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Heat flux and temperature field cloaks for arbitrarily shaped objects

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Abstract

We apply transformation optics (TO) theory to investigate two-dimensional heat flux cloaks for arbitrarily shaped objects. The TO theory is applied to design a device through which heat flux travels around objects with arbitrary shapes, which greatly improves the flexibility of the cloak applications. The proposed theory is verified by numerical results, showing that the proposed method is capable of controlling the diffusive heat flow and cloaking a region with arbitrary geometries of interest.

1. Introduction

Transformation optics (TO) has been proposed as a powerful tool to control wave propagation. The concept was initially proposed by Pendry \cite{1} and Leonhardt \cite{2} in the context of electromagnetics. One of the exciting applications of TO is ‘cloak’, a coating shell can guide the propagation of light and acoustic waves. As a result, a region inside the shell becomes invisible.

Later, this concept inspired many theoretical and experimental approaches for controlling other waves, such as acoustics \cite{3–7}, dc magnetic field \cite{8}, elastic wave \cite{9–11} and matter waves \cite{12}. Very recently, TO theory was extended to thermodynamics \cite{13–16}. Although heat flux is not a real wave because it does not transport energy as waves normally do, the thermal cloak by using TO has still been achieved. This is because the heat conduction equation, also known as the heat diffusion equation, is invariant under coordinate transformation. These pioneer works theoretically open possibilities for cloaking and focusing heat flux. Later, such a thermal cloak was experimentally confirmed by Narayana and Sato \cite{17} using the multilayered composite cylindrical material. The artificially engineered thermal material can shield, concentrate and invert heat current to particular paths of interest. To the best of our knowledge, all the previous heat flux cloaks are limited to regular and symmetrical geometry, such as spherical and circular shapes. There is no report about the theory of heat flux cloaks with arbitrary shape which may be interesting in other applications. Thus, this paper may improve the designing flexibility of the thermal cloak.

In this paper, we propose two-dimensional heat flux cloaks for arbitrarily shaped objects. The general expressions of the complex material parameters for the cloaks’ structure are derived. The interactions of the designed cloaks with incident heat flux are studied based on finite element method. The numerical results verify the invisibility properties of the designed cloaks and effectiveness of the proposed method.

2. Theoretical model

The material parameters of the thermal cloak for arbitrary geometries will be derived based on the TO theory. Thermal conduction is the movement of heat flux flowing from a high-temperature region toward a low-temperature region. Here we start from the thermal conduction equation without the source term

\begin{equation}
\nabla (-\kappa \nabla T) = 0,
\end{equation}

where \(\kappa\) is the thermal conductivity and \(T\) is the temperature. In this study, the analysis is restricted to a two dimensional case. Milton et al \cite{18} have proved that this equation is invariant in its form under coordinate transformation.

In physical space, the polar coordinates \((r, \theta)\) can be converted to the Cartesian coordinates \((x, y)\) by using the...
According to the TO theory, the transformed thermal conductivity tensor $\tilde{\kappa}$ can be calculated by using the following relationship

$$\tilde{\kappa}(x') = \frac{A\kappa(x)A^T}{\det(A)},$$

where $A$ is the Jacobian transformation matrix with elements defined by

$$A = \frac{\partial(x', y', z')}{\partial(x, y, z)} = \begin{bmatrix} \frac{\partial x' / \partial x}{\partial y' / \partial y} & \frac{\partial x' / \partial y}{\partial y' / \partial y} & \frac{\partial x' / \partial z}{\partial y' / \partial y} \\ \frac{\partial y' / \partial x}{\partial z' / \partial y} & \frac{\partial y' / \partial y}{\partial z' / \partial y} & \frac{\partial y' / \partial z}{\partial z' / \partial y} \\ \frac{\partial z' / \partial x}{\partial z' / \partial y} & \frac{\partial z' / \partial y}{\partial z' / \partial y} & \frac{\partial z' / \partial z}{\partial z' / \partial y} \end{bmatrix}.$$ (8)

By application of transformation ((5a)–(5c)), the heat conduction equation is mapped into:

$${\nabla(-\tilde{\kappa}\nabla T)} = 0,$$ (9)

where the transformed thermal conductivity tensor $\tilde{\kappa}$ of the coated material can be calculated by using (9), the result is

$$\tilde{\kappa}_{xx} = \left[r' - \tau R_0(\theta')\right]^2 \cos^2 \theta' + \tau^2 \left[\frac{dR_0(\theta')}{d\theta'}\right]^2 \cos^2 \theta'$$

$$- 2\tau r' \frac{dR_0(\theta')}{d\theta'} \sin \theta' \cos \theta' + r'^2 \sin^2 \theta' \right]$$

$$\times \left[r'[r' - \tau R_0(\theta')]\right]^{-1}$$ (10a)

$$\tilde{\kappa}_{yy} = \tilde{\kappa}_{zz} = \frac{1}{r'} \left[ r'[r' - \tau R_0(\theta')] \right]^{-1}$$ (10b)

$$\tilde{\kappa}_{zz} = \left(\frac{1}{1 - \tau}\right)^2 \left[r'[r' - \tau R_0(\theta')] / r'\right]$$ (10c)

in which $dR_0(\theta') / d\theta'$ is the first order derivative of $R_0(\theta')$ with respect to $\theta'$. Therefore, equations (10a)–(10d) are the general expression of the required material parameter for a cloak with outer boundary $R_0(\theta')$ and inner boundary $\tau R_0(\theta')$. Although the transformed thermal conductivity has to be highly anisotropic, such a cloaking device can be implemented by using an approximate method. First, since $R_0(\theta')$ can be chosen as closed contours with arbitrary shapes, they can be generally expressed by a Fourier series with periodic $2\pi$. Then, based on effective media theory, such required anisotropic material can be constructed by discretizing the overall continuous material as a network of thermal elements in series. This method was experimentally confirmed by Narayana and Sato [17].
3. Numerical simulation and discussion

Full-wave simulations are carried out to demonstrate the effectiveness of the designed cloaks based on finite element method. Different arbitrary geometries without symmetry are chosen to describe the flexibility of the approach. Without losing the generality, the irregular boundaries of the cloak are defined on some different contour equations

\[
\text{Case A: } R_0(\theta) = \frac{[12 + 2 \cos(\theta) + \sin(2\theta) - 2 \sin(3\theta)]}{200}.
\]

\[
\text{Case B: } R_0 = 1 + 0.1 \sin(\theta) + 0.2 \sin(3\theta).
\]

\[
\text{Case C: } R_0(\theta) = 0.7 + 0.1 \sin(\theta) + 0.1 \sin(3\theta) + 0.1 \cos(5\theta).
\]

Figure 2 shows the temperature profiles for a thermal cloak for case A. The arrows denote the pathway of the total heat flux and the solid lines represent the isothermal lines. It is clearly seen that the heat fluxes travel around the inner domain and eventually return to their original pathway. The isothermal lines are smoothly bent around the inner region. In contrast, the temperature field inside the inner region does not interact with the shell region. Therefore, the object inside the inner domain is protected from the invasion of the external heat flux. However, a small variation of temperature fluctuation can be still observed. This is partially due to the small numerical error of discretization in finite element method, and can be minimized by using smaller meshes. On the other hand, the singularity of the transformed material parameter, as shown in equations (10a)–(10d), may also lead to imperfect invisibility effect since it is not easy to deal with ideally in numerical simulations.

Figures 3 and 4 show the cloaking performance for Case B and Case C, respectively. It is seen that the heat fluxes are smoothly guided around inner region and then return in the same propagating direction as if they had passed through the enclosed irregular regions. The isothermal lines are excluded from these regions.

As the last example, we show the thermal cloak around a perfect cylinder which is expressed by contour equation \( R_0(\theta) = 1 \), as shown in figure 5. It is seen that the internal cloak manipulates the incoming heat flux around a thermal forbidden perfect cylinder where hidden objects can be thermally protected.

Thus, such a thermal cloak for arbitrarily shaped objects can be achieved. In contrast, the idea of putting an insulating object, such as polyurethane, in the path and blocking some of the heat flux is straightforward and too simple. The approach presented in this paper is made possible such that metamaterial can reroute the heat flow flux around the region with arbitrary geometries which may lead to practical applications.
Figure 5. Temperature field profile around a perfect cylinder.

4. Conclusions

In summary, the reported theoretical results in this paper suggest that by using transformation optics technique it is possible to design a two-dimensional heat flux cloak for an arbitrarily shaped object.

The transformation optics theory is utilized to develop the material parameters. We show that heat flux is guided round the arbitrary shaped objects and the isotherms are smoothly bent around the invisibility region. Such heat flux cloaking should lead to unprecedented heat flux control and novel applications.

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