On the local nature of the strain field calculation method for measuring heterogeneous deformation of cellular materials

Shenfei Liao\textsuperscript{a,b}, Zhijun Zheng\textsuperscript{a,\ast}, Jilin Yu\textsuperscript{a}

\textsuperscript{a}CAS Key Laboratory of Mechanical Behavior and Design of Materials, University of Science and Technology of China, Hefei, Anhui 230026, PR China
\textsuperscript{b}National Key Laboratory of Shock Wave and Detonation Physics, Institute of Fluid Physics, CAEP, Mianyang, Sichuan 621900, PR China

\textbf{A R T I C L E  I N F O}

Article history:
Received 3 December 2012
Received in revised form 23 September 2013
Available online 23 October 2013

Keywords:
Cellular material
Heterogeneous deformation
Local strain
Cut-off radius
Shock wave

\textbf{A B S T R A C T}

A strain field calculation method based on the optimal local deformation gradient technique has been developed to calculate the ‘local’ strain tensor of cellular materials using cell-based finite element models. The local nature and accuracy of this method may be strongly dependent on the cut-off radius, which is introduced to collect the effective nodes for determining the optimal local deformation gradient of a node. Two different schemes are first analyzed to determine the suitable cut-off radius by characterizing the heterogeneous deformation of Voronoi honeycombs under uniaxial compression and we suggest that in Scheme 1, the cut-off radius defined based on the reference configuration is about 1.5 times the average cell radius; in Scheme 2, the cut-off radius defined based on the current configuration is about 0.5 times the average cell radius. Then, Scheme 3, a combined scheme of the two former schemes, is further suggested. It is demonstrated that the optimal cut-off radius in Scheme 3 characterizes the local strain reasonable well whether the compression rate is low or high. Finally, the strain field calculation method with the optimal cut-off radius is applied to reveal the evolution of the heterogeneous deformation of two different configurations of double-layer cellular cladding under a linear decaying blast load. The 2D fields and the 1D distributions of local engineering strain are calculated. These results interpret the shock wave propagation mechanisms in both claddings and provide useful understanding in the design of a double-layer cellular cladding.

© 2013 Elsevier Ltd. All rights reserved.

\textbf{1. Introduction}

Man-made cellular materials, such as metallic foams and honeycombs, have been widely used in lightweight structural and multifunctional applications (Gibson and Ashby, 1997). There exist at least two length scales in the characterization of a cellular material due to its cellular nature: the mesoscopic concept (cell level) and the macroscopic scale (specimen level). At the mesoscopic scale, the individual response of a cell strongly depends on its morphology and location in the material. For example, three possible deformation mechanisms at the cell level were revealed in Bastawros et al. (2000). Distortions and rotations arising at multiple cells develop the localized deformation bands of approximately one-cell width, which have been observed in cellular materials under both quasi-static compression (Bastawros et al., 2000; Bastawros and Evans, 2000; Jang and Kyriakides, 2009; Tan et al., 2002) and dynamic compression (Lee et al., 2006; Radford et al., 2005; Tan et al., 2012; Zou et al., 2009). At the macroscopic scale, the response of the cellular material is an average of the responses of cells at the mesoscopic scale. A continuum concept, nominal engineering stress–strain relationship, is commonly used to characterize the macroscopic response of cellular materials without considering the cellular nature. However, deformation localization is a typical feature of the response of cellular materials, especially in dynamic cases, and obviously it cannot be represented by nominal stress–strain relationship (Liu et al., 2009). Due to the cellular nature, the concept of ‘local’ strain should be introduced to characterize the deformation of cells at the mesoscopic scale. Here, the terminology of ‘local’ refers to a small region but not to a mathematical point. Zheng et al. (2012) suggested that the local strain in cellular materials should be defined as statistical average measured over the range of at least one cell size. Following this guide, we aim to evaluate the local nature and accuracy of a local strain in a cellular material in this study.

Digital image correlation (DIC) technique based on camera system and image processing system has been used in a considerable number of experimental studies on cellular materials to capture the heterogeneous deformation at the mesoscopic scale. For example, this technique was employed by Bastawros et al. (2000) to monitor the evolution of plastic deformation in a closed-cell aluminum foam under quasi-static compression: at the onset of nonlinear response, localized deformation bands initiate and have width of approximately one-cell diameter; on further straining to plateau...
regime, deformation bands contact with each other and develop a characteristic spacing of 3–4 cell diameters. Local deformation in ultra-light open-cell foams (Wang and Cuitio, 2002) and honeycombs (Khan et al., 2012) under quasi-static compression was also revealed by using the DIC technique. Besides the applications in quasi-static loading case, the DIC technique was used by Elnaeei et al. (2007) to perform a quantitative measurement of the strain fields during the crushing of metallic cellular structures under impact loading, by means of which shock front propagation was observed and shock front speed was measured through a simplified analysis.

Some numerical methods for characterizing the local strain in cellular materials have been proposed in the literature. For regular honeycombs, Zou et al. (2009) presented a definition of local engineering strain based on relative displacement between two neighboring cross-sections. From the strain distribution, shock front propagating through the honeycomb was captured. Nevertheless, the strain behind the shock front suffers high fluctuations and thus is difficult for quantitative analysis. Besides, this definition of local strain masks the gross behavior over the transverse direction, recognizing the limitation of being more suitable for a high impact velocity. For irregular cellular materials, a local strain map algorithm, similar to the DIC technique, was developed by Mangipudi and Onck (2011) for characterizing the strain fields at the mesoscopic scale. In their work, Voronoi honeycomb cells were triangulated into triangles, in which local strain was assumed to be constant and calculated with displacements of cell nodes by using standard finite element method. The local strain was defined by Cauchy strain formula, which is only appropriate for small-strain states (Mangipudi and Onck, 2011). Recently, Liao et al. (2013a) developed a strain field calculation method based on the optimal local deformation gradient technique for cellular materials. In this approach, Voronoi honeycomb cells in the numerical simulation were discretized into a series of nodes and discrete local deformation gradients at representative nodes, in a least squares sense, were calculated by the relative motions of the representative nodes and their neighboring nodes. Then, a local strain tensor related to the optimal local deformation gradient can be calculated through the continuum mechanics theory and it allows considering general finite-strain states of both regular and irregular cellular materials.

The cut-off radius, which is used to determine the neighboring nodes of a representative node for calculating the optimal local deformation gradient, is a key parameter in the strain field calculation method presented in our previous work (Liao et al., 2013a). The local nature and accuracy of a local strain may be strongly dependent on the cut-off radius for a cellular material. Thus, we aim to extend our previous work (Liao et al., 2013a) by investigating the effect of the cut-off radius on the local strain and to find an optimal choice of the cut-off radius. A brief introduction of the formulation of the local strain tensor with different schemes for the definition of the cut-off radius is presented in Section 2. The effect of the cut-off radius on the local nature and accuracy of the local strain is analyzed and an optimal choice of the cut-off radius is suggested in Section 3. As an application of the strain field calculation method, heterogeneous deformations of Voronoi honeycomb cores in two different configurations of double-layer cellular cladding under blast load are characterized in Section 4, followed by conclusions in Section 5.

2. Formulation of local strain

2.1. Local strain tensor

According to the continuum mechanics theory, large deformation can be represented by the Lagrangian or Green strain tensor, \( E \), given by

\[
E = \frac{1}{2} (\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I}),
\]

where \( \mathbf{F} \) is the deformation gradient, \( \mathbf{I} \) the identity matrix and superscript \( T \) denotes the transpose of a matrix. Once a local deformation gradient is determined, a local strain tensor is obtained. However, due to the cellular nature, the deformation gradient cannot be directly constructed in a cellular material. Therefore, a discrete local deformation gradient determined by the method of least squares defined in Li and Shimizu (2005), Gullett et al. (2008) and Zimmerman et al. (2009) was introduced in our previous work (Liao et al., 2013a) to develop a strain field calculation method for characterizing the local deformation of a cellular material. To perform the strain field calculation, the locations of nodes are monitored and output from the cell-based finite element model. For example, a Voronoi honeycomb is discretized into a series of corner nodes (vertices of Voronoi honeycomb cells) and other nodes on the cell walls, as illustrated in Fig. 1. The deformation of the Voronoi honeycomb is reflected by the relative motions of all nodes. The interior strain of the Voronoi honeycomb is then assumed to be discretely represented by the local strains at all nodes. For the purpose of saving computational cost, we sample the local strains at corner nodes rather than at all nodes to discretely represent the interior strain of a Voronoi honeycomb in the present work. The reasonableness of this simplification will be evaluated later.

Two nodal configurations, namely the reference (undeformed) configuration \( \Omega_0 \) and the current (deformed) configuration \( \Omega_t \), are needed to calculate the local deformation gradient. For node \( i \) and its neighboring node \( j \), their relative position vectors are \( \mathbf{u}_i = \mathbf{x}_i - \mathbf{x}_0 \) and \( \mathbf{u}_j = \mathbf{x}_j - \mathbf{x}_0 \) in configurations \( \Omega_0 \) and \( \Omega_t \), respectively, where \( \mathbf{x} \) and \( \mathbf{x}_0 \) are the position vectors of a node in \( \Omega_0 \) and \( \Omega_t \), respectively, as illustrated in Fig. 1. They are all considered to be column vectors. It is assumed that there exists an optimal local deformation gradient \( \mathbf{F}_i \) defined at node \( i \), which best maps node \( i \) and all of its neighboring nodes from \( \Omega_0 \) to \( \Omega_t \) by \( \mathbf{u}_0 = \mathbf{F}_i \cdot \mathbf{u}_i \). In other words, the optimal local deformation gradient \( \mathbf{F}_i \) minimizes the least squares mapping error of node \( i \) defined as (Li and Shimizu, 2005)

\[
\varphi_i = \sum_{j=1}^{N} (\mathbf{F}_i \cdot \mathbf{u}_{ij} - \mathbf{u}_{ij})^T \cdot (\mathbf{F}_i \cdot \mathbf{u}_{ij} - \mathbf{u}_{ij}),
\]

where \( N \) is the number of neighboring nodes of node \( i \) to ensure the local nature of the local strain, a cut-off radius \( R_c \) is introduced to determine the neighboring nodes of node \( i \) and will be discussed later. Based on the method of least squares, the optimal local deformation gradient \( \mathbf{F}_i \) is determined by

\[
\frac{\partial \varphi_i}{\partial \mathbf{F}_i} = 2 \sum_{j=1}^{N} (\mathbf{F}_i \cdot \mathbf{u}_{ij} - \mathbf{u}_{ij}) \cdot \mathbf{u}_{ij}^T = 0.
\]

The solution of Eq. (3) is given by Li and Shimizu (2005)

\[
\mathbf{F}_i = \mathbf{W}_i \cdot \mathbf{V}_i^{-1},
\]

where matrix \( \mathbf{V}_i \) and \( \mathbf{W}_i \) are

\[
\mathbf{V}_i = \sum_{j=1}^{N} \mathbf{u}_{ij} \cdot \mathbf{u}_{ij}^T, \quad \mathbf{W}_i = \sum_{j=1}^{N} \mathbf{u}_{ij} \cdot \mathbf{u}_{ij}^T.
\]

2.2. Possible choices of the cut-off radius

The choice of neighboring nodes plays an important role in the calculation of the discrete local deformation gradient at a node. Intuitively, neighboring nodes within a certain proximity can
influence the motion of a node, but those far away should have little contribution to ensure the local nature of strain states. In this study, a cut-off radius, \( R_c \), was introduced to specify a domain of a node, only the neighboring nodes located within which were assumed to contribute to the formulation of the discrete local deformation gradient. A correct choice of cut-off radius will make the local strains calculated have the local nature at the mesoscopic scale.

We first consider two different schemes for the definition of the cut-off radius: Scheme 1 based on the \( \Omega_0 \) and Scheme 2 based on the \( \Omega_1 \), as illustrated in Fig. 1. After discussion, an optimal choice of the cut-off radius, denoted as Scheme 3, will be suggested. In Scheme 1, the neighboring nodes of any node are determined in the \( \Omega_0 \) with a cut-off radius and so they remain unchanged during deformation. In Scheme 2, the neighboring nodes of any node are determined by the two schemes with a certain cut-off radius are the same. The effect of the cut-off radius on the local nature and accuracy of local strain will be investigated in Section 3.

### 2.3. Local engineering strain and strain field

Suppose that the deformation is considered in a rectangular Cartesian coordinate system \((X_1, X_2, X_3)\), the diagonal terms of Lagrangian strain tensor \( \mathbf{E} \) at a given material point, \( E_{aa} \) \((a = 1, 2, 3)\), which are known as the normal strains, have a relationship with the stretch of a curve element in the direction of \( X_a \) at the given point (Reddy, 2008):

\[
E_{aa} = \frac{1}{2} (\lambda_a^2 - 1),
\]

where \( \lambda_a \) is defined as the ratio of deformed length of the curve element to its original length. Thus, the normal components of the local engineering strains, representing the ratio of the change in the length of a curve element to its original length, at a given material point are given by

\[
e_{aa} = \sqrt{1 + 2E_{aa}} - 1.
\]

In this study, we will only consider \( e_{11} \), which is the local engineering strain in the loading direction of cellular specimens used later. To treat the cellular specimens as continuum, a numerical scheme performing scattered data interpolation based on an underlying Delaunay triangulation is then used to achieve continuous strain field from the data of discrete strains.

### 3. Effect of the cut-off radius on the local strain

The cut-off radius, \( R_c \), determines the size of the local domain over which the local strain at a node characterized by enough neighboring nodes. On the other hand, it should be small enough to satisfy the local nature of strain. In this section, we first conduct a parametric study of the effect of the cut-off radius defined in the two schemes on the local strain. Then, an optimal choice of the cut-off radius is suggested and the accuracy of the measurement is discussed.

Voronoi honeycombs constructed by 2D random Voronoi technique (Zheng et al., 2005) with cell irregularity of 0.6 and relative density of 0.1 were taken as examples to demonstrate the local strain calculation in cellular materials, as illustrated in Fig. 2. The Voronoi honeycomb specimen was supported by a fixed rigid plate at one end and crushed by a rigid plate with a constant velocity \( V \) at the other end. The length along the loading direction \((X_1)\) and the width in the transverse direction \((X_2)\) of the specimen constructed with 1400 nuclei are \( L = 242.49 \text{ mm} \) and \( W = 240 \text{ mm} \), respectively. The average cell size of the specimen, which is
defined as the diameter of a circle whose area is equal to the average area of Voronoi honeycomb cells, is \( d = 2r = 7.28 \) mm, where \( r \) is the average cell radius. The cell-wall material was assumed to be elastic, perfectly plastic with Young’s modulus 66 GPa, Poisson’s ratio 0.3, yield stress 175 MPa and density \( \rho_x = 2700 \) kg/m\(^3\). Numerical simulations were performed by using finite element code ABAQUS/Explicit. The cell walls of specimens were modeled with S4R (4-node doubly curved, reduced integration) shell elements as used in Zheng et al. (2005, 2013), of which the size was set to be about 0.6 mm in plane and 1 mm out of plane through a mesh sensitivity analysis. General contact with slight friction was defined to consider all possible contacts, as done in Zheng et al. (2005). All the nodes were constrained in the out-of-plane direction to simulate an in-plane strain state. Thus, the strain field considered later is two-dimensional (2D).

Local compressive engineering strain in the loading direction, \( \varepsilon_{11} \) (taken as positive in compression hereafter), is calculated and employed to investigate the effect of the cut-off radius. To quantitatively demonstrate the suitability of the strain field calculation method to capture the heterogeneous deformation in cellular materials, the accuracy of the calculated local strain should be verified. The direct verification of a local strain is infeasible due to the lack of a reference quantity. Here, we carry out the verification through comparing the following two strains. One is the nominal strain, \( \varepsilon_{\text{nom}} \), which is defined as

\[
\varepsilon_{\text{nom}} = \frac{\Delta L}{L},
\]

where \( \Delta L \) is the total crushing of the specimen and \( L \) is its original length. The other is the average strain, \( \varepsilon_{\text{avg}} \), determined from the strain field, as defined below. The 1D distribution of local engineering strain in the loading direction, \( \varepsilon_1(X_1) \), can be obtained by averaging \( \varepsilon_{11} \) in the 2D strain field along the transverse direction \( X_2 \) axis as

\[
\varepsilon_1(X_1) = \frac{1}{W} \int_0^W \varepsilon_{11}(X_1, X_2) \, dX_2.
\]

The average strain in the loading direction, \( \varepsilon_{\text{avg}} \), is an average of \( \varepsilon_1(X_1) \) along the loading direction and can be determined by

\[
\varepsilon_{\text{avg}} = \frac{1}{L} \int_0^L \varepsilon_1(X_1) \, dX_1.
\]

By taking \( \varepsilon_{\text{nom}} \) as a reference quantity, the accuracy of the calculated local strain can be estimated by the relative error between \( \varepsilon_{\text{avg}} \) and \( \varepsilon_{\text{nom}} \), given by

\[
\delta = \left| \frac{\varepsilon_{\text{avg}} - \varepsilon_{\text{nom}}}{\varepsilon_{\text{nom}}} \right|.
\]

The deformation of Voronoi honeycombs under uniaxial constant-velocity compression is complex; however, it can be classified into three modes according to the impact velocity, i.e. the Homogeneous Mode, the Transitional Mode and the Shock Mode (Liu et al., 2009). Some deformation patterns at the impact velocities of 1, 40 and 100 m/s are shown in Fig. 2. To ensure that the local strains are correctly calculated at different loading rates, the effect of the cut-off radius on the local strain in all the three deformation modes is considered.

3.1. Scheme 1: Cut-off radius defined based on the reference configuration

Consider the cut-off radius is defined based on the \( \Omega_c \) as illustrated in Fig. 1a. The relationships between the average strain \( \varepsilon_{\text{avg}} \) and the cut-off radius \( R_c \) normalized with respect to the average cell radius \( r \), i.e. \( R_c/r \), at nominal strain \( \varepsilon_{\text{nom}} \) of 0.2 and 0.5 for different impact velocities are shown in Fig. 3. Similar features of the curves are found for any nominal strain and for any impact velocity. The impact-velocity independence of the relationship between \( \varepsilon_{\text{avg}} \) and \( R_c/r \) indicates that the variation of \( \varepsilon_{\text{avg}} \) with \( R_c/r \) is independent of the deformation modes. The average strain \( \varepsilon_{\text{avg}} \) generally increases with increasing \( R_c/r \) and asymptotes to \( \varepsilon_{\text{nom}} \) for all impact velocities. When the cut-off radius is small, say \( R_c/r < 1 \), \( \varepsilon_{\text{avg}} \) is much lower than \( \varepsilon_{\text{nom}} \) which indicates that the strain calculated over a range of one cell size produces a large underestimation. This is caused by the small number of neighboring nodes within the cut-off radius, inside which the local deformation gradient is estimated. As \( R_c/r \) increases, \( \varepsilon_{\text{avg}} \) has a significant increase until \( R_c/r \) reaches near 1.5, then it slowly gets close to \( \varepsilon_{\text{nom}} \). It is noted that \( \varepsilon_{\text{avg}} \) is always lower than \( \varepsilon_{\text{nom}} \) for a finite \( R_c/r \). To get small error between \( \varepsilon_{\text{avg}} \) and \( \varepsilon_{\text{nom}} \), the domain specified by the cut-off radius for determining neighboring nodes should be large enough. However, a too large domain, over which the local strain is calculated, will introduce severe averaging effect and thus the local strain will lose
its desired nature. Therefore, the cut-off radius plays a crucial role in calculating the local strain. From Fig. 3, in which the cut-off radius is defined based on the \( \Omega_0 \), the normalized cut-off radius \( R_c / r \) is suggested to be 1.5, with which the relative error \( \delta \) between \( \varepsilon_{\text{avg}} \) and \( \varepsilon_N \) is limited to about 10%.

### 3.2. Scheme 2: Cut-off radius defined based on the current configuration

Consider the cut-off radius is defined based on the \( \Omega_1 \), as illustrated in Fig. 1b. The relationships between the average strain \( \varepsilon_{\text{avg}} \) and the normalized cut-off radius, \( R_c / r \), at nominal strain \( \varepsilon_N \) of 0.2 and 0.5 for different impact velocities are shown in Fig. 4. Generally, \( \varepsilon_{\text{avg}} \) increases as \( R_c / r \) increases until it reaches a maximum and then decreases and asymptotes to \( \varepsilon_N \) due to few neighboring nodes and \( \varepsilon_{\text{avg}} \) with a too large \( R_c / r \) suffers severe averaging effect. However, it is noteworthy that \( \varepsilon_{\text{avg}} \) with a small \( R_c / r \) is lower than \( \varepsilon_N \) while that with a large \( R_c / r \) is higher than \( \varepsilon_N \) which is irrespective of the impact velocity and \( \varepsilon_N \). It means that there exists a critical cut-off radius with which \( \varepsilon_{\text{avg}} \) can be very close to \( \varepsilon_N \). From Fig. 4, at \( \varepsilon_N = 0.2 \), the critical normalized cut-off radius can be chosen to be \( R_c / r = 0.6 \), with which \( \varepsilon_{\text{avg}} \) agrees with \( \varepsilon_N \) to within 2.5% for all impact velocities considered; at \( \varepsilon_N = 0.5 \), the critical normalized cut-off radius is \( R_c / r = 0.4 \), with which \( \varepsilon_{\text{avg}} \) agrees with \( \varepsilon_N \) to within 1%. Nevertheless, a consistent and reliable critical cut-off radius should not be dependent upon the nominal strain. Consequently, when the cut-off radius is defined based on the \( \Omega_1 \), it is suggested that the critical normalized cut-off radius can be approximately chosen as \( R_c / r = 0.5 \), with which \( \varepsilon_{\text{avg}} \) agrees with \( \varepsilon_N \) at a relative error within 10%. Note that this critical normalized cut-off radius is not very large, which ensures the local nature of the local strain.

### 3.3. Local strain at individual nodes

The deformation of cellular materials is heterogeneous due to the inherent cellular nature, as the deformation of Voronoi honeycomb specimen shown in Fig. 2. This heterogeneous nature of deformation requires that the local strain should satisfy its local nature when it is applied to characterize the deformation. To gain a further understanding about the local nature of the local strain, a parametric study of the effect of the cut-off radius on the local strain calculation at individual nodes is conducted. As the deformation pattern at nominal strain \( \varepsilon_N = 0.2 \) at impact velocity \( V = 1 \) m/s shown in Fig. 2, there are three kinds of representative nodes—nodes located in a large strain region (in a crushing band), nodes...
located in a small strain region (far away from the crushing bands) and nodes located near a large strain region (neighboring to a crushing band). Thus, nodes A, B and C are selected as representative nodes for parametric studies, see Fig. 5 for their locations. The variations of the local engineering strain $\varepsilon_{11}$ at nodes A, B and C with the normalized cut-off radii defined in Schemes 1 and 2 are shown in Fig. 6.

At node A (located in a crushing band, see Fig. 5b), the local engineering strain $\varepsilon_{11}$ generally first increases with the increase of $R_c/r$ as shown in Fig. 6. Similar behavior is observed in the relationship between $\varepsilon_{11}$ and $R_c/r$ in Scheme 2. However, it is noted that the local strain $\varepsilon_{11}$ at node A reaches a high value even with a small $R_c/r$ in Scheme 2. For example, $\varepsilon_{11}$ with $R_c/r = 0.5$ in Scheme 2 reaches the level of $\varepsilon_{11}$ with $R_c/r = 1.5$ in Scheme 1. The nodes in the neighborhood of node A involved in the local strain calculation with different schemes and $R_c/r$ ratios provide visual explanation for the variations of the local strain, as illustrated in Fig. 7. In Scheme 2, the local strain at node A is primarily characterized by nodes located in the large strain region even with the smallest considered $R_c/r$ as indicated in Fig. 7b1, therefore it reaches a high value. As $R_c/r$ increases, additional nodes, including nodes located in the large strain region and those located in the relatively smaller strain region, make contributions. Note that the number of the former is greater than that of the latter as seen in Figs. 7b1–b3, and therefore the local strain continues to increase as the cut-off radius increases. In Scheme 1, however, some nodes located in the relatively smaller strain region are involved in the calculation, while those located in the large strain region are not when $R_c/r$ is not large enough, see Figs. 7a1–a3. Consequently, the cut-off radius defined in Scheme 2 is able to collect effective nodes located in a region of comparable strain to calculate the local strain. It indicates that the local strain calculated with Scheme 2 satisfies the local nature better than that calculated with Scheme 1. In Scheme 1, as $R_c/r$ increases from 1.5 to 2.8, it is evident that many additional neighboring nodes, which are located in a small strain region, are added in the calculation of the local strain at node A, see Figs. 7a3–a4. As a result, the local strain decreases. It is clearly demonstrated that the local strain calculated with a too large cut-off radius cannot accurately characterize the local deformation since it loses its local nature. Similar behavior is also observed in Scheme 2, as shown in Figs. 7b3–b4.

At node B (located far away from the crushing bands, see Fig. 5b), the local strain $\varepsilon_{11}$ first increases with the increase of $R_c/r$, and then decreases and finally increases slowly in both schemes, as shown in Fig. 6. When the cut-off radius $R_c$ is small, the local strains calculated with the two schemes are consistent.

This is because the deformation around node B is small so the effective nodes collected for calculating the optimal local deformation gradient by the two schemes are almost the same, as illustrated in Figs. 8a1–a2 and b1–b2. As $R_c$ continues to increase, the regions of larger deformation are included and so the local strains calculated with the two schemes are different, as illustrated in Figs. 8a3–a4 and b3–b4. As shown in Fig. 8b1, the local domain determined by $R_c/r = 0.5$ defined in Scheme 2, over which the effective nodes are collected, is much smaller than that suggested by Zheng et al. (2012), which is of at least one cell size. The local strain measured over this small domain may not accurately characterize the local heterogeneous deformation. As a result, the small cut-off radius defined in Scheme 2, $R_c/r = 0.5$, which is suggested in Section 3.2, may not be suitable for small deformation states. In contrast, the cut-off radius defined in Scheme 1, suggested as $R_c/r = 1.5$ in Section 3.1, is more applicable for the local strain calculation in this situation.

At node C (located near a crushing band, see Fig. 5b), two stages in the local strain $\varepsilon_{11}$ vs. normalized cut-off radius $R_c/r$ curves, i.e. increasing stage and decreasing stage, are also observed in both schemes, as shown in Fig. 6. However, $\varepsilon_{11}$ calculated with Scheme 2 suffers a rapid change once $R_c/r$ exceeds a certain value (say 0.4), while $\varepsilon_{11}$ calculated with Scheme 1 increases much slowly in the increasing stage. As illustrated in Fig. 9b, the local strain at node

---

**Fig. 5.** Locations of nodes A, B and C in (a) the reference configuration and (b) the current configuration corresponding to nominal strain of 0.2 and impact velocity of 1 m/s.

**Fig. 6.** Variations of the local engineering strain $\varepsilon_{11}$ at nodes A, B and C with the normalized cut-off radii $R_c/r$ defined in Schemes 1 and 2, when the nominal strain is 0.2 and the impact velocity is 1 m/s.
C in Scheme 2 is primarily characterized by nodes located in a small strain region with $R_c/r < 0.4$, see Fig. 9b1; while significant contributions are made by nodes located in the large strain region as $R_c/r$ increases to 0.5, as seen in Fig. 9b2, which results in a steep rising in the local strain. In Scheme 1, the effective nodes collected for calculating the optimal local deformation gradient at node C are determined in the $X_0$. Therefore, as the cut-off radius $R_c$ increases the nodes that are near to node C are collected firstly, as shown in Figs. 9a1–a4, while the nodes located in the large strain region, which are located far from node C in the $X_0$, are collected only when $R_c$ is large enough. Thus, the local strain at node C calculated with Scheme 1 increases much slowly with the increase of $R_c/r$, compared with that calculated with Scheme 2.

In Scheme 2, the cut-off radius $R_c$ has a great effect on the accuracy of local strain at a node neighboring to a large strain region (e.g. node C), which is primarily characterized by nodes located in the large strain region once $R_c/r$ exceeds a certain value and therefore the local strain is overestimated. This is probably the reason why $\varepsilon_{avg}$ with a relatively large $R_c/r$ is higher than the nominal strain $\varepsilon_{nom}$ observed in Fig. 4. In view of this point, it is expected that a more uniform local deformation results in a smaller degree of overestimation of $\varepsilon_{avg}$. The deformation patterns given in Fig. 2 show that the deformation, at the same nominal strain, tends to localize closer to the impact end at a higher impact velocity, which leads to a more uniform local deformation in both the large deformation region and the small deformation region. This deformation...
The local deformation of a cellular material at the mesoscopic scale is irregular and non-uniform due to the inherent cellular nature. This deformation feature brings difficulty in determining the local strains. The two schemes mentioned above have shown that a small cut-off radius leads to a lack of accuracy of the local strains and a large cut-off radius makes the local strains lose the local nature. Thus, the local strain at a node should be characterized by enough neighboring nodes with a suitable cut-off radius. In scheme 2, the accuracy of the average strain during deformation compared to the nominal strain is found to be slightly sensitive to the cut-off radius. For practical purpose, we expect that the cut-off radius for each scheme is a constant no matter with the nominal strain and the impact velocity. There exists a critical cut-off radius for each scheme with which the average strain is in reasonable agreement with the nominal strain.

With the understandings of the effect of cut-off radius defined in the two schemes on the local strains, we suggest that in Scheme 1, the normalized cut-off radius \( R_c/r \) is about 1.5, while in Scheme 2, \( R_c/r = 0.5 \) seems to be a reasonable choice to satisfy the local nature, although suffered a limitation in small deformation states. Consequently, it is expected that a combination of the two schemes may include enough neighboring nodes and satisfy the local nature of the local strain. Thus, we suggest a combination scheme, denoted as Scheme 3, that the neighboring nodes, which characterize the local strain at any node, consist of the two parts determined by Scheme 1 with \( R_c/r = 1.5 \) in the \( \Omega_0 \) and by Scheme 2 with \( R_c/r = 0.5 \) in the \( \Omega_1 \) but removing duplicates. In other words, a set made up by the neighboring nodes in Scheme 3 is the union of the two sets of nodes from Schemes 1 and 2. The relative error \( \delta \) between the average strain \( \varepsilon_{avg} \) calculated with Scheme 3 and \( \varepsilon_N \) is shown in Fig. 10. It is clearly demonstrated that most of the average strains agree with the corresponding nominal strains to within 5% for the range of impact velocities considered. Therefore, Scheme 3 indeed characterizes the local strain reasonable well, and can be employed to conduct studies in cellular materials.

The distributions of local engineering strain \( \varepsilon_l \) calculated with three schemes in the loading direction for an impact velocity of 100 m/s are shown in Fig. 11. They clearly characterize the deformation of the Voronoi honeycomb specimen in the Shock Mode, as illustrated in Fig. 2. It is evident that there exists a steep drop ("discontinuity") in the strain distribution, which separates the compacted region of large strain and the uncompacted region of small strain. The slope of the strain "discontinuity" calculated with...
Scheme 2 is steeper than that with Scheme 1. The relative error \( \delta \) between the average strain \( \varepsilon_{\text{avg}} \) calculated with Scheme 1 and the nominal strain \( \varepsilon_N \) is 12.1%, while that between \( \varepsilon_{\text{avg}} \) calculated with Scheme 2 and \( \varepsilon_N \) is only 1.7%. Note that the primary difference between the distributions of strain calculated with Schemes 1 and 2 lies in the strain calculation in the compacted region, as shown in Fig. 11. These results mean that Scheme 2 with \( R/c = 0.5 \) is more suitable for the local strain calculation in a large strain region, ensuring both accuracy and local nature, while Scheme 1 with \( R/c = 1.5 \) is more suitable for a small strain region, ensuring a larger number of neighboring nodes, which is reflected in the strain calculation in the uncompacted region. Scheme 3 takes full advantage of Scheme 2 in a large strain region and Scheme 1 in a small strain region, as shown in Fig. 11, and therefore it characterizes the local strain better than Schemes 1 and 2, obtaining a smallest relative error of 0.8% between \( \varepsilon_{\text{avg}} \) and \( \varepsilon_N \). It is also worth noting that \( \varepsilon_{\text{avg}} \) calculated with Scheme 1 is lower than \( \varepsilon_N \) and \( \varepsilon_{\text{avg}} \) calculated with Schemes 2 and 3 are slightly higher than \( \varepsilon_N \).

As stated previously, local strains at corner nodes rather than at all nodes are taken as samples to discretely represent the interior strain of a Voronoi honeycomb to save the computational cost. To examine the influence of this simplification, the relative error \( \delta \) between the average strain \( \varepsilon_{\text{avg}} \) calculated from local strains at all nodes with Scheme 3 and the nominal strain \( \varepsilon_N \) is given in Fig. 12. It is observed from Fig. 12 that most of the average strains calculated from local strains at all nodes agree with the corresponding nominal strains to within 5% for different impact velocities; similarly in Fig. 10 with local strains defined at corner nodes. Therefore, it is reasonable to adopt the local strains at corner nodes as representatives of the interior strains.

### 4. Application

Cellular materials have been widely used as protective structures since they absorb energy and attenuate impact/blast loads at an approximately constant stress level. In the application, a cellular material is usually employed as a core and sandwiched by two cover plates to construct a sacrificial cladding. Due to the limitations of single-layer foam cladding in finite foam layer thickness, plateau stress and densification strain, a double-layer foam cladding has been suggested by Ma and Ye (2007) and its energy absorption capacity under blast load has been analytically investigated based on an idealized foam model – a rate independent, rigid–perfectly plastic–locking idealization (Reid and Peng, 1997). It was found that the propagation of the shock wave in the double-layer foam cladding has a direct effect on the energy absorption capacity (Ma and Ye, 2007). To provide a further understanding of the dynamic response of a double-layer foam cladding, the heterogeneous deformations of a double-layer cellular cladding under blast load are characterized in this section by using the strain field calculation method for local strain characterization with the optimal choice of the cut-off radius presented in Section 3.4.

Voronoi honeycombs are used to model the cellular cores in a double-layer cellular cladding, which is supported by a fixed surface at one end and subjected to a blast load in the \( X_1 \) direction at the other end, as illustrated in Fig. 13a. The proximal layer and the distal layer, each of which has a cellular core and a cover plate, are assumed to be bonded perfectly. A Voronoi honeycomb is constructed in an area of 263.27 \( \times \) 132 mm\(^2\) with 836 nuclei. It is employed to produce two honeycomb blocks with different relative densities by changing the cell-wall thickness, i.e. Block 1 has a relative density of \( \rho_1 = 0.10 \) and Block 2 has \( \rho_2 = 0.15 \). The cell-wall material of the two blocks and mesh density used in the numerical simulations are the same as that used in Section 3. The quasi-static nominal stress–strain curves of the two honeycomb blocks are shown in Fig. 13b, indicating that both of them exhibit strain hardening. The yield stress (marked with an empty circle in Fig. 13b) and plateau stress defined in Tan et al. (2005) of Block 1 are \( \sigma_y = 0.72 \) MPa and \( \sigma_y = 0.93 \) MPa, and those of Block 2 are \( \sigma_y = 1.68 \) MPa and \( \sigma_y = 2.09 \) MPa, respectively. Both the cover plates have a mass of \( m = 0.1 \) g and are modeled as rigid bodies. The blast load is simplified as a triangular pressure pulse as done in Hansen et al. (2002), Ma and Ye (2007) and Main and Gazonas (2008). In this paper, the initial peak pressure of the triangular pulse is taken to be 15 MPa, which is 16 and 7 times the plateau stress of Blocks 1 and 2, respectively. The duration of the blast load is 1 ms. Two different configurations of double-layer cellular cladding, denoted as C1-2 and C2-1, are investigated. C1-2 has Block 1 in the proximal layer and Block 2 in the distal layer while C2-1 has Block 2 in the proximal layer and Block 1 in the distal layer.

The time histories of stress on the support surface for C1-2 and C2-1 are shown in Fig. 14. For both configurations, the impact energy is completely absorbed by the two Voronoi honeycomb cores.
Clearly, the applied blast force is attenuated to a much lower stress level on the support surface, which leads to potential applications of the double-layer cellular cladding in protective structures. From a design perspective, C2-1 tends to be more suitable for protective structures compared with C1-2 because the pressure transferred to the support surface (protected structure) in C2-1 remains at a low level for most of the time. Ma and Ye (2007) reported that the blast resistant capacity of double-layer cellular cladding like C2-1 was smaller than that like C1-2 because the proximal layer in the former was not fully utilized. It is notable that the thicknesses of the proximal layer and distal layer in cladding like C2-1 in Ma and Ye (2007) are equal. In fact, the blast resistant capacity of cladding like C2-1 can be improved through an optimal design of the thicknesses of the proximal layer and distal layer. This optimal design issue may be solved in the future.

The deformation patterns and corresponding local engineering strain fields of C1-2 and C2-1 are shown in Figs. 15 and 16, respectively. Under the blast load, deformation localization is clearly demonstrated in the local engineering strain fields of both the clad- dings. The plastic strain in the localization region reaches to a much higher level than that in the rest of the cladding. C1-2 deforms progressively from the proximal layer to the distal layer like a propagation of a shock front, across which there is a steep drop in strain as clearly observed in the local engineering strain fields in Fig. 15. Due to the much different densification behavior of Blocks 1 and 2, there is also an obvious steep drop in strain between the proximal layer and the distal layer. For C2-1, the localization of large plastic strain also occurs first in the proximal layer, though the plateau stress of Block 1 in the distal layer is lower than the counterpart of Block 2 in the proximal layer. At the early stage, the localization region propagates through the proximal layer in a ‘shock’ like manner while little deformation is observed in the...
distal layer. However, before the shock front in the proximal layer reaches the distal layer, large plastic strain in the shear crushing bands occurs in the distal layer, see the pattern and the local engineering strain field at $t = 1.8$ ms in Fig. 16. It is worth noting that the localization of large plastic strain in the distal layer of C2-1 initially is not in the manner of a shock-type, but interestingly, it eventually develops to be like a shock front, see the local engineering strain field at $t = 2.85$ ms in Fig. 16. Thereafter, little deformation is observed in the proximal layer and the distal layer tends to be completely compacted earlier than the proximal layer, see the local engineering strain field at $t = 6.0$ ms in Fig. 16.

The distributions of local engineering strain $e_{11}$ of C1-2 and C2-1, plotted against the Lagrangian location $X_1$, are shown in Figs. 17 and 18, respectively. A nearly step change in the compression strain is observed for both the claddings. For C1-2, the position of the step change propagates through the cladding from the proximal layer to the distal layer, which clearly demonstrates the propagation of the shock front. The strain distribution in the proximal layer stays unchanged once the shock front has reached the distal layer, see Fig. 17 for $t \geq 1.95$ ms. Before the plastic shock front reaches the distal layer, the stress on the support surface reaches approximately twice as large as the yield stress of Block 1 due to

Fig. 15. Deformation patterns (left) together with their corresponding local engineering strain fields of $e_{11}$ (right) of C1-2 at different times.

Fig. 16. Deformation patterns (left) together with their corresponding local engineering strain fields of $e_{11}$ (right) of C2-1 at different times.
unchanged, as seen in Fig. 18 for 1.8
the distal layer, the strain distribution in the proximal layer stays
proximal layer while localization in the distal layer develops to
imal layer eventually vanishes before it reaches the end of the
increases as the shock front propagates through the proximal layer,
develops in the distal layer near the cover plate and its magnitude
the case in C1-2. However, as the time increases, plastic strain
of Block 2, as shown in Fig. 14 at
of the thicknesses of cellular cores (Liao et al., 2013b).
These mechanistic insights provide further understanding in the
gate in C2-1: one is also due to inertia and the other is mainly
gates in C1-2, which is due to inertia. Double shock fronts propa-
results interpret the shock wave propagation mechanisms
in the two configurations of cladding. A single shock front propa-
gate relative densities are used as cores in the double-layer cellular
The effects of the cut-off radius with respect to different configura-
ations on the local nature and accuracy of the local strain in the
loading direction are investigated by characterizing the heteroge-
neous deformation of Voronoi honeycombs under uniaxial compression with different impact velocities. In Scheme 1, the
average strain obtained from the strain field approaches to the
nominal strain with the increase of the cut-off radius, but when
the normalized cut-off radius $R_c/r$ is much larger than 1.5 the local
nature of the ‘local’ strain is lost. In Scheme 2, the average strain
first increases and then decreases as approaching to the nominal
strain with the increase of cut-off radius, and when the normalized
cut-off radius $R_c/r$ is about 0.5, the average strain is approximate to
the nominal strain. To ensure the local nature of the ‘local’ strain at
the mesoscopic scale and the accuracy of the average strain, the
normalized cut-off radius $R_c/r$ in Scheme 1 is suggested to be 1.5
and that in Scheme 2 to be 0.5. And then, an optimal choice of the
cut-off radius by combining Schemes 1 and 2, denoted as
Scheme 3, is strongly suggested to make the local strain reasonable
and effective whether the compression rate is low or high. It
should be noted that the critical cut-off radius is probably prob-
lem-dependent. The suggestions in the present paper are mainly
for the type of problem about uniaxial compression/impact of 2D
cellular materials. The choice of other type of problems can be
obtained through similar analysis. A self-adaptive choice of cut-off ra-
dius may be worthy of future study.
The strain field calculation method with the optimal choice of
the cut-off radius in Scheme 3 is applied to characterize the defor-
mation of two different configurations of double-layer cellular
cladding under a linear decaying blast load. Two Voronoi honey-
comb blocks of different plateau stress levels with respect to differ-
ent relative densities are used as cores in the double-layer cellular
claddings. The 2D fields and the 1D distributions of local engineer-
ing strain are calculated for the two configurations of cladding.
These results interpret the shock wave propagation mechanisms
in the two configurations of cladding. A single shock front propa-
gates in C1-2, which is due to inertia. Double shock fronts propa-
gate in C2-1: one is also due to inertia and the other is mainly
caused by the strength difference between the two cellular cores.
These mechanistic insights provide further understanding in the
design of a double-layer cellular cladding, e.g. the optimal design
of the thicknesses of cellular cores (Liao et al., 2013b).

5. Conclusions
The optimal local deformation gradient technique is introduced
to develop a strain field calculation method for determining the
‘local’ strain in a cellular material. Two different schemes for the
definition of the cut-off radius involved in this strain field calcula-
tion method are first presented: Scheme 1 based on the reference
configuration and Scheme 2 based on the current configuration.
The effects of the cut-off radius with respect to different configura-
tions on the local nature and accuracy of the local strain in the
loading direction are investigated by characterizing the heteroge-
neous deformation of Voronoi honeycombs under uniaxial compression with different impact velocities. In Scheme 1, the
average strain obtained from the strain field approaches to the
nominal strain with the increase of the cut-off radius, but when
the normalized cut-off radius $R_c/r$ is much larger than 1.5 the local
nature of the ‘local’ strain is lost. In Scheme 2, the average strain
first increases and then decreases as approaching to the nominal
strain with the increase of cut-off radius, and when the normalized
cut-off radius $R_c/r$ is about 0.5, the average strain is approximate to
the nominal strain. To ensure the local nature of the ‘local’ strain at
the mesoscopic scale and the accuracy of the average strain, the
normalized cut-off radius $R_c/r$ in Scheme 1 is suggested to be 1.5
and that in Scheme 2 to be 0.5. And then, an optimal choice of the
cut-off radius by combining Schemes 1 and 2, denoted as
Scheme 3, is strongly suggested to make the local strain reasonable
and effective whether the compression rate is low or high. It
should be noted that the critical cut-off radius is probably prob-
lem-dependent. The suggestions in the present paper are mainly
for the type of problem about uniaxial compression/impact of 2D
cellular materials. The choice of other type of problems can be
obtained through similar analysis. A self-adaptive choice of cut-off ra-
dius may be worthy of future study.
The strain field calculation method with the optimal choice of
the cut-off radius in Scheme 3 is applied to characterize the defor-
mation of two different configurations of double-layer cellular
cladding under a linear decaying blast load. Two Voronoi honey-
comb blocks of different plateau stress levels with respect to differ-
ent relative densities are used as cores in the double-layer cellular
claddings. The 2D fields and the 1D distributions of local engineer-
ing strain are calculated for the two configurations of cladding.
These results interpret the shock wave propagation mechanisms
in the two configurations of cladding. A single shock front propa-
gates in C1-2, which is due to inertia. Double shock fronts propa-
gate in C2-1: one is also due to inertia and the other is mainly
couced by the strength difference between the two cellular cores.
These mechanistic insights provide further understanding in the
design of a double-layer cellular cladding, e.g. the optimal design
of the thicknesses of cellular cores (Liao et al., 2013b).

Acknowledgements
This work is supported by the National Natural Science Founda-
tion of China (Projects Nos. 11002140, 90916026, 11372308 and
10932011) and the Fundamental Research Funds for the Central
Universities (WK2090050023).

References
Bastawros, A.F., Bart-Smith, H., Evans, A.G., 2000. Experimental analysis of
Solids 48, 301–322.
enhancement of cellular structures under impact loading: Part I Experiments. J.


