inequality we construct is devised by the method in [17]. The inequality is given by

$$\hat{A}_0 \hat{B}_0 \hat{C}_0 \hat{D}_0 + \hat{A}_0 \hat{B}_1 \hat{C}_1 \hat{D}_1 + \hat{A}_1 \hat{B}_0 \hat{C}_1 \hat{D}_0 - \hat{A}_1 \hat{B}_1 \hat{C}_0 \hat{D}_1 \leq 2,$$

(3)

where the local observables $\hat{A}_i, \hat{B}_i, \hat{C}_i$ and $\hat{D}_i$ have the form: $\hat{X}_i = x_i \cdot \sigma$, where $x_i = (x_{i1}, x_{i2}, x_{i3}) \in \mathbb{R}^3$ with $|x_i| = 1$ (i=0,1), Pauli operator $\sigma = (\sigma_x, \sigma_y, \sigma_z)$.

By numerical computation, the maximal expectation value of Eq. (3) is given by 3.054 for Dicke state, while the coefficients of the local observables are given in Table 1:

By adjusting the angles of the QWP and the HWP before the PBS, we can realize the measurement of the local observable, and the detections of the two outputs of PBS are corresponding to the projection measurement of its two eigenstates respectively. The 16 (four
Fig. 3. Results. Each term’s values are combinations of 16 coincidence rates. The values for $a_0b_0c_0d_0$, $a_0b_1c_1d_1$, $a_1b_0c_1d_0$, $a_1b_1c_0d_1$ are $0.6443 \pm 0.0168$, $0.4683 \pm 0.0160$, $0.6315 \pm 0.0165$ and $-0.6588 \pm 0.0167$, respectively. By substitution of these into the Bell inequality, we obtained a value of $2.4029 \pm 0.0330$, which violates the local realism threshold by more than $12\sigma$.

Table 1. The coefficients of the local observables.

<table>
<thead>
<tr>
<th></th>
<th>$j = 1$</th>
<th>$j = 2$</th>
<th>$j = 3$</th>
<th>$j = 1$</th>
<th>$j = 2$</th>
<th>$j = 3$</th>
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</thead>
<tbody>
<tr>
<td>$a_{0j}$</td>
<td>-0.0817</td>
<td>0.2953</td>
<td>-0.9519</td>
<td>$a_{1j}$</td>
<td>-0.2300</td>
<td>0.8310</td>
</tr>
<tr>
<td>$b_{0j}$</td>
<td>-0.0815</td>
<td>0.2945</td>
<td>-0.9522</td>
<td>$b_{1j}$</td>
<td>0.2332</td>
<td>-0.8426</td>
</tr>
<tr>
<td>$c_{0j}$</td>
<td>-0.2301</td>
<td>0.8316</td>
<td>0.5055</td>
<td>$c_{1j}$</td>
<td>-0.0813</td>
<td>0.2937</td>
</tr>
<tr>
<td>$d_{0j}$</td>
<td>0.0815</td>
<td>-0.2945</td>
<td>0.9522</td>
<td>$d_{1j}$</td>
<td>-0.2601</td>
<td>0.9400</td>
</tr>
</tbody>
</table>

photon coincidence) measurements of D1 (D2), D3 (D4), D5 (D6) and D7 (D8) are combined together to obtain the values of the four terms in Eq.(3), which are given by $0.6443 \pm 0.0168$, $0.4683 \pm 0.0160$, $0.6315 \pm 0.0165$ and $-0.6588 \pm 0.0167$ (Fig. 3). When plugging them into the left of the inequality, we get a result of $2.403 \pm 0.033$. The threshold for the local realistic model of these correlations is less or equal to 2, and our experiment violates it by $12\sigma$. Errors are mainly due to the time uncertainty of the photon pairs generated in BBO. The coincidence counts obey a Poisson distribution.

4. Conclusions

In summary, we have prepared a four-photon symmetric Dicke state with high fidelity by non-linear SPDC process and high-visibility HOM interference. Then we demonstrate the nonlocal realism of the Dicke state. The confidence of violation is $12\sigma$ in our experiment. The new Bell-type inequality we use only involves four correlation functions, i.e. we need the least measurements compared to other inequalities [26]. And the term number will remain unchanged with the increasing of the size of system, so it is efficient for multi-photon experiment.
Acknowledgments

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