The overconfident trader does not always overreact to his information\textsuperscript{☆}

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\textbf{A R T I C L E  I N F O}

Article history:
Accepted 8 February 2015
Available online 25 February 2015

Keywords:
Outsider
Shared information
Inside trading
Public information
Overconfidence
Nash equilibrium

\textbf{A B S T R A C T}

This article develops a strategic trading model in which the outsider is overconfident on the shared information. Our result shows that a more confident outsider underreacts to his information in the sense that he trades less aggressively on his information, leading to a less profit in the trading. However, the insider trades more aggressively on the shared information and less aggressively on the private information when he faces a more overconfident outsider. Also, the overconfidence of the outsider leads to a larger insider’s expected profits, an increased expected loss of noise trader, and a less efficient and less stable market.

\textsuperscript{☆} The authors would like to thank the Editor and anonymous referees for many helpful comments and suggestions.

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\textsuperscript{1} This author is grateful for financial support of National Natural Science Foundation of China No. 11201060 and No. 11471069.

\textsuperscript{2} Holden and Subrahmanyam (1994), Zhang (2004), etc give the risk-averse informed trader case based on the model of Kyle (1985). Kyle (1985) has led to a large literature that is covered well in Vives (2010).

1. Introduction

Abundant evidences show that investors and managers are prone to be overconfident in the sense that they overestimate the precision of their information. Wang (1998), Glaser and Weber (2007), etc. have predicted that traders will trade more when they are overconfident on their information while, Benos (1998) and Kyle and Wang (1997) find that a rational insider trades less when he faces an overconfident opponent. However, we find that a more overconfident outsider trades less aggressively on his information, leading to a less profit in the trading. Also, when the rational insider’s overconfident opponent is the outsider who shares some information with him, he would like to trade more aggressively on the shared information but less aggressively on his private information.

The pioneering model proposed by Kyle (1985) investigates the optimal trading strategies of a rational and risk-neutral insider with a precise signal about the risky asset’s fundamental value\textsuperscript{2}. Based on this model, Holden and Subrahmanyam (1992) and Foster and Viswanathan (1996) consider a market with multiple competing insiders, and they show that competition among insiders accelerates the release of their private information. Huddart et al. (2001) present an insider’s equilibrium trading strategy in a multi-period rational expectations framework based on Kyle (1985), given the requirement that the insider must publicly disclose his stock trades after the fact. Gong and Liu (2012) and Zhang (2008) extend Huddart et al. (2001) by allowing for competition among identical informed agents and the existence of outsiders, respectively. All the papers mentioned above have the assumption that the insider is rational while, Wang (1998) extends Kyle’s (1985) model by assuming that there is an irrational insider in the market, and finds that heterogeneous prior beliefs lead to a large increase in the trading volume. In addition, Kyle and Wang (1997), Wang (1997) and Harris and Raviv (1993) all consider the case of heterogeneous prior beliefs. A consistent finding is that the more overconfident investors overreact to their private information, the more aggressive the trade is while, in this paper we try to give a model to show that the overconfident outsider does not always overreact to his information. Also, we study the trading behavior of a rational insider facing an overconfident outsider and analyze the impacts on the market efficiency, the market depth and the profits of each trader.

Using an extension of the framework of Kyle (1985), this paper analyzes a strategic trading model with the presence of overconfident outsider and the public information. Strictly speaking, the outsider shares some information with the insider, but he overestimates its precision. Market makers have a knowledge of a public signal concerning the liquidity value and set the price conditional on their information, including the order imbalance and the public information, in a semi-strong efficient way. We find that the rational insider puts a positive weight on private information and puts a bigger weight on the shared information than the overconfident outsider does. This is different from the result of Liu and Zhang (2011),
which finds that the insider and the outsider put the same weight on the shared information. We also show that the risk-neutral market makers use the public information correctly, while neither the insider nor the outsider considers about the public information when formulating their trading strategies. This distinguishes our model from Zhou (2011), which finds that the rational insider trades manipulatively on public information when he trades with overconfident market makers. Also, a more overconfident outsider trades less aggressively on his shared information while, the insider trades less aggressively on his private information and put a bigger weight on their shared information when he faces a more confident outsider. Intuitively, the rational insider would like to trade more aggressively on the shared information to take advantage of “misvalued” opportunities made by the overconfident outsider. Furthermore, a more confident outsider can get less profit and lead to a smaller marker depth, a less efficient market, more profits of the insider and a more loss of the noise traders.

This paper is structured as follows. In Section 2, we present the model. In Section 3, we identify the unique Nash equilibrium of the model. Section 4 concludes. In Appendix A we provide proofs.

2. The model

A single risky asset is traded in the market among four kinds of risk-neutral traders: an informed insider, an outsider, noise traders and competitive market makers. The ex-post liquidation value of the risky asset is a random variable \( v = \bar{v} + \bar{s} + \bar{\epsilon} \), normally distributed with mean \( \mu_0 \) and variance \( \Omega^2 \). i.e. \( v \sim N(\mu_0, \Omega^2) \). The first part, \( \bar{v} \), is related to the private information known only to the insider. Prior to trading, the insider learns the value of the security by observing the signal \( \bar{x}, \bar{u} \) and \( \bar{\epsilon} \). The second part, \( \bar{s} \), which is normally distributed with zero mean and variance \( \sigma_{\bar{s}}^2 \), is related to the shared information obtainable by the outsider (but not by market makers and noise traders). We set \( \sqrt{E(\sigma_{\bar{s}}^2)}^{-1} \) as the precision of the shared information. However, the overconfident outsider overestimates it as \( k(\sqrt{E(\sigma_{\bar{s}}^2)})^{-1} \). In other words, for shared information, the insider has the rational belief: \( \bar{s} \), while the outsider has the overconfident belief: \( k \bar{s} \). And the third part, \( \bar{\epsilon} \), which is normally distributed with zero mean and variance \( \sigma_{\bar{s}}^2 \), is public information known to all traders but not to noise traders. And random variables \( \bar{x}, \bar{s}, \bar{u}, \bar{\epsilon} \) and \( \bar{s} \) are mutually independent.

We conform the trading process to Luo (2001) and Liu and Zhang (2011). There are two periods, period 0 and period 1, in the economy. At period 0, the information (public and private) is released and the trading takes place. After the announcement of the information, the insider submits an order \( \bar{x} = X(\bar{v}, \bar{s}, \bar{\epsilon}) \) based on his information \( \bar{v}, \bar{s}, \bar{\epsilon} \), and the outsider chooses his trading strategy by submitting an order \( \bar{y} = Y(\bar{s}, \bar{\epsilon}) \) based on the information \( \bar{s}, \bar{\epsilon} \). The market makers receive these orders along with those of noise traders whose exogenously generated total demand \( \bar{u} \) is normally distributed with mean zero and variance \( \sigma_{\bar{u}}^2 \). After receiving the total order \( \bar{w} = \bar{x} + \bar{y} + \bar{u} \) (but not each individual’s) and the public information \( \bar{\epsilon} \), the market makers set the price \( \bar{p} = P(\bar{w}, \bar{\epsilon}) \) of the risky asset in a semi-strong efficient way such that they expect to earn a zero profit. At period 1, the uncertainty is resolved and the risky asset payoff is realized.

Let \( \bar{n}_i(X, P) = (\bar{v} - \bar{p}) \bar{x}, \bar{n}_0(Y, P) = (\bar{v} - \bar{p}) \bar{y} \), denote the resulting trading profits of the insider and that of the outsider, respectively. Use \( E_n, E_0 \) to denote the insider’s and market makers’ expectation conditional on their information, respectively. And \( E_0 \) denotes the expectation of the overconfident outsider conditional on the overestimated information.

**Definition 1.** An equilibrium consists of the insider’s and outsider’s trading strategies and market makers’ pricing rule \((X, Y, P)\), such that the following two conditions hold:

1. **Profit maximization:** for any alternate trading strategy \( X' \) of the insider,

\[
E_0[\bar{p}(X, P)|\bar{v}, \bar{s}, \bar{\epsilon}] \geq E_0[\bar{p}(X', P)|\bar{v}, \bar{s}, \bar{\epsilon}] .
\]

For any alternate trading strategy \( Y' \) of the outsider,

\[
E_0[\bar{p}(Y, P)|\bar{s}, \bar{\epsilon}] \geq E_0[\bar{p}(Y', P)|\bar{s}, \bar{\epsilon}] .
\]

2. **Market efficiency:** \( P(\bar{\epsilon}, \bar{w}) = E_M(\bar{v}|\bar{\epsilon}, \bar{w}) \).

3. The unique linear equilibrium

The concept of linear equilibrium is similar to the that of Liu and Zhang (2011). And the linear equilibrium satisfying the following:

**Theorem 3.1.** There exists a unique linear Nash equilibrium, in which \( X, Y, P \) are linear functions, with the constants \( a, b, c, \alpha, \beta, \theta, \gamma, \delta, \eta \) and \( \lambda \) such that\(^3\)

\[
x = X(\bar{v}, \bar{s}, \bar{\epsilon}) = a + \alpha \bar{\epsilon} + \beta (\bar{v} - \mu_0) + \theta \bar{u},
\]

\[
y = Y(\bar{s}, \bar{\epsilon}) = b + \gamma \bar{s} + \delta \bar{\epsilon},
\]

\[
p = P(\bar{\epsilon}, \bar{w}) = c + \mu_0 + \eta \bar{\epsilon} + \lambda \bar{w},
\]

with

\[
a = b = c = 0,
\]

\[
\alpha = -\gamma = 1 \frac{1}{2 \lambda},
\]

\[
\beta = 1 \frac{2}{2 \lambda},
\]

\[
\theta = -\frac{k}{2(4-k)\lambda},
\]

\[
\gamma = k \frac{4-k}{(4-k)\lambda},
\]

\[
\delta = 0,
\]

\[
\eta = 1,
\]

\[
\bar{w} = \bar{x} + \bar{y} + \bar{u}.
\]

**Proof.** See the appendix.

Since the insider’s information is \((\bar{v}, \bar{s}, \bar{\epsilon})\), i.e. \((\bar{\epsilon}, \bar{s}, \bar{\epsilon})\), the equilibrium trading strategy of the insider can be rewritten as \( x = (\alpha + \beta) \bar{\epsilon} + \beta \)

\((\text{when } M = 1)\).
\((\hat{s} - p_0) + (\beta + \theta)\hat{s}\). Eqs. (3.6), (3.7), (3.8) and (3.9) imply that \(\beta > 0\), \(\alpha + \beta = \delta = 0\) and \(\theta + \beta = \frac{\gamma}{2} - \frac{\xi}{2\eta} > \gamma > 0\). These mean that the insider puts a positive weight on private information and puts a bigger weight on the shared information \(\hat{s}\) than the overconfident outsider does. This is different from the result of Liu and Zhang (2011), which finds that the insider and the outsider put the same weight on the shared information. In other words, the overconfident outsider trades less aggressively on his private information than the rational trader does. Eq. (3.11) shows that the risk-neutral market makers use the public information correctly while, the insider and the outsider all do not consider the public information when formulating their trading strategies.

As to the overconfident effects on the pricing rule and trading strategies, we find the following result:

**Proposition 3.1.** From Eqs. (3.5), (3.6), (3.7), (3.8), (3.10), we know that

\[
\frac{\partial \beta}{\partial k} > 0, \quad \frac{\partial (\beta + \theta)}{\partial k} < 0, \quad \frac{\partial \gamma}{\partial k} > 0, \quad \frac{\partial \delta}{\partial k} < 0.
\]

(3.12)

**Proof.** See the appendix.

The proposition means that a more confident outsider trades less aggressively on his shared information while, the insider trades less aggressively on his private information and puts a bigger weight on his shared information when he faces a more confident outsider. Intuitively, the rational insider knows that he can get more profits from shared information when the outsider predicts it incorrectly.

As in Kyle (1985), the depth of the market which is measured by \(\lambda\) is the order flow necessary to induce the price to rise or fall by one unit. \(\frac{\partial \lambda}{\partial k} < 0\) means that a more confident outsider leads to a smaller market depth.

To obtain a measure of the informativeness of price, we define

\[
\Sigma = \text{var} (\bar{Y} | \bar{P}),
\]

which is a measure of the residual information after the information is incorporated into the price. Thus

\[
\Sigma = \sigma^2_\nu = \left[ \frac{t_\nu}{4-k} + \frac{1}{2} (1-t_e-t_s) \right] \sigma^2_\nu.
\]

Let \(I(t_\nu, t_e, k) = \text{var} (\bar{Y}) - \Sigma\) denote how much information has incorporated into the equilibrium price.

**Proposition 3.2.** In the equilibrium,

\[
I(t_\nu, t_e, k) = \left[ \frac{t_\nu}{4-k} + \frac{1}{2} (1-t_e-t_s) \right] \sigma^2_\nu,
\]

and

\[
\frac{\partial I(t_\nu, t_e, k)}{\partial k} > 0.
\]

**Proof.** See the appendix.

From the above proposition, we know that \(I(t_\nu, t_e, k)\) is an increasing function of \(k\), which means that a more overconfident outsider leads to a less efficient market. However, \(I(t_\nu, t_e, k) \geq I(t_\nu, 0, k)\) means that the existence of public information leads to more informativeness of the price.

Next we consider the insider’s and outsider’s equilibrium profits, and the noise trader’s expected loss.

**Proposition 3.3.** In the equilibrium, the expected profit conditional on his information of the insider is

\[
E_l [\bar{Y} | \bar{v} = \bar{v}, \bar{e} = \bar{e}, \bar{s} = \bar{s}] = \frac{\sigma_\nu^2}{4\lambda \sigma_\nu \sqrt{\frac{4-2k}{(4-k)^2} t_\nu + \frac{1}{4} (1-t_e-t_s)}},
\]

the expected profit conditional on his information of the outsider is

\[
E_o [\bar{Y} | \bar{v} = \bar{v}, \bar{e} = \bar{e}, \bar{s} = \bar{s}] = \frac{k^2 \sigma_\nu^2}{(4-k)^2 \sigma_\nu \sqrt{\frac{4-2k}{(4-k)^2} t_\nu + \frac{1}{4} (1-t_e-t_s)}} s^2.
\]

Their ex ante expected profits are respectively

\[
E_l [\bar{L}] = \frac{\sigma_\nu^2 \sigma_\nu \left[ \frac{4-2k}{(4-k)^2} t_\nu + \frac{1}{4} (1-t_e-t_s) \right]}{4\lambda \sigma_\nu \sqrt{\frac{4-2k}{(4-k)^2} t_\nu + \frac{1}{4} (1-t_e-t_s)}},
\]

and

\[
E_o [\bar{L}] = \frac{k^2 \sigma_\nu^2 \sigma_\nu t_\nu}{(4-k)^2 \sigma_\nu \sqrt{\frac{4-2k}{(4-k)^2} t_\nu + \frac{1}{4} (1-t_e-t_s)}}.
\]

The ex ante expected loss of noise traders is:

\[
E [\bar{L}] = \lambda \sigma_\nu^2 = \sigma_\nu^2 \left[ \frac{4-2k}{(4-k)^2} t_\nu + \frac{1}{4} (1-t_e-t_s) \right].
\]

**Proof.** See the appendix.

**Corollary 3.1.** From the Proposition 3.3, we know that

\[
\frac{\partial E_l [\bar{Y}]}{\partial k} < 0, \quad \frac{\partial E_o [\bar{Y}]}{\partial k} > 0, \quad \frac{\partial E [\bar{L}]}{\partial k} < 0.
\]

**Proof.** See the appendix.

The corollary implies that insider’s profit and the noise traders’ loss are all decreasing functions of \(k\). This means that a more overconfident outsider leads to a more profit of insider and a more loss of the noise traders. The profit of the outsider is an increasing function of \(k\). And this means that a more overconfident outsider gets less profit.

4. Conclusion

We investigate Kyle’s (1985) extended model with the setting of shared information, public information and heterogeneous prior beliefs. We find that the more overconfident the insider is the more aggressively the insider trades on the shared information and the less aggressively on the private information while, a more confident outsider trades less aggressively on his information (the shared information). Also, the overconfidence of the outsider leads to an increased insider’s profits, an increased loss of noise trader, and a less efficient and less stable market. Moreover, a more overconfident outsider can profit less in the trading.
Appendix A. Proofs

Now we state a well known regression result that will be used later.

**Lemma 4.1.** Let $X_1$ and $X_2$ have joint normal distribution, $(X_1, X_2) \sim N(\mu, \Sigma)$ with $\mu = \left(\mu_1, \mu_2\right)$, $\Sigma = \left(\Sigma_{11}, \Sigma_{12}; \Sigma_{21}, \Sigma_{22}\right)$. Then the random variable $X_1$ conditional on $X_2$ has a normal distribution, and

$$E[X_1 | X_2] = \mu_1 + \Sigma_{12} \Sigma_{22}^{-1} (X_2 - \mu_2), \text{Var}(X_1 | X_2) = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}. $$

**Proof of theorem 3.1.** We conjecture that the linear equilibrium is given by

$$\bar{x} = X(\bar{v}, \bar{s}, \bar{\xi}) = a + \alpha \bar{e} + \beta (\bar{v} - p_0) + \delta \bar{e}, \quad \bar{y} = Y(\bar{s}, \bar{\xi}) = b + \gamma \bar{s} + \delta \bar{e}, \quad \bar{p} = P(\bar{e}, \bar{\xi}) = c + p_0 + \eta \bar{e} + \lambda \bar{w},$$

where the parameters $a, b, c, \alpha, \beta, \theta, \gamma, \delta, \eta$ and $\lambda$ are constants that need to be determined. We will identify the parameters and verify the conjecture.

Since $\bar{x}$ is a measurable function of $\bar{v}, \bar{s}$ and $\bar{\xi}$, the insider’s expected profit conditional on his information is

$$E_{\bar{\xi}} E[\bar{v} | \bar{v}, \bar{x}, \bar{s}, \bar{\xi}] = E_{\bar{\xi}} [\bar{v} | \bar{v} - c - \frac{c - \lambda b}{2\lambda} (\bar{v} - p_0) - \frac{\gamma s}{2} (\eta + \lambda d) - \frac{\eta + \lambda d}{2\lambda}] = E_{\bar{\xi}} E[\bar{v} | \bar{v} - c - \frac{c - \lambda b}{2\lambda} (\bar{v} - p_0) - \frac{\gamma s}{2} (\eta + \lambda d) - \frac{\eta + \lambda d}{2\lambda}]$$

By the first order condition, we have

$$\bar{x} = -\frac{c - \lambda b}{2\lambda} (\bar{v} - p_0) - \frac{\gamma s}{2} (\eta + \lambda d) - \frac{\eta + \lambda d}{2\lambda}.$$ (A5)

And by the second order condition, $\lambda > 0$. Comparing Eqs. (A5) and (A1), we have

$$\begin{cases} a = -\frac{c - \lambda b}{2\lambda}, \\ \alpha = \frac{-\gamma s}{2} (\eta + \lambda d) - \frac{\eta + \lambda d}{2\lambda}, \\ \beta = \frac{1}{2\lambda}, \\ \theta = \frac{-\gamma}{2}. \end{cases}$$ (A6)

Since $\bar{y}$ is a measurable function of $\bar{s}$ and $\bar{\xi}$, the outsider’s expected profit conditional on his information is

$$E_{\bar{\xi}} E[\bar{v} | \bar{v}, \bar{s}, \bar{\xi}] = E_{\bar{\xi}} [\bar{v} | \bar{v} - c - \frac{c - \lambda b}{2\lambda} (\bar{v} - p_0) - \frac{\gamma s}{2} (\eta + \lambda d) - \frac{\eta + \lambda d}{2\lambda}] = E_{\bar{\xi}} E[\bar{v} | \bar{v} - c - \frac{c - \lambda b}{2\lambda} (\bar{v} - p_0) - \frac{\gamma s}{2} (\eta + \lambda d) - \frac{\eta + \lambda d}{2\lambda}]$$

By the first order condition, we have

$$\bar{y} = \frac{1}{2\lambda} [\bar{v} - c - \lambda a - (\alpha + \eta) \bar{e}] + (1 - \lambda \beta) \bar{e} - \lambda \theta \bar{d} + (1 - \lambda \beta) \bar{k}.$$ (A8)

Comparing Eqs. (A8) and (A2), we have

$$\begin{cases} b = -\frac{c - \lambda a}{\lambda}, \\ \gamma = (1 - \lambda \beta - \lambda \theta) \frac{k}{\lambda}, \\ \delta = 1 - \lambda \beta - \lambda \gamma - \eta \frac{k}{\lambda}, \end{cases}$$ (A9)

Now by the semi-strong efficient condition of the market, we have

$$P(\tilde{w}, \tilde{e}) = E_M [\bar{v} | \bar{v} + \bar{y} + \tilde{u}, \bar{e}] = E_M [\bar{v} | \bar{a} + (\alpha + \beta + \delta) \bar{e} + (\beta + \theta + \gamma) \bar{s} + \beta (\bar{v} - p_0) + \bar{u}, \bar{e}]$$

Therefore

$$\begin{pmatrix} \bar{v} + \bar{y} + \tilde{u} \\ \bar{a} + \bar{b} + (\alpha + \beta + \delta) \bar{e} + (\beta + \theta + \gamma) \bar{s} + \beta (\bar{v} - p_0) + \bar{u} \end{pmatrix} \sim N \left( \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \right).$$ (A11)

where

$$\begin{align*} \Sigma_{11} &= \sigma_v, \\ \Sigma_{12} &= \Sigma_{21} = (\alpha + \beta + \delta) \sigma_v^2, \\ \Sigma_{22} &= (\alpha + \beta + \delta)^2 \sigma_v^2 + (\beta + \theta + \gamma)^2 \sigma_v^2 + \beta (1 - \theta + \theta) \sigma_v^2, \\ \Sigma_{32} &= (\alpha + \beta + \delta) \sigma_v^2, \\ \Sigma_{33} &= \sigma_v^2. \end{align*}$$

By Lemma 4.1, we have

$$E_M (\bar{v} | \bar{v} + \bar{y} + \tilde{u}, \bar{e}) = p_0 + (\Sigma_{12}, \Sigma_{23}) \cdot \left( \begin{pmatrix} \Sigma_{12} & \Sigma_{23} \end{pmatrix} \right)^{-1} \left( \begin{pmatrix} \bar{v} + \bar{y} + \tilde{u} \end{pmatrix} \right).$$ (A12)

let

$$D = \left( \begin{pmatrix} \Sigma_{12} & \Sigma_{23} \end{pmatrix} \right).$$ (A13)

The determinant of $D$ is

$$|D| = \sigma_v^2 \left( (\beta + \theta + \gamma)^2 \sigma_v^2 + 3 (1 - \theta + \theta) \sigma_v^2 + \sigma_v^2 \right).$$ (A14)

Now Eq. (A12) becomes

$$E_M (\bar{v} | \bar{v} + \bar{y} + \tilde{u}, \bar{e}) = p_0 + \frac{1}{|D|^2} \left( \begin{pmatrix} \Sigma_{12} & \Sigma_{23} \end{pmatrix} \right) \left( (\beta + \theta + \gamma)^2 \sigma_v^2 + 3 (1 - \theta + \theta) \sigma_v^2 + \sigma_v^2 \right) \tilde{w}$$

$$+ \frac{1}{|D|^2} \left( (\alpha + \beta + \delta) \sigma_v^2 \left( (\beta + \theta + \gamma)^2 \sigma_v^2 + 3 (1 - \theta + \theta) \sigma_v^2 + \sigma_v^2 \right) \right) \tilde{p}.$$ (A15)
So we get
\[ \begin{align*}
\lambda &= \left[ \frac{1}{\xi(t_s) - t_e} \right] \frac{\beta + \gamma t_s}{\xi(t_s) - t_e} + \beta(1 - t_e - t_s) \eta^2 \\
\eta &= \left[ \frac{1}{\xi(t_s) - t_e} \right] \left\{ - (\alpha + \beta + \delta) t_s \eta^2 + (\beta + \gamma) t_s \eta^2 + \beta(1 - t_e - t_s) \eta^2 \right. \\
&\quad \left. + \eta + (\beta + \gamma)^2 t_s \eta^2 + \beta^2 (1 - t_e - t_s) \eta^2 \right\}.
\end{align*} \]  
(A16)

Plug \( \epsilon = 0 \) into Eqs. (A16) and (A9), and we immediately have
\( a = b = 0 \), from Eqs. (A6) and (A9) we also have
\( \beta = \frac{1}{2k} \), \( \gamma = \frac{k}{(4-k)\lambda} \), \( \theta = -\frac{k}{2(4-k)\lambda} \).
(A17)

also from Eqs. (A6) and (A9), we have
\( \delta = \frac{1 - \eta}{3\lambda} \), \( \alpha = -\frac{1 + 2\eta}{6\lambda} \).
(A18)

Consequently,
\[ \beta + \theta + \gamma = \frac{2}{(4-k)\lambda}, \alpha + \beta + \delta = \frac{2 - 2\eta}{3\lambda}, \lambda(\alpha + \beta + \delta) + \eta = \frac{2 + 3\eta}{3}. \]

Substituting Eqs. (A17) and (A19) into Eq. (A16), and taking into account the second order condition of the insider's optimization problem, we get
\[ \lambda = \frac{\sigma_y}{\sigma_u} \frac{4 - 2k}{(4-k)^2} t_s + \frac{1}{4(1-t_e-t_s)}. \]
(A20)

Plug Eqs. (A17) and (A19), into Eq. (A16), we get
\[ \eta = 1, \]
(A21)
then we got the equilibrium.

Proof of Proposition 3.1. From the expression of \( \lambda, \gamma \), it is easy to know that \( \frac{\partial \eta}{\partial \epsilon} > 0 \). Since
\[ \frac{\partial \eta}{\partial \epsilon} = \frac{\sigma_u}{\sigma_y} \frac{4 - 2k}{(4-k)^2} t_s + \frac{1}{4(1-t_e-t_s)}. \]

Proof of Proposition 3.2.
\[ \Sigma = \text{var}(\overline{v}_p) = \text{var}(v_p) + \lambda \eta \gamma \text{var}(\overline{v}_p) + \theta \gamma (t_s - t_e) + \beta \gamma \lambda \text{var}(\overline{v}_p) + \beta \gamma \lambda \text{var}(\overline{v}_p) + \theta \gamma \lambda \text{var}(\overline{v}_p) + \beta \gamma \lambda \text{var}(\overline{v}_p) + \beta \gamma \lambda \text{var}(\overline{v}_p). \]

Then, it is easy to have
\[ I(t_s, t_e, k) = \sigma_e^2 - \text{var}(\overline{v}_p) = \left[ t_s + \frac{2}{4-k} t_s + \frac{1}{4} (1-t_e-t_s) \right] \sigma_e^2, \]
and

$$\frac{\partial l(t_e, t_c, k)}{\partial k} = \frac{\partial}{\partial k} \left[ \epsilon + \frac{2}{4-k} t_e + \frac{1}{2} (1-t_e-t_c) \right] \sigma_v^2$$

$$= \sigma_v^2 - \frac{2(1-k)}{(4-k)^2} t_e = 2t_e \sigma_v^2 > 0$$

(A26)

Obviously, $l(t_e, t_c) \geq l(0, t_c)$, and $l(t_e, t_c) \geq l(t_e, 0)$.

**Proof of Proposition 3.3.** From Theorem 3.1 and Eq. (A4), the condition expected profit of the insider is

$$E_{\tilde{t}}[\tilde{v} | \tilde{v} = \tilde{v}] = \frac{\partial}{\partial k} \left[ (1-t_e-t_c) \tilde{v} \right]$$

$$= \frac{\partial}{\partial k} \left[ \epsilon \tilde{v} + \frac{2}{4-k} \tilde{v} + \frac{1}{2} (1-t_e-t_c) \tilde{v} \right] \sigma_v^2$$

$$= \frac{\partial}{\partial k} \left[ \epsilon \tilde{v} + \frac{2}{4-k} \tilde{v} + \frac{1}{2} (1-t_e-t_c) \tilde{v} \right] \sigma_v^2$$

Substituting Eq. (A19), the equation above is

$$E_{\tilde{t}}[\tilde{v} | \tilde{v} = \tilde{v}] = \frac{1}{4k} \left[ \epsilon \tilde{v} + \frac{2}{4-k} \tilde{v} + \frac{1}{2} (1-t_e-t_c) \tilde{v} \right]$$

Hence

$$E_{\tilde{t}}[\tilde{v}] = \frac{\epsilon \tilde{v} + \frac{2}{4-k} \tilde{v} + \frac{1}{2} (1-t_e-t_c) \tilde{v}}{4k}$$

From Theorem 3.1 and Eq. (A7), the conditional expected profit of the outsider is

$$E_{\tilde{t}}[\tilde{v}] = \frac{\epsilon \tilde{v} + \frac{2}{4-k} \tilde{v} + \frac{1}{2} (1-t_e-t_c) \tilde{v}}{4k}$$

Therefore the outsider’s ex-ante expected profit is

$$E_{\tilde{t}}[\tilde{v}] = \frac{k^2 \sigma_v^2 \sigma_t t_e}{(4-k)^2 t_e + \frac{1}{4} (1-t_e-t_c)}.$$

The loss of noise traders is

$$L = (\tilde{p} - \tilde{v}) \tilde{v} = [\tilde{p} + \gamma \tilde{v}] - \lambda \tilde{v} [\tilde{v} - \tilde{p}] + \tilde{v} + \gamma \tilde{v} - \tilde{v} [\tilde{v} - \tilde{p}] + \tilde{v}$$

Their expected loss is therefore

$$E[L] = \lambda \sigma_v^2 = \sigma_v^2 \sqrt{\frac{4-2k}{(4-k)^2} t_e + \frac{1}{4} (1-t_e-t_c)}.$$

**Proof of Corollary 3.1.**

$$\frac{\partial}{\partial k} \left[ \sigma_t \sigma_v \sqrt{\frac{4-2k}{(4-k)^2} t_e + \frac{1}{4} (1-t_e-t_c)} \right]$$

$$= \frac{\sigma_t \sigma_v}{4k} \left[ \frac{4-2k}{(4-k)^2} t_e + \frac{1}{4} (1-t_e-t_c) \right]$$

and

$$\frac{\partial}{\partial k} \left[ \frac{k^2 \sigma_v^2 \sigma_t t_e}{(4-k)^2 t_e + \frac{1}{4} (1-t_e-t_c)} \right]$$

$$= \frac{\sigma_t \sigma_v}{4k} \left[ \frac{4-2k}{(4-k)^2} t_e + \frac{1}{4} (1-t_e-t_c) \right]$$

and

$$\frac{\partial}{\partial k} \left[ \frac{k^2 \sigma_v^2 \sigma_t t_e}{(4-k)^2 t_e + \frac{1}{4} (1-t_e-t_c)} \right]$$

$$= \frac{\sigma_t \sigma_v}{4k} \left[ \frac{4-2k}{(4-k)^2} t_e + \frac{1}{4} (1-t_e-t_c) \right]$$

References


