Investigation of interference effect during ionization of hydrogen atom in bichromatic laser fields of circular polarization

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ABSTRACT

We theoretically investigate the strong-field ionization of hydrogen atom in bichromatic laser fields of circular polarization by calculating the momentum distributions. We find that the momentum distributions consist of a series of arc fringes, which correspond to the different above-threshold ionization peaks, and the momentum distributions generally are inversion asymmetric owing to the interference happening between the different ionization channels. But if only one ionization channel is considered, the momentum distributions are perfect and concentric rings, and as the number of ionization channel increases, the asymmetry of the momentum distribution becomes more and more notable.

1. Introduction

Over the past few decades, the studies of interaction between strong laser fields with atoms and molecules have dramatically advanced our understanding on the strong field phenomena, such as above-threshold ionization (ATI), high-order harmonic generation (HHG), and nonsequential double ionization (NSDI) [1–6]. Construct and control of the electric field waveform of laser pulses, however, have opened a new way to study and control electron dynamics in strong-field processes due to the generation of the laser pulses with duration as short as few optical cycles [7–9]. For atoms and molecules irradiated by few-cycle laser pulses, the ionization rates in opposite directions are not always identical, which is termed as inversion asymmetry. According to our earlier work [11,10], due to the interference effect happening between the different ionization channels, the photoelectron angular distribution is asymmetric. The photoelectron angular distribution, however, only displays a part of the interference feature. Now an open question is how the ionization channels are responsible for the specific interference effect. In this paper, we will answer this question. To avoid the interference effect coming from the two-center feature of molecule [12–16], we choose H atom in 1S state as target atom, which is isotropic and will benefit our study on the interference happening only between the different ionization channels. In order to reduce the difficulties in analyzing the interference effect, in this paper, we will use a bichromatic laser field of circular polarization, which is specially modulated such that it can be used to mimic a sequence of one-cycle laser pulses [11,10].

2. Ionization-rate formula

In the study of photoionization of atoms and molecules in intense laser fields, the strong field approximation (SFA) is widely used [17,18]. In this treatment, the Coulomb attraction of the parent ion to the ionized electron is neglected and the state of the electron in the intense laser field is described by a Volkov state [19]. Within the SFA, the ionization-rate formula of photoelectron with a given kinetic energy in the bichromatic laser field is given by [11,20] (the units \( c = \hbar = 1 \) are used throughout this paper)

\[
\frac{dW}{d\Omega_\tau} = \frac{(2m_0^2\omega_0^3)^{1/2}}{(2\pi)^2} \sum_{n=n_0}^{\infty} (n-u_{p1}-u_{p2}+\epsilon_0)^{1/2} \times (n-u_{p1}-u_{p2})^2 \sum_{j_1,j_2} X_{j_1,j_2}(\zeta_1,\zeta_2) \left| j_1 \right|^2 \left| \phi(\mathbf{P}_j) \right|^2,
\]

where \( d\Omega_\tau = \sin \theta_\tau \, d\theta_\tau \, d\phi_\tau \) is the differential solid angle of the final momentum of photoelectron \( \mathbf{P}_j \), \( \theta_\tau \) and \( \phi_\tau \) are its emission and azimuthal angles; \( j_i \) (\( i = 1, 2 \)) is the number of photon absorbed from the \( i \)th mode. Here we identify one set of \( j_i \) that...
satisfies \( j_1 + j_2 = n \) for a fixed \( n \), which determines the final kinetic energy of photoelectron and denotes the ATI order, and \( n \) is the laser frequency of the \( j \)th mode and \( n_0 = 2n_0 \) for a given ATI peak. Each pair of \( (j_1, j_2) \), which satisfies \( j_1 + 2j_2 = n \) for a given ATI peak, is a transition channel. \( \epsilon_j = E_j/\omega_0 \) is the binding energy of H atom in the unit of \( \omega_0 \). The ponderomotive parameters of the laser field are defined as

\[
u_{p1} = \frac{\epsilon_j^2 I}{16 \pi \epsilon_j \epsilon_0}, \quad \nu_{p2} = \frac{\nu_{p1}}{32},
\]

where \( I \) is the peak intensity of laser pulse. The two-mode generalized phased Bessel (GPB) function \( \chi_{j_1,j_2}(\xi_1, \xi_2, \zeta) \) in Eq. (1) is given by \([20, 21]\)

\[
\chi_{j_1,j_2}(\xi_1, \xi_2, \zeta) = \sum_{q=-\infty}^{\infty} \epsilon_j X_{j_1,q}(\xi_1) X_{j_2,q}(\xi_2) \chi_{-q}(\zeta),
\]

(2)

where

\[
\epsilon_j = \left( \frac{2u_{p1}}{m_0 \omega_0} \right)^{1/2}, \quad \epsilon_j \xi_1, \xi_2 = \left( \frac{u_{p2}^2}{m_0 \omega_0} \right)^{1/2} \epsilon_j \xi_1, \xi_2 = \nu_{p1} \epsilon_1 \epsilon_2.
\]

(3)

Here \( \epsilon_1, \epsilon_2 \) is the polarization vector of the \( j \)th mode and is defined as

\[
\epsilon_1 = \epsilon \cos \frac{\pi}{2} + i \epsilon \sin \frac{\pi}{2}, \quad \epsilon_2 = \epsilon \epsilon^\phi
\]

and \( \xi = \pi/2 \) is the polarization degree for circular polarization. \( \phi \) is the relative phase difference between two modes and \( \phi = -\phi_0 \), which is the carrier-envelop (CE) phase [22].

In Eq. (1), \( \Phi(\mathbf{P}) \) is the Fourier transform of the initial wave function of H atom in 1S state and has the following form \([17]\):

\[
\Phi(\mathbf{P}) = \frac{2^j a_0^{1/2} \epsilon_j^{1/2}}{(1 + q^2 P^2)^{j/2}},
\]

(4)

where \( a_0 \) is the Bohr radius and \( P_j = |\mathbf{P}_j| \).

3. Numerical results and discussion

As for the laser field we assume that it is circularly polarized, and the central wavelength is 800 nm. According to Eq. (1), we calculate the momentum distribution as function of \( (p_x, p_y) \) in polarization plane, that means the emission angle is \( \theta_f = \pi/2 \).

Fig. 1 depicts the momentum distribution as functions of \( (p_x, p_y) \) in polarization plane for different CE phases at laser intensity of \( 10^{14} \) W/cm\(^2\). From Fig. 1, we find that the momentum distributions are inversion asymmetric and vary with the CE phase. For \( \phi_0 = 0 \), the electrons escape predominantly in the \( p_x < 0 \) direction and are located at the second and third quadrants, while for \( \phi_0 = \pi \), the electrons escape predominantly in the \( p_x > 0 \) direction and are located at the first and fourth quadrants as shown in Fig. 1(c). The maximal ionization rates for \( \phi_0 = 0 \) and \( \pi \) are located in the opposite directions, and the spectra recorded by the left-hand and the right-hand detectors are identical. When \( \phi_0 = \pi/2 \), the momentum distributions still are inversion asymmetric, but the electrons emit mainly in the \( p_y < 0 \) direction and are symmetric with respect to the \( p_x = 0 \) axis as shown in Fig. 1(b). This feature that the inversion asymmetry of the momentum distribution varies the CE phases has been observed experimentally in the ionization of helium by a few-cycle circularly polarized laser pulse [23] and shown theoretically by solving the three-dimensional time-dependent Schrödinger equation for three-cycle laser pulse as shown in Ref. [24, Fig. 2]. For an arbitrary CE phase, the momentum distributions can be got through rotating the momentum distributions for other CE phase. Here we do not show this.

We discuss now the interesting phenomena of the momentum distributions shown in Fig. 1. The momentum distributions consist of several fringes which correspond to the different ATI peaks, and the number of fringe is same for the different CE phases. In the momentum distribution, the innermost and the outermost fringes, which correspond the photoelectrons with the lowest and the highest final kinetic energies, respectively, show that the ionization rate is very low. The intermediate fringes correspond to the photoelectrons with middle kinetic energy, which have the highest ionization rate and dominate the total ionization rate. In order to clearly display this, we calculate the photoelectron energy spectra as function of \( p_y \) for \( \phi_0 = 0 \) and setting \( p_x = 0 \) as shown in Fig. 2. In Fig. 2, we see that the envelope of photoelectron energy spectra has a bell shape, that means that the ionization rate of photoelectron with the lowest and highest final kinetic energy is suppressed, and the photoelectron energy spectra have several peaks, which correspond to the fringes in the momentum distribution. For each peak in Fig. 2, the ionization yield gradually increases and sharply drops, and then a new peak appears with the increasing of the energy of photoelectron. This is easily understood when we recall Eqs. (2) and (3). For a given ATI order, \( n \) satisfies \( n = j_1 + 2j_2 \) and is determined by

\[
n = \left[ \frac{n_0^2}{(2n_0)} + \frac{u_{p1}^2 + u_{p2}^2}{E_0}/n_0 \right]^{1/2}.
\]

(5)

So \( n \) increases with the energy of photoelectron, but does not continuously increase. The values of arguments in GPB, \( \xi_1, \xi_2 \), in Eq. (2) enlarge as the energy of photoelectron increases. And then the value of GPB increases as the energy of photoelectron increases, before \( n \) reaches to the next integer for the next ATI order. Once \( n \) reaches to and goes beyond the next integer when the energy of photoelectron increases, the value of GPB becomes about zero then increases again from the zero. As a result, the peak in Fig. 2 gradually increases and sharply drops, and each peak has a certain width. From Fig. 2, we can also know that for a given ATI peak, the photoelectrons with largest kinetic energy have main contribution to the ionization rate.

Fig. 1. The momentum distribution as functions of \( (p_x, p_y) \) in polarization plane for different CE phases: (a) \( \phi_0 = 0 \), (b) \( \phi_0 = \pi/2 \), (c) \( \phi_0 = \pi \) at laser intensity of \( 10^{14} \) W/cm\(^2\). Note that every plot has been normalized by their maximum value.
In Fig. 1, we have presented the inversion asymmetry of the momentum distribution. According to our earlier work [11], the interference effect happening between the different ionization channels leads to the inversion asymmetry. To clearly understand how the ionization channels affect the interference, we calculate the momentum distributions with different number of ionization channel as shown in Fig. 3 for \( \phi_0 = 0 \). Before presenting the results, we prefer to recall how to define one ionization channel. In Eq. (1), we had identified one set of \( j_i (i = 1, 2) \) as the number of photons absorbed from the \( i \)th mode. Each pair of \((j_1, j_2)\), which satisfies \( j_1 + 2j_2 = n \) for a given ATI peak, is a transition channel and each pair of \((j_1, j_2)\) is a transition channel. In Fig. 3(a), the momentum distributions are for \( j_1 = 0 \), which means that there is a ionization channel \((j_1 = 0, 2j_2 = n)\). There is no interference happening during ionization process, because the electron absorbs photon only from the second laser mode. So the momentum distributions are perfect and isotropic rings, that is to say, the ionization rate in random direction is same. From Fig. 3(a), we also find that there are three concentric circles with different radii, which stand for different ATI orders \( n \). In Fig. 3(b), we present the momentum distributions with \( j_1 = 2 \). For this case, three ionization channels may occur: \((j_1 = 0, 2j_2 = n)\), \((j_1 = 1, 2j_2 = n - 1)\) and \((j_1 = 2, 2j_2 = n - 2)\). The increasing of ionization channel makes the momentum distributions more complex. Fig. 3(b) displays that at least four circles can be seen and the circles are not so perfect and slightly asymmetric along the \( p_x \) axis, especially for the intermediate brightest circle, which means that more electrons are emitting into the \( p_x < 0 \) region than into the \( p_x > 0 \) region. In Fig. 3(c) it is noted as the momentum distributions for \( j_1 = 5 \). For \( j_1 = 5 \), there may be six ionization channels: \((j_1 = i, 2j_2 = n - i)\), here \( i = 0, 1, 2, 3, 4, 5 \). Under these ionization channels, the momentum distributions have more than five fringes and the momentum distributions obviously become inversion asymmetric along the \( p_x \) axis. In Fig. 3(d), we present the momentum distributions for \( j_1 = 8 \), for which there may be nine ionization channels: \((j_1 = i, 2j_2 = n - i)\), here \( i = 0, 1, 2, 3, 4, 5, 6, 7, 8 \). The number of fringe reaches to eight in the momentum distributions and the inversion asymmetry further becomes clear as shown in Fig. 3(d). If the ionization channel further increases, the results in Fig. 1(a) can be reproduced again. Here we only take \( \phi_0 = 0 \) as an example to show how the interference happens during the ionization process. If \( \phi_0 = \pi \) or

![Fig. 2. The photoelectron energy spectra as functions of \( p_x \) axis for \( \phi_0 = 0 \). Laser intensity is \( I = 10^{14} \text{ W/cm}^2 \). Note that the values have been normalized by the maximum value. The negative momentum stands for anti-direction of \( p_x \).](image1)

![Fig. 3. The momentum distributions as a function of \((p_x, p_y)\) in polarization plane for several ionization channels: (a) \( N = 1 \), (b) \( N = 3 \), (c) \( N = 6 \), (d) \( N = 9 \). Laser intensity is \( I = 10^{14} \text{ W/cm}^2 \) and the CE phase is \( \phi_0 = 0 \). Note that the values have been normalized by the maximum value for each plot.](image2)
other values, the above-mentioned results can be got again after rotating an angle. So we do not discuss this here. The phenomenon that the inversion asymmetry becomes more prominent with the increasing of ionization channel has not been seen in Ref. [24, Fig. 4a], where Martiny et al. have shown that the momentum distributions of H atom in 1S state have concentric ring structures at the same intensity used in this paper. The reason is that in our paper, the laser field used in this paper is one-cycle laser pulse for which the electric field varies with the CE phase; while in Ref. [24, Fig. 4a] the number of optical cycle is twenty, for which the electric field is almost constant during the dominant cycles of the field. So the momentum distributions of H atom in 1S state (Ref. [24, Fig. 4a]) are completely symmetric distribution.

4. Conclusions

In summary, we have theoretically investigated the strong-field ionization of H atom in bichromatic laser fields of circular polarization by calculating the momentum distributions. We found that the momentum distributions consist of a series of fringes, which generally are inversion asymmetric, owing to the interference happening between the different ionization channels. In order to verify how the ionization channels are responsible for the interference, we have calculated the momentum distributions with the different number of the ionization channel. The results show that if only one ionization channel is included, the momentum distributions consist of several concentric rings, which are perfect and isotropic, that is to say, no interference happens. Hence the asymmetry of the momentum distribution cannot be seen. As the number of ionization channel increases, the number of ring or fringe in the momentum distribution also increases, and the asymmetry of the momentum distribution becomes more and more notable.

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