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Revised lattice Boltzmann model for traffic flow with equilibrium traffic pressure

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A revised lattice Boltzmann model concerning the equilibrium traffic pressure is proposed in this study to tackle the phase transition phenomena of traffic flow system. The traditional lattice Boltzmann model has limitation to investigate the complex traffic phase transitions due to its difficulty for modeling the equilibrium velocity distribution. Concerning this drawback, the equilibrium traffic pressure is taken into account to derive the equilibrium velocity distribution in the revised lattice Boltzmann model. In the proposed model, a three-dimensional velocity-space is assumed to determine the equilibrium velocity distribution functions and an alternative, new derivative approach is introduced to deduct the macroscopic equations with the first-order accuracy level from the lattice Boltzmann model. Based on the linear stability theory, the stability conditions of the corresponding macroscopic equations can be obtained. The outputs indicate that the stability curve is divided into three regions, i.e., the stable region, the neutral stability region, and the unstable region. In the stable region, small disturbance appears in the initial uniform flow and will vanish after long term evolution, while in the unstable region, the disturbance will be enlarged and finally leads to the traffic system entering the congested state. In the neutral stability region, small disturbance does not vanish with time and maintains its amplitude in the traffic system. Conclusively, the stability of traffic system is found to
be enhanced as the equilibrium traffic pressure increases. Finally, the numerical outputs of the proposed model are found to be consistent with the recognized, theoretical results.

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1. Introduction

Traffic flow system presents significant impact on social economic and urban environment [1,2]. In the past decades, many traffic flow models have been developed to reproduce all kinds of traffic jam phenomena, but no perfect model achieved yet [3].

The system of traffic is open and giant complex, the modeling thought based on lattices present certain advantages for its convenience, such as lattice hydrodynamic models [4,5] in macroscopic level and cellular automaton models in microscopic level [6,7]. For example, introducing revolutionary games [8,9] to cellular automaton Biham–Middleton–Levine (BML) model [7] results in a premature occurrence of traffic jams and thus unnecessarily burdens the transportation system.

In this paper, the study focuses on the lattice Boltzmann traffic model, which is one of kinetic-type models in mesoscopic level. The drawbacks of the kinetic-type models are identified [10]. Despite the presumed assumptions such as vehicle chaos assumption, there are still number of unknown parameters and empirical relations need to be estimated via observations. Furthermore, the mathematically integral–differential-type equations are usually difficult to be solved with either numerical methods or analytical methods.

Meng et al. [10] introduced the Bhatnagar–Gross–Krook-type approximation to Boltzmann equation and proposed a lattice Boltzmann model to study traffic phenomena. The model can simulate traffic flow efficiently and provide certain meaningful results explaining the physical phenomenon. However, seeking relevant equilibrium velocity distribution function is still a challenging task as no momentum and energy conservations exists in traffic flow. Hence, in this study, a revised lattice Boltzmann model, considering the traffic pressure and the velocity–density relation, is proposed.

2. Model development

2.1. Lattice Boltzmann model

The governing equation of lattice Boltzmann model for traffic flow, proposed by Meng et al. [10], is given as below:

\[
\frac{f_i(x + v_i \delta t, t + \delta t) - f_i(x, t)}{\delta t} = \omega \left[ f_i^{eq}(x, t) - f_i(x, t) \right]
\]

where \(f_i(x, t)\) is the phase-space density, it denotes the distribution of vehicles moving with velocity \(v_i\) at location \(x\) and time \(t\), \(f_i^{eq}\) is the equilibrium phase-space density, namely, the distribution of vehicles under the local equilibrium state, which describes the local equilibrium state resulting from the competition between two opposite sides in a local region, i.e., the drivers’ effort toward their desired speeds and the interactions with other vehicles, namely, the equilibrium phase-space density. \(\omega\) the dimensionless relaxation factor, \(\omega = \delta t / \tau\), \(\delta t\) a small time clearance, and \(\tau\) the relaxation time. Similar to fluid and granular media, it is known that the phase-space density also depends on their gradients, which is a consequence of the finite adaptation time required to reach local equilibrium.

In hydrodynamics, one can derive the appropriate equilibrium distribution function \(f_i^{eq}\) according to the discretization method of phase space. But such function usually contains certain unknown coefficients, which need to be determined based on the conservation laws of fluid mechanics. However, there is no such conservation principle in traffic flow system, which causes dilemma in most traffic studies. Fortunately, it is possible to determine \(f_i^{eq}\) through the empirical observation under certain assumptions, similar to the velocity–density relation in the Lighthill–Whitham–Richards macroscopic model [11] for closing the mathematical formation.

By empirical and/or field measurements, the velocity distributions of traffic flow with a small fraction of trucks follow the Gaussian distribution [12,13]. However, if the sampling intervals are too large, the bimodal distribution may be observed [14,15], which reflect the transition from free to congested traffic. Helbing [3] summarized some empirical data from the literature [12–17] and stated that vehicle velocities are more or less Gaussian distributed. Hence, establishing proper velocity distribution function plays a key role in traffic flow modeling. A specific equilibrium phase-space density function [5] is given as follows,

\[
f^{eq} = \rho \exp \left\{ -\left( v - V_{eq} \right)^2 / (2\theta_{eq}) \right\} / (2\pi \theta_{eq})^{1/2}
\]

where \(V_{eq}\) is the equilibrium average velocity, \(\theta_{eq}\) is the equilibrium velocity variance, and \(\theta_{eq} = A(\rho) V_{eq}^2\), \(A(\rho)\) is the density-dependent variance prefactor, its empirical form proposed by Shvetsov and Helbing [18] is complicated. It is difficult to extend these functions to more complex traffic cases or numerical simulation.
Meng et al. [10] also suggested one special form of equilibrium phase-space density,

$$f_{i}^{eq} = \frac{u_{i}^{2} \exp \left(-\frac{v_{i}^{2} \rho^{2}}{1-\rho^{2}}\right) \rho_{n}}{1 + \sum_{i=1}^{6} u_{i}^{2} \exp \left(-\frac{v_{i}^{2} \rho^{2}}{1-\rho^{2}}\right)},$$

(3)

$$f_{0}^{eq} = \frac{\rho_{n}}{1 + \sum_{i=1}^{6} u_{i}^{2} \exp \left(-\frac{v_{i}^{2} \rho^{2}}{1-\rho^{2}}\right)},$$

(4)

$$\rho' = \frac{\sum_{i=0}^{5} \rho_{n+i}}{6}.$$  

(5)

Assuming that a single-lane road can be divided into a one dimensional array of $N$ sites, $\rho_{n}$ denote the density at the $n$th site. $\rho'$ is the average density on the six sites including the $n$th site and five sites ahead. For the velocity coordinate, each vehicle move with an integer velocity $v_{i} \in \{0, \ldots, v_{\text{max}}\}$ and $v_{\text{max}}$ is set to be 5, which similar to the work of cellular automaton model [6,10]. Combining the equilibrium phase-space density function (Eqs. (3)–(5)) and the lattice Boltzmann equation (1), phase transition from free flow to congested traffic can be captured by simulation.

However, in order to investigate complex traffic states, there is no approach on establishing another reasonable equilibrium phase-space density function.

Recently, considering above achievements, we proposed one approach [19] which can derive the equilibrium phase-space density functions based on the relations between the mesoscopic quantities (such as the equilibrium phase-space density) and the macroscopic quantities (such as the density or flow).

Namely, Expanding the exponent function into power series in Eq. (2) and maintaining the top two items, a kind of equilibrium phase-space density is established,

$$f^{eq} = \rho \left[ 1 - \eta (v - V_{eq})^{2} \right]$$

(6)

where $\eta$ is the undetermined coefficient. According to the lattice Boltzmann method, considering the velocity–space dimensions, Eq. (6) can be rewritten by,

$$f_{i}^{eq} = \rho \left[ 1 - \eta (v_{i} - V_{eq})^{2} \right].$$

(7)

Substituting Eq. (7) into the following relations

$$\sum_{i=0}^{v_{\text{max}}} f_{i}^{eq} = \rho$$

(8)

$$\sum_{i=0}^{v_{\text{max}}} v_{i} f_{i}^{eq} = \rho V_{eq}.$$  

(9)

Solving Eqs. (7)–(9), only two coefficients can be determined, in consequence, a kind of 2-dimensional equilibrium phase-space density function is acquired,

$$f_{0}^{eq} = \rho \left( 1 - V_{eq} \right)$$

(10)

$$f_{1}^{eq} = \rho V_{eq}.$$  

(11)

where $v_{\text{max}} = 1$. Applying Eqs. (10)–(11) to the lattice Boltzmann equation (1), it can not only reproduce the phase transition from free flow to congestion, but also study another complex traffic cases, such as combination road traffic between the elevated road and the plane road. For the detail see Ref. [19]. However, the model in Ref. [19] does not consider traffic pressure.

In the following paragraphs, we would derive the macroscopic expression of traffic pressure, consider the relation between the macroscopic traffic pressure and the equilibrium phase-space density functions, and derive a new kind of 3-dimensional equilibrium phase-space density function. Applying the new equilibrium phase-space density to the lattice Boltzmann equation (1) and establish the new lattice Boltzmann traffic model with traffic pressure. The study focuses on the impact of traffic pressure on the traffic flow.

Concerning above points, we aim to establish the equilibrium velocity distribution based on macroscopic variables, e.g., the fitted empirical velocity–density relation. Under the equilibrium traffic state, the following basic relations must be complied:

$$\sum_{i=0}^{v_{\text{max}}} f_{i}^{eq} = \rho$$

(12)
where $P_{eq}$ is the so-called equilibrium traffic pressure, introduced by Meng et al. [10]. Based on Eqs. (12)–(14), the equilibrium phase-space densities $f_i^{eq}$ can be derived by using the macroscopic fitted empirical velocity–density relation $V_{eq}(\rho)$ and the traffic pressure $P_{eq}$. Based on the modeling of lattice Boltzmann method of traffic flow [10], the value of each $v_i$ is integer. For three-dimensional velocity space, the maximum velocity value is set to be $v_{max} = 2 \ (i = 2)$, which denotes the maximum velocity state; while $v_i = 0 \ (i = 0)$ denotes the stopped state and $v_i = 1 \ (i = 1)$ the medium velocity state. Meanwhile, the resultant, corresponding three equations can be rewritten as follows:

\begin{align}
\sum_{i=0}^{v_{max}} v_i f_i^{eq} &= \rho V_{eq} \\
\sum_{i=0}^{v_{max}} v_i^2 f_i^{eq} &= \rho V_{eq}^2 + P_{eq}
\end{align}

where $f_i^{eq}$ is the so-called equilibrium traffic pressure, introduced by Meng et al. [10]. Based on Eqs. (12)–(14), the equilibrium phase-space densities $f_i^{eq}$ can be derived by using the macroscopic fitted empirical velocity–density relation $V_{eq}(\rho)$ and the traffic pressure $P_{eq}$. Based on the modeling of lattice Boltzmann method of traffic flow [10], the value of each $v_i$ is integer. For three-dimensional velocity space, the maximum velocity value is set to be $v_{max} = 2 \ (i = 2)$, which denotes the maximum velocity state; while $v_i = 0 \ (i = 0)$ denotes the stopped state and $v_i = 1 \ (i = 1)$ the medium velocity state.

In realistic traffic situation, the following constraints must be satisfied:

\begin{align}
0 &\leq f_0^{eq} \leq \rho_{jam} \\
0 &\leq f_1^{eq} \leq \rho_{jam} \\
0 &\leq f_2^{eq} \leq \rho_{jam}
\end{align}

where $\rho_{jam}$ is the jam density. Inserting Eqs. (18)–(20) into formulae (21)–(23), the constraints for the equilibrium traffic pressure $P_{eq}$ can be obtained as below:

\begin{align}
3\rho V_{eq} - \rho V_{eq}^2 - 2\rho &\leq P_{eq} \leq 3\rho V_{eq} - \rho V_{eq}^2 - 2\rho + 2\rho_{jam} \\
2\rho V_{eq} - \rho V_{eq}^2 - \rho_{jam} &\leq P_{eq} \leq 2\rho V_{eq} - \rho V_{eq}^2 \\
\rho V_{eq} - \rho V_{eq}^2 &\leq P_{eq} \leq \rho V_{eq} - \rho V_{eq}^2 + 2\rho_{jam}.
\end{align}

Physically, the equilibrium traffic pressure $P_{eq}$ must satisfy the above three inequalities simultaneously. That is, one should find the maximum of the left hand sides and the minimum of the right hand sides for inequalities (24)–(26). After calculating carefully, the equilibrium traffic pressure $P_{eq}$ must comply to following constraints:

\begin{align}
\rho V_{eq} - \rho V_{eq}^2 &\leq P_{eq} \leq 2\rho V_{eq} - \rho V_{eq}^2, \quad \text{for } V_{eq} \leq 1 \\
3\rho V_{eq} - \rho V_{eq}^2 - 2\rho &\leq P_{eq} \leq 2\rho V_{eq} - \rho V_{eq}^2, \quad \text{for } V_{eq} > 1.
\end{align}

It can be seen that the gas-kinetic model is sensitive to the selection of specific parameters like traffic pressure and velocity–density relation, the appropriate expression of traffic pressure must be handled carefully and demand further investigation.

2.2. Expressions of specific parameters

Here, we focus on exploring the proper expression for macroscopic velocity–density relation $V_{eq}$ and traffic pressure $P_{eq}$ under the equilibrium traffic state. The earliest version of fitted empirical linear velocity–density relation was proposed by Greenshields [20]. In past decades, more studies on velocity–density relations were reported [21–28], where field measurements and observations were mainly used. Meanwhile, some specific expressions of traffic pressure were also proposed [3,15,29–33]. However, based on our experiences, previous expressions for traffic pressure found in the literature
are not suitable to the constraints of inequalities (i.e., formulae (27a), (27b)). To solve such problem, we, hereby, propose an alternative expression for traffic pressure, complying to formulae (27a) and (27b), listed below:

\[ P_{eq} = (1 + \beta) \rho V_{eq} - \frac{1}{2} (1 + \beta) \rho V_{eq}^2 \]  

(28)

where \( \beta \), \( \beta \in [0, 1] \) is the stress level of traffic pressure corresponding to specific vehicle density \( \rho \) and equilibrium average velocity \( V_{eq} \).

The fitted empirical velocity–density relationship for \( V_{eq} \) is selected from Drake’s model [25], which has priority of less parameters and higher fitting degree with the measured data in expressway:

\[ V_{eq} = V_0 \exp \left[ -\frac{1}{2} \left( \frac{\rho}{\rho_0} \right)^2 \right] \]  

(29)

where \( \rho_0 \) denotes the critical density corresponding to the maximum flow, which can be obtained by field measurement and \( \rho \) the local density. Using Eqs. (28) and (29), the relationships between the equilibrium traffic pressure and the traffic density can be obtained and presented in Fig. 1 under different stress levels of \( \beta = 0, 0.2, 0.5, 0.8, 1.0 \) respectively.

From Fig. 1, it is interesting to notice that the equilibrium traffic pressure \( P_{eq} = 0 \) occurs at the density \( \rho = 0 \) and \( \rho = \rho_{jam} (\rho_{jam} = 1) \). The outputs shown in Fig. 1 indicate the advantage of our model, i.e., no vehicle accelerations appear in congested region \( (P_{eq} = 0 \ as \ \rho \rightarrow 1) \), which are not shown in Phillip’s work [33]. In addition, the peaks of traffic pressure rise step by step as the coefficient \( \beta \) increases, i.e., different traffic pressure peaks denote different traffic states as well as the corresponding equilibrium velocity distributions.

Based on above, the relation between the equilibrium velocity distribution \( \lambda_{i}^{eq} \) (i = 0, 1, 2) and the equilibrium phase-space density can be expressed as:

\[ f_{i}^{eq} = \rho \lambda_{i}^{eq}. \]  

(30)

The equilibrium velocity distribution can be further derived as:

\[ \lambda_{0}^{eq} = 1 + \frac{1}{4} (1 - \beta) V_{eq}^2 - \frac{1}{2} (2 - \beta) V_{eq} \]  

(31a)

\[ \lambda_{1}^{eq} = (1 - \beta) V_{eq} - \frac{1}{2} (1 - \beta) V_{eq}^2 \]  

(31b)

\[ \lambda_{2}^{eq} = \frac{1}{4} (1 - \beta) V_{eq}^2 + \frac{1}{2} \beta V_{eq}. \]  

(31c)

Fig. 2 presents curves of velocity distributions and phase-space density at different density levels according to Eqs. (29)–(31) under the condition of \( \beta = 0.5 \). From Fig. 2, some characters, in agreement with Meng’s work [10], can be drawn as follows:

(1) The lower the vehicles density, the higher the probability of the maximum velocity does, i.e., as \( \rho \rightarrow 0, f_{\nu_{max}}^{eq} \gg f_{\nu_{eq}}^{eq} \), but the phase-space density corresponding to the maximum probability should not beyond the local density, i.e., as \( f_{\nu_{max}}^{eq} \rightarrow \rho, \lambda_{\nu_{max}}^{eq} \rightarrow 1 \).

(2) The higher the vehicles density, the higher the probability of the zero-velocity is. When the density approaches the jam density, the stop probability is significantly greater than other probabilities, i.e., as \( \rho \rightarrow \rho_{jam}, f_{\nu_{0}}^{eq} \gg f_{\nu_{eq}}^{eq}, \) and the stop probability is close to the local density, i.e., as \( f_{\nu_{0}}^{eq} \rightarrow \rho, \lambda_{\nu_{0}}^{eq} \rightarrow 1 \).

Besides, situations different from previous studies are observed. First, an additional peak point on the velocity distribution \( \lambda_{1}^{eq} \) for \( \nu_l = 1 \) is observed and shown in Fig. 2. This implies that the maximum point also exists with medium velocity condition, which follows the normal distribution [3]. To further understand the situation with medium velocity, we explore the impact of stress level \( \beta \) on the velocity distribution \( \lambda_{1}^{eq} \) and shown in Fig. 3. It depicts the general decrease of the peaks with the increase of stress levels. Under the same density, all corresponding values of \( \lambda_{1}^{eq} \) reduce as the \( \beta \)-level increases and approach to zero when \( \beta = 1.0 \).

Fig. 4 represents the evolving patterns of velocity distributions \( \lambda_{1}^{eq} \) and the phase-space density profiles with different densities under the maximum stress level \( \beta = 1.0 \). The difference between Figs. 2 and 4 occurs on the curve \( \lambda_{1}^{eq} \), in which constant values, i.e., \( \lambda_{1}^{eq} = 0 \) are observed under all densities for \( \beta = 1.0 \). Such state implies that the maximum probability, occurring in other stress levels (\( \beta \neq 1.0 \)), disappears as \( \beta = 1.0 \). In contrast to Fig. 2, three peaks change to two peaks, which is similar to the bimodal distribution [14]. Consequently, under the high traffic pressure (\( \beta = 1.0 \)), the vehicle velocity tends to two extremes, i.e., either stopping or running.

2.3. Macroscopic dynamic characteristics

Traditionally, Taylor–Chapman–Enskog expansions [10,34–37] are used to solve Eq. (1) to validate the discretization method of phase space and to investigate the macroscopic dynamics of respective model used. Similar processes can be
The equilibrium traffic pressure \( P_{eq} \) with density variations \( \rho \) considering different stress levels of traffic pressure (\( \beta \)). The peaks of equilibrium traffic pressure rise step by step with traffic pressure increasing.

Fig. 1. The single-peak curves of the equilibrium traffic pressure \( P_{eq} \) with density variations \( \rho \) considering different stress levels of traffic pressure (\( \beta \)).

Repeatedly applied to deduce the formulae with first and second order accuracies, which are also same to Meng’s work. The so-called acceleration equation, i.e., the first order accuracy, is derived as:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V)}{\partial x} &= 0 \\
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} &= -\frac{1}{\rho} \left[ \frac{\partial P}{\partial x} \right] + \omega [V_{eq} - V].
\end{align*}
\]

In which the following relations are used,

\[ \rho V = \sum_i f_i v_i, \quad V_{eq} = \sum_i \left( v_i \frac{f_{eq}^i}{\rho} \right), \quad \langle v^2 \rangle = \frac{1}{\rho} \sum_i v_i^2 f_i \]

and the definition \([3,10]\) of traffic pressure as \( P = -\rho \{ v^2 \} - \rho V^2 \). The corresponding formula with second order accuracy level is listed below:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V_{eq})}{\partial x} &= \left( \frac{1}{\omega} - \frac{1}{2} \right) \frac{\partial}{\partial x} \left[ \frac{\partial P_{eq}}{\partial x} + \frac{\partial (\rho V_{eq}^2)}{\partial x} \right] - \left( V_{eq} + \rho \frac{\partial V_{eq}}{\partial \rho} \right) \frac{\partial (\rho V_{eq})}{\partial \rho}.
\end{align*}
\]
The equilibrium velocity distribution

\[ \lambda_{eq}^{0} = 0 \]
\[ \lambda_{eq}^{1} = 0.2 \]
\[ \lambda_{eq}^{2} = 0.5 \]
\[ \lambda_{eq}^{3} = 0.8 \]
\[ \lambda_{eq}^{4} = 1.0 \]

**Fig. 3.** The impact curves of stress level \( \beta \) on the medium velocity distribution \( \lambda_{eq}^{1} \). Under the same density, all corresponding values of \( \lambda_{eq}^{1} \) reduce as the \( \beta \)-level increases and approach to zero when \( \beta = 1.0 \) extremely.

\[ \Omega = P'_{eq}(\rho_e) - \left[ V'_{eq}(\rho_e) \right]^2 \rho_e^2 \]  

(34)

where \( \rho_e \) denotes the average vehicle density on the entire road, i.e., \( V'_{eq}(\rho_e) = dV_{eq}(\rho_e) / d\rho \), and \( P'_{eq}(\rho_e) = dP_{eq}(\rho_e) / d\rho \).

When \( \Omega > 0 \), the system is stable, i.e., homogeneous state, where \( \rho(x, t) = \rho_e \) and \( V(x, t) = V_{eq}(\rho_e) \). When \( \Omega < 0 \), the system becomes unstable.

**Fig. 4.** (a) The velocity distributions \( \lambda_{eq}^{i} \) curves or (b) the equilibrium phase-space density \( f_{eq}^{i} \) curves with density increasing are similar to the bimodal distribution at \( \beta = 1.0 \).

Moreover, we carry out the linear stability analysis to both first-order accuracy (i.e., Eqs. (32a), (32b)) and second-order accuracy cases (i.e., Eq. (33)), and compare the results with Kerner’s work [31] and Meng’s work [10] respectively. Same justification of system stability condition, under vehicle conservation and periodic boundary condition, can be obtained as below:

**Fig. 5** presents the stability curves \( \Omega \) vs. densities under different stress levels according to Eq. (34). From **Fig. 5**, all \( \Omega \)-curves can be divided into three sections, i.e., stable region (\( \Omega > 0 \)), neutral stable region (\( \Omega = 0 \)) and unstable one (\( \Omega < 0 \)). The positive \( \Omega \) region means that the traffic system is in stable state, i.e., the perturbation triggered at initial uniform flow would disappear as time extending. Whilst the negative \( \Omega \) range denotes the unstable state of the traffic system, i.e., the disturbance enlargement would occur as time expanding in case triggered initially and potential traffic congestion caused. For critical state, i.e., \( \Omega = 0 \), the system is in neutral stable and the added disturbance would remain its original strength. From **Fig. 5**, it can be seen that the stability increases as the stress level \( \beta \) increases in this research, which is not detected in previous studies.
3. Simulation and discussion

The numerical simulation is performed in following steps:

Step 1. Computing the local equilibrium velocity using Eq. (29). The local density \( \rho \) covers the current lattice and the two adjacent lattices in front, that is,

\[
\rho = \frac{1}{3} (\rho_n + \rho_{n-1} + \rho_{n-2}).
\]  

(35)

Step 2. Computing the equilibrium velocity distribution \( \lambda^\text{eq}_i \) via Eq. (28) and Eqs. (31a)–(31c);

Step 3. Solving the phase-space density \( f^\text{eq}_i \) in the current lattice using Eq. (30);

Step 4. Conducting the “collision” process based on the lattice Boltzmann equation (1);

Step 5. Controlling boundary condition: To avoid the occurrence of nonphysical density, it is necessary to take one site as a “virtual boundary” varying with time in the simulation [10,38]. Such setting guarantees that the following car will not cross-over the current, occupied site and move forward under priority justification in real case.

Step 6. Updating and repeating. Based on above steps, simulation is carried out to explore the dynamic behavior of traffic flow in single lane under periodic boundary condition. The total number of lattice is set at \( N = 1000 \) and the relaxation factor is \( \omega = 0.9 \) (as numerical stability requests \( 0 \leq \omega \leq 2 \) [39]). Initially, all lattice densities distribute randomly and each lattice velocity distribution is under the equilibrium state. Ten samples of evolution are performed, and each requests 10 000 time steps of evolution. The average vehicle flow is statistically averaged during the last 2000 time steps (\( T = 2000 \)) with the following formula:

\[
\langle q \rangle = \frac{1}{T} \sum_{t=1}^{T} \left( \frac{1}{N} \sum_{n=1}^{N} \left( \sum_{i=0}^{1} v_n f^t_{n,i} \right) \right) = \frac{1}{T} \sum_{t=1}^{T} \left( \frac{1}{N} \sum_{n=1}^{N} f^t_{n,1} \right) = \frac{1}{TN} \sum_{t=1}^{T} \sum_{n=1}^{N} f^t_{n,1}.
\]  

(36)

The fundamental diagram (i.e., average flow–density relation) of simulation process is shown in Fig. 6, where \( \rho_G \) is the global density of whole road and the stress level of equilibrium traffic pressure \( \beta = 1.0 \). It can be seen that our simulations agree well with Drake’s outputs [25]. The comparisons fully indicate the feasibility of introducing the traffic pressure expression to the lattice Boltzmann method.

The traffic phase transition is further investigated and presented visually in Fig. 7. Regarding such simulation, the relevant parameters are specified as: (a) initial uniform lattice density, (b) selecting 50 pairs of successive lattices randomly with minor density adjustment of \( \pm 0.05 \) between two adjacent lattices, (c) capturing the snapshot of lattice positions [500, 600] at 10 000 time steps of evolution in terms of \( \rho_n = \sum_{i=0}^{2} f_i, V_n = \frac{\rho_n}{\rho_0} = \frac{1}{\rho_0} \sum_{i=0}^{2} v_i f_i \) From Fig. 7, referring to Fig. 5, three sets of simulation results are presented corresponding to the cases of \( \rho_G = 0.1, 0.4, \) and 0.8. With \( \rho_G = 0.1 \), the system is in stable region, all small disturbances vanish with time evolution and the system maintain the free flow state at the constant, average velocity of 1.6. For \( \rho_G = 0.4 \), the unstable situation observed, the amplitudes of density and velocity fluctuate in wide range of 0.2–0.45 and 0.02–0.5 respectively, which indicate the enlargement of disturbance in the system and, stop-and-go traffic emerges. When \( \rho_G = 0.8 \), the majority of lattice maintains high vehicle densities, few small fluctuations of lattice density appear and keep the same levels of initial disturbance, i.e., the gap moving phenomena.

4. Conclusions

This study tackles the challenging task to construct the equilibrium velocity distribution and propose a revised lattice Boltzmann model considering the traffic pressure impact. The revised model provides an alternative approach to establish
the equilibrium velocity distribution of traffic flow system and also put forwards the expression of traffic pressure, which can be used and compared with the real traffic situations conveniently. A new, derivative process from mesoscopic lattice Boltzmann equation to macroscopic continuity and velocity equations, at the first-order accuracy level, is obtained. In terms of the proposed parameter, i.e., stress level $\beta$, a new justification method is created and can be applied to assess the stability of traffic system. The performance of the proposed model has been verified and in good agreement with the published results in the literature. Conclusively, the stability of traffic system is enhanced with the increase of equilibrium traffic pressure.

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