Research paper

On phase transformation behavior of porous Shape Memory Alloys

Bingfei Liu\textsuperscript{a}, Guansuo Dui\textsuperscript{a,}\textsuperscript{*}, Yuping Zhu\textsuperscript{b}

\textsuperscript{a}Institute of Mechanics, Beijing Jiaotong University, Beijing 100044, China
\textsuperscript{b}Institute of Mechanics and Engineering, Jiangsu University, Zhenjiang 212013, China

\textbf{A R T I C L E I N F O}

Article history:
Received 2 June 2011
Received in revised form 28 September 2011
Accepted 29 September 2011
Published online 8 October 2011

Keywords:
Porous Shape Memory Alloys
Phase transformation
Constitutive model

\textbf{A B S T R A C T}

This paper is concerned on the phase transformation mechanism of porous Shape Memory Alloys (SMAs). A unit-cell model is adopted to establish the constitutive relation for porous SMAs, the stress distributions, the phase distributions and the martensitic volume fractions for the model are then derived under both pure hydrostatic stress and uniaxial compression. Further, an example for the uniaxial response under compression for a porous Ni–Ti SMA material considering hydrostatic stress is supplied. Good agreement between the theoretical prediction of the proposed model and published experimental data is observed.

Crown Copyright © 2011 Published by Elsevier Ltd. All rights reserved.

1. Introduction

In the past two decades, the number of innovative applications for advanced materials has been rapidly increasing (Wang et al., 2009, 2010; Liu and Han, 2010). Recently, porous Shape Memory Alloys (SMAs) have been widely used in many engineering applications such as sensors and actuators (Teppei et al., 2005; Zhao et al., 2006), biomedical devices (Greiner et al., 2005), surgical implant materials as well as surgical instruments (Ayers et al., 2007; Ponsonnet et al., 2006), and micro-electro-mechanical systems (Starovetsky and Gotman, 2001) due to their interesting behaviors such as good biocompatibility, unique shape memory properties, mechanical properties, superior damping capability, excellent corrosion resistance and wear resistance.

Up to now, many porous Ni–Ti SMAs with different pore structures have been successfully produced by combustion synthesis with a selfpropagating wave (Whitney et al., 2008), metal injection molding (Krone et al., 2004), hot isostatic pressing (Krone et al., 2004), and spark plasma sintering (Shearwood et al., 2005). The transformation behaviors of dense SMAs have been studied and reported extensively (Calloch et al., 2005; Huang and Zhu, 2000; Lexcellent et al., 2002, 2005; Lexcellent and Blanc, 2004; Lexcellent and Schlomerkemper, 2007; Qidwai and Lagoudas, 2000; Taillard et al., 2006). However, it should be noted that few literature have been reported to study the detailed phase transformation mechanism for porous SMAs. Recently, several authors (Entchev and Lagoudas, 2002, 2004; Qidwai et al., 2001; Zhao et al., 2005) have used the micromechanical averaging technique to investigate the mechanical response of porous SMAs. In those analyses, both Mori–Tanaka and

\textsuperscript{*}Corresponding author. Tel.: +1 86 1051688437; fax: +1 86 1051682094.
E-mail address: Gsdui@center.njtu.edu.cn (G. Dui).

1751-6161/$ - see front matter Crown Copyright © 2011 Published by Elsevier Ltd. All rights reserved.
self-consistent method have been used as averaging schemes for the prediction of the macroscopic response of the heterogeneous porous material. The effect of the hydrostatic stress, however, is not considered. It has been reported by Liu et al. (2011) that such impact factor is important. Additionally, since the micromechanical methods based on the averaging theory without considering the definite phase distribution, the prediction of the critical phase transformation stress is higher than the real one (Qidwai et al., 2001; Zhao et al., 2005). As a result, the traditional micromechanical models cannot be used without modification in porous SMAs because of ignoring the effect of hydrostatic stress. Therefore, it is necessary to analyze the phase transformation mechanism for porous SMAs considering the effect of hydrostatic stress.

In this work, the different transformation distributions under both pure hydrostatic stress and uniaxial compression are first analyzed, respectively. Using the dense SMA's parameters, an example for the uniaxial response is then supplied to illustrate one possible application of the full set of constitutive equation with the J2 – I1 type transformation function is chosen. The modeling result is in good agreement with the published experimental data by Zhao et al. (2005).

2. Phase transition for porous SMAs

A representative volume element of the material under investigation is developed in this part and taken to be a hollow sphere with SMA matrix and the porosity taken to be a representative volume element (RVE) (a) with being subject to a macroscopic stress \( \Sigma_{ij} \) and strain at a rate \( \dot{E}_{ij} \). The composite is made of an assembly of composite spheres of various sizes, with a pore of radius \( a \) and a shell of thickness \( b - a \) comprised of the SMA matrix material in each composite sphere. The size distribution of the composite spheres must be such that the entire space of the composite is fully occupied by the composite spheres. According to the works (Christensen and Lo, 1979b; Wang et al., 1996), we isolate a typical spherical element of material as shown in Fig. 1(b)–(c), and assume that this element experiences the same stress and strain as in the remote field. Regardless the size of the composite sphere, the ratio of \( a/b \) for all spheres in the composite is such that the pore volume fraction (PVF) \( f \) in each composite sphere is that of the entire composite

\[
f = \frac{a^3}{b^3}. \tag{1}
\]

It is well known that porous SMAs have been synthesized using many different methods such as combustion synthesis, hot isostatic processing and so on. Material constants for porous SMA are unstable and the materials in different experiments are different, and it is hard to get the parameter values for porous SMAs with different porosities to theoretical analysis. Therefore, in order to adapt to different porosities, we choose the parameters of dense SMA because it is more stable to get and more applicable than the porous ones. Using the stable given parameters of dense SMA, the constitutive model in this work can be used to predict the stress–strain curves for porous SMA materials with different porosities, and it can also be degenerated to model the dense SMA.

2.1. Phase transformations for porous SMAs under pure hydrostatic stress

As shown in Fig. 1(b), the phase transformation of the model under pure hydrostatic pressure \( P = I_1/3 \) on its external surface is first analyzed. Let \( (r, \theta, \phi) \) be the polar–spherical coordinates with the origin of axes located at the center of the SMA shell, the geometrical equations for SMA matrix are given by

\[
\varepsilon_\tau = \frac{du}{dr}, \quad \varepsilon_\theta = \frac{e}\tau = \frac{u}{r} \tag{2}
\]

which lead to the compatibility relation as follows

\[
r \frac{d\varepsilon_\theta}{dr} = \varepsilon_\tau - \varepsilon_\phi. \tag{3}
\]

The only equilibrium equation which should be considered as

\[
d\sigma_r + 2 \varepsilon_r \sigma_r - \sigma_\theta = 0 \tag{4}
\]

where \( \sigma_r \) and \( \sigma_\theta \) are the orthonormal components of stress tensor.

According to the constitutive equations for dense SMAs in Appendix, the effective stress is given by

\[
\sigma_e = \sigma_r - \sigma_\phi. \tag{5}
\]

For forward transformation, it is easy to obtain from \( (A.2) \) and \( (A.3) \) that

\[
\varepsilon_\tau = \frac{\sigma_r - 2\nu\sigma_\theta}{E(\xi)} + H_{\max}\xi \tag{6}
\]

\[
\varepsilon_\theta = \frac{(1 - \nu)\sigma_r - \nu\sigma_\theta}{E(\xi)} = \frac{1}{2}H_{\max}\xi. \tag{7}
\]

The following governing equations can be obtained by combining \( (3)–(7) \)

\[
d\sigma_r = -\frac{2}{r} \sigma_e \tag{8}
\]

\[
d\sigma_\theta = -f(\sigma_e, \sigma_r) \frac{1}{r} \tag{9}
\]

\[
f(\sigma_e, \sigma_r) = \frac{3}{2}H_{\max}\xi + \frac{3}{2}E(\xi)\sigma_\theta
\]

\[
- \left[ \frac{1}{2} H_{\max} + \frac{1}{2} \nu(1 - \nu) \sigma_r - (1 - \nu)\sigma_\theta \left( \frac{1}{E_A} - \frac{1}{E_M} \right) \right] \xi. \tag{10}
\]

Considering the boundary conditions \( \sigma_r|_{r=a} = 0; \sigma_r|_{r=b} = -P \), the distribution of the effective stress can be easily derived by \( (8)–(10) \). Together with the yield criterion: when
\[ \sigma_e = \sigma_{MS}, \] forward transformation initiates, when \( \sigma_e = \sigma_{MF} \), finish the martensite phase transformation, the phase distribution under pure hydrostatic stress can be easily obtained.

The stress distributions are obtained with a max value at the inner edge and a min value at the outer edge. The phase distributions of the model under pure hydrostatic stress are obtained in Fig. 2. As shown in Fig. 2, under isothermal condition that is above the austenite finishing temperature, the material is initially in its austenitic state; with the max effective stress at the inner edge reaches the critical stress, the material begins to transform into martensite; upon further loading, transformation spreads and the material transforms from partially to all into martensite.

For any casual point of the SMA matrix, the martensite volume fraction \( \xi(r) \) can be expressed as \( A \) (5). The average volumetric fraction of martensite phase \( \varsigma_1 \) for porous SMA can be calculated by

\[ \varsigma_1 = \frac{V_M}{V} = \frac{3}{(b^3 - a^3)} \int_a^b \xi(r)^2 dr \] (11)

where \( V_M \) is the volume of the transformation part and \( V \) is the total volume.

The relationship between the martensitic volume fraction of porous SMA and the hydrostatic stress can be easily got from (11), which is shown as the dot curve in Fig. 3. It can be observed that, in the former part of the transformation phase, the relationship between the martensitic volume fraction evolution and the hydrostatic stress can be approximated as linear.

### 2.2. Phase transformations for porous SMAs under uniaxial compression

In order to get the relationship between the martensitic volume fraction of porous SMA and the second stress deviator invariant \( J_2 \), the phase transformation with \( J_2 \) flow rule is then analyzed for the model under uniaxial compression. As shown in Fig. 1(c), the 2-D model like the work of Gurson (1977) is considered here for simplicity. Let \( (r, \theta) \) be the polar coordinates with the origin of axes located at the center of

Fig. 2 – Transformation distribution for a porous SMA under hydrostatic stress.

Fig. 3 – The relationship between the martensitic volume fraction and the pure hydrostatic stress.

In the shell, the geometrical equations and the compatibility relations are given by

\[ \varepsilon_r = \frac{du}{dr}, \quad \varepsilon_\theta = \frac{u}{r} + \frac{1}{r} \frac{du_\theta}{dr}, \] (12)

\[ \gamma_{r\theta} = \frac{1}{r} \frac{du_r}{dr} - \frac{u_\theta}{r} + \frac{1}{r} \frac{du_\theta}{dr}, \] (13)

The solution involves normal stresses \( \sigma_r, \sigma_\theta \) and shear stress \( \tau_{r\theta} \), which are all functions of \( r \) and \( \theta \). The equilibrium equations are expressed as

\[ \frac{d\sigma_r}{dr} + \frac{1}{r} (\sigma_r - \sigma_\theta) + \frac{1}{r} \frac{d\tau_{r\theta}}{dr} = 0 \] (14)

\[ \frac{1}{r} \frac{d\sigma_\theta}{dr} + 2 \frac{\tau_{r\theta}}{r} + \frac{d\tau_{r\theta}}{dr} = 0. \] (15)

According to the constitutive model for dense SMAs in Appendix, the effective stress \( \sigma_e \) of the SMA matrix can be expressed as the same format of (5). It is really hard to get the analytic solution to calculate the stress distribution of the model under uniaxial load. According to Christensen' book (Christensen and Lo, 1979a) and Gurson’s model (Gurson, 1977), the shear stress is assumed as a constant value for simplify, \( \tau_{r\theta} = \tau \), so that the analysis solution of the stress distribution can be easily got. And then we can derive from (A.2) and (A.3) as

\[ \varepsilon_r = \frac{\sigma_r - \sigma_\theta}{E(\xi)} + \frac{\sqrt{3}}{2 \sigma_{Hmax}} \xi \left[ 1 - \frac{3\tau^2}{\sigma_e^2} \right] \] (16)

\[ \varepsilon_\theta = \frac{\sigma_\theta - \sigma_r}{E(\xi)} + \frac{\sqrt{3}}{2 \sigma_{Hmax}} \xi \left[ 1 - \frac{3\tau^2}{\sigma_e^2} \right] \] (17)

\[ \gamma_{r\theta} = \frac{(1 + \nu) \tau}{E(\xi)} + \frac{3}{2 \sigma_{Hmax}} \frac{r \sigma_e}{\sigma_e}. \] (18)

Combining (14)–(18), the stress distribution can be obtained by the following governing equations

\[ \frac{d\sigma_r}{dr} = \frac{-\sigma_e}{r} \] (19)

\[ \frac{\sqrt{3} \sigma_{Hmax}}{2} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial}{\partial \theta} \xi \right) + \sqrt{3} \xi \left( \frac{\partial}{\partial r} \xi \right) + \frac{\partial}{\partial \theta} \right] \]
The porous SMA can be calculated by

\[ \sigma = \frac{1}{E(\xi)} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( (1 - \nu) \frac{\partial \sigma_r}{\partial r} + \frac{\partial \sigma_\theta}{\partial \theta} \right) \right] 
+ \frac{2(1 - \nu)}{E_A} \frac{\partial \sigma_\sigma}{\partial r} 
+ \frac{2(1 - \nu) \sigma_r + \nu \sigma_\sigma}{1 + \nu} \left( \frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} \right) 
+ \left( \frac{1}{E_M} + \frac{1}{E_A} \right) \frac{\partial \theta}{\partial r} \left( \frac{1}{r} \frac{\partial \sigma_\theta}{\partial r} - \frac{1}{r} \frac{\partial \sigma_r}{\partial \theta} \right) \right]. \tag{20} 

Considering \( \tau_\theta |_{\theta=\pm \pi/2} = \Sigma_{ij} \sin 2\theta/2, \tau_\sigma |_{\theta=\pm \pi/2} = 0; \sigma_r |_{\theta=\pm \pi/2} = 0 \) and \( \sigma_r |_{\theta=\pm \pi/2} = -\Sigma_{ij}/2 = -\Sigma_{ij} \cos 2\theta/2 \), the distribution of the effective stress can be easily derived by (19) and (20), and the phase distribution can be obtained in the same method as part 2.1.

The distributions of the stress are obtained with a max value at the inner edge at \( \theta = \pm \pi/2 \) and a miner value in the outer edge at \( \theta = \pm \pi. \) The approximate phase distributions of the model under uniaxial compression are easily obtained in Fig. 4. As shown in Fig. 4, when the max effective stress in the inner edge reaches the critical stress, the phase transformation happens. Upon further loading, the transformation spreads and the model goes through the phase transition states from (a) to (b) in Fig. 4 and then all transformed into martensite.

In the same way, for any casual point of the SMA matrix, the martensitic volume fraction \( f(t, \theta) \) can be calculated by A (5). The average volumetric fraction of martensite phase \( \varepsilon_2 \) for the porous SMA can be calculated by

\[ \varepsilon_2 = S_{\text{martensite}} \frac{r \sin \theta \, dS}{S} \tag{21} \]

where \( S_M \) is the area of the transformation part and \( S \) is the whole area.

The average volumetric fraction of martensite phase can be calculated by (21). The relationships between the average volumetric fraction of martensite phase and the second stress deviator invariant for different porosities are shown as the dot curves in Fig. 5. It can be observed that, since the transformation beginning, the relationship between the martensitic volume fraction evolution and the second stress deviator invariant can be approximated as linear.

3. The constitutive model for porous SMAs

In order to build a simple model to describe the mechanical behaviors of porous SMAs, the mechanical properties of dense SMA should be used adequately. Most of the dense SMA constitutive models based on \( J_2 \) invariant type transformation function are capable of mechanical behavior. Since porous material is significant affected by hydrostatic stress, the transformation function should be assumed to depend on both the first stress \( I_1 \) and the second stress deviator invariant \( J_2 \) (Qidwai and Lagoudas, 2000). Let transformation function be defined by

\[ \Phi = \eta \sqrt{3J_2} + \omega I_1 - \sigma_0 \tag{22} \]

where \( \sigma_0 \) is the critical stress of phase transformation. The model parameters \( \eta \) and \( \omega \) will be determined by the threshold conditions of phase transformation. The relationship between the martensitic volume fraction and the hydrostatic
stress can be approximated as a linear function, which is shown by the solid curve in Fig. 3 and the linear function is
\[
\dot{\varsigma}_1 = \frac{f t}{3(1 - 2f)\sigma_{Mf} - 2(1 - f)\sigma_{Ms}} - \frac{2(1 - f)\sigma_{Ms}}{3(1 - 2f)\sigma_{Mf} - 2(1 - f)\sigma_{Ms}}.
\] (23)

In the same way, the relationship between the martensitic volume fraction and the second stress deviator invariant can be approximated as a linear function, which is shown by the solid curve in Fig. 5 and the function is
\[
\dot{\varsigma}_2 = \frac{\sqrt{3}J^0}{\sqrt{6f^2 - 4f + 1}}\left(\frac{1}{\sigma_{Mf} - \sigma_{Ms} - f(3\sigma_{Mf} - 2\sigma_{Ms})} - \frac{\sigma_{Mf} - \sigma_{Ms} - f(3\sigma_{Mf} - 2\sigma_{Ms})}{f(3\sigma_{Mf} - 2\sigma_{Ms})}\right).
\] (24)

According to (23), (24) and the threshold conditions of phase transformation, the model parameters \(\eta\) and \(\omega\) can be calculated by
\[
\eta = \frac{1 - 4f}{6f^2 - 4f + 1}, \quad \omega = \frac{f}{6f^2 - 4f + 1}.
\] (25) \quad (26)

Under isothermal conditions, the overall stress–strain relation of the system is given by
\[
\dot{E}_{ij} = M^p \dot{E}_{ij}^p + \dot{E}_{ij}^e.
\] (27)

By considering the existence of spherical pores and using Mori–Tanaka mean-field theory (Tanaka, 1986), the overall compliance tensor of porous SMA can be fully represented by (Mochida et al., 1991)
\[
M^p = \frac{7 - 7f}{7 + 8f}(M_A + \varsigma(M_M - M_A))
\] (28)

where \(M_A\) and \(M_M\) are the compliance tensor of pure austenite and pure martensite for the SMA matrix, respectively.

The increment of overall transformation strain during the forward or reverse transformation can be expressed by the following equation
\[
\dot{E}_{ij}^e = \lambda \frac{\partial \phi}{\partial \Sigma_{ij}}
\] (29)

where \(\lambda\) is the Lagrange multiplier given by the consistency conditions \(\phi = 0\)
\[
\frac{\partial \phi}{\partial \Sigma_{ij}} \dot{\Sigma}_{ij} + \frac{\partial \phi}{\partial t} \dot{t} + \frac{\partial \phi}{\partial \varsigma} \dot{\varsigma} = 0.
\] (30)

It is assumed that the effective transformation strain is proportional to the maximum transformation strain of porous SMA during an uniaxial compression test as
\[
\dot{E}_e = \frac{H_{\text{max}} \dot{\varsigma}}{1 - f},
\] (31)

where \(\dot{E}_e\) is called the effective transformation strain rate for porous SMA, and provides a scalar measure of the total transformation strain. This quantity is defined as
\[
\dot{E}_e = \sqrt{\frac{2}{3} \dot{E}_{ij}^e \dot{E}_{ij}^e}.
\] (32)

The evolution of the martensitic volume fraction, \(\dot{\varsigma}_e\), can be calculated from (29)–(32). Substituting Eq. (29) into Eq. (32) and using Eq. (31), the increment of the transformation strain for porous SMA is
\[
\dot{E}_e^p = \frac{H_{\text{max}} \dot{\varsigma}}{1 - f} \frac{\partial \phi}{\partial \Sigma_{ij}}.
\] (33)

The overall strain behaviors of porous SMAs can be obtained by using (25)–(27), (33) and (22).

The above developed theory will be applied in predicting the constitutive response of porous SMAs. One usually studies porous SMAs under the uniaxial compression, because it is hard to analyze the phase distributions under various loadings. Hence, we analyze the model under uniaxial compression for simple and the cases under other loading conditions will be discussed in the future work.

To testify the feasibility and correctness of the proposed model, as an example, we predict the stress–strain response for the porous SMA with 13% porosity under uniaxial compression. To be able to compare the modeling prediction result with the obtained experimental data published by Zhao et al. (2005), here we use the same material parameters of the dense Ni–Ti SMA published by Zhao et al. (2005) in Table 1.

Fig. 6 shows a comparison under uniaxial compression between the predicted result of the stress–strain response for 13% porosity and the experimental data (Zhao et al., 2005) respectively. The dotted curve represents the experimental result published by Zhao et al. (2005). The dashed curve is the model result of Zhao without considering the effect of hydrostatic stress. The solid curve corresponds to the present model. As shown in Fig. 6, the present result agrees very well with the experimental one, and the point where the transformation starts in the present model is closer to the experimental data than Zhao’s model. The behavior of the material in the minor loop is also correctly reproduced and verifies the validity of the present model. When \(f = 0\), the model can be degenerated to model the dense SMA.

4. Conclusion

Utilizing micromechanics and elastic mechanics, the phase distributions and the constitutive model for porous SMA is
obtained. The effect of hydrostatic stress, not accounted for in previous models, has been considered in the present model. Numerical result has been compared with the experimental data (Zhao et al., 2005) has and show good agreement. Importantly, the transformation initiation stress is much closer to the experiment result than simulated by Zhao et al. (2005).

**Acknowledgments**

The authors acknowledge the financial support of National Natural Science Foundation of China (No. 11172033, 10772021 and 10972027) and National Basic Research Program of China (973 Program) (2010CB7321004).

**Appendix. Constitutive model for dense SMAs**

Within the framework of small deformations under isothermal conditions, the total strain consists of two parts, an elastic part caused by stress and a transformation strain caused by phase transformation, and is given as,

\[ \varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^t. \]  

(A.1)

\[ \varepsilon_{ij}^e = \frac{(1+\nu)\sigma_{ij} - \nu\sigma_{kk}}{E(\xi)}, \]  

(A.2)

where \(\xi, E(\xi)\) and \(\nu = \nu_A = \nu_M\) are the volume fraction of the martensite phase, the Young's modulus and the Poisson's ratio for dense SMA, respectively. Using J2 rule, assume that the effective stress \(\sigma_e = \sqrt{\sigma'_{ij}\sigma'_{ij}/2}1/2\). Then the transformation strain may be associated with volumetric phase fraction of martensite \(\xi\) in a rate from (Lagoudas et al., 1996)

\[ \varepsilon_{ij}^t = \begin{cases} \frac{3H_{\max}\sigma'_{ij}}{2\sigma_e} \xi, & \text{when } \xi > 0 \\ \frac{H_{\max}\varepsilon_{ij}^t}{\varepsilon_{ij}^t}, & \text{when } \xi < 0 \end{cases} \]  

(A.3)

where \(\sigma'_{ij}\) is the stress deviator and \(H_{\max}\) is the maximum transformation strain for dense SMA. \(\varepsilon_{ij}^t\) is the transformation strain at the reversal of transformation and the effective strain \(\varepsilon_{ij}^t = (2\varepsilon_{ij}^t/3)^{1/2}\). Furthermore, it is assumed that elastic compliance for dense SMA with mixture rule so that

\[ \frac{1}{E(\xi)} = \xi \left( \frac{1}{E_M} - \frac{1}{E_A} \right) + \frac{1}{E_A} \]  

(A.4)

where \(E_A\) and \(E_M\) are the Young's moduli of pure austenite and pure martensite for dense SMA, respectively. The first equation in (A.3) for forward phase transformation is similar as the J2 flow rule in the classical plasticity theory. It is worth noting when \(\xi > 0, \varepsilon_{ij}^t = 0\) which implies that forward transformation triggers no volume dilation and pressure sensitivity effect is ignored. The underlying assumption is that forward transformation strains are coaxial with the stress deviator. For completeness of the constitutive model, a kinetic relationship between martensitic volume fraction and effective stress has to be supplemented. To simplify the current analysis, we assumed that \(\xi\) is a linear function of the effective stress, which is different from the popular models

\[ \xi = \begin{cases} 0 & \sigma_e \leq \sigma_{Ms} \\ \left(\sigma_e - \sigma_{Ms}\right)/\left(\sigma_{Mf} - \sigma_{Ms}\right) & \sigma_{Ms} \leq \sigma_e \leq \sigma_{Mf} \\ 1 & \sigma_e \geq \sigma_{Mf} \end{cases} \]  

(A.5)

where \(\dot{\xi} = d\xi/d\sigma_e, \sigma_{Ms}, \sigma_{Mf}\) is the threshold stress at the forward transformation for dense SMA. A similar expression may be constructed in the case of the reverse transformation. With the aid of Eqs. (A.1)–(A.5), it is easy to obtain the response of SMA under applied stress.

**References**


