Autapse is an unusual type of synapse generated by a neuron on itself. The effect of autapse connected to a neuron is often described by using a self-feedback forcing current in a close loop, and the electric activities of neuron can be regulated by the autapse greatly. Generally, positive feedback in the autapse can excite the quiescent neuron while negative feedback often calms down the excitable neuron. In this paper, the Hindmarsh–Rose neuron model is used to define the local kinetics of each node in the neuronal network, and the distribution of autapses in the network is considered to investigate the emergence of emitting wave induced by autapse (in electrical type) with negative feedback. In the case of ring network, it is found that pulse can be blocked by the neurons with negative feedback in autapse, and the pulse also can keep alive in the ring network stably under appropriate coupling intensity. Furthermore, target wave can be induced in the two-dimensional square array, and the nodes adjusted by negative electrical feedback type in autapses can emit target-like waves to regulate the collective behaviors of neurons, this is a new type of wave formation results from diffraction. It concludes that local distribution of autapse with negative feedback type can generate ‘defects’ in the network, the diversity in excitability accounts for the emergence of emitting wave from these defects. Finally, the functional switch between negative and positive feedback in autapse is discussed, it is claimed that positive feedback autapse plays an important role in cheering up quiescent neurons, while the negative feedback in electric autapse can contribute to slow down the excitable neurons.

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noise and external noise also plays an important role in regulating the electrical activity of a single neuron, or neurons in a network, for example, Ref. [36] investigated the transition of spiral wave in neuronal networks induced by channel noise. The channel noise can be altered when the conductance of ion channels is changed and this can result in a loss of stable behavior due to blocking in ion channels [37]. One may then ask, why investigate spiral waves in neuronal networks? In fact, some experimental results have confirmed that spiral wave can emerge in the cortex of brain, and these spiral waves seem to regulate the collective behaviors of neurons as a pacemaker [38], as a result, Refs. [34–37] presented detailed discussion about the potential formation mechanism for spiral wave (one arm or multi-armed type), and transition of ordered wave induced by changeable connection probability in the network, and their previous results claimed that ion channel blocking in some neurons may account for the emergence of spiral wave in the network.

As we know well, target wave can occur in a reaction–diffusion system and a two-dimensional array under local heterogeneity or external periodical forcing in local area, and the continuous emitting wave can regulate the collective behavior of the media or coupled oscillators. Indeed, spiral wave also can be generated from heterogeneity after collision between ordered waves, for example, Ref. [39] reported the formation of spiral wave from fractal heterogeneity, Ref. [40] suggested that the emitting wave from heterogeneity induced by rotating electric field can be used to remove spatiotemporal turbulence in the media. Ref. [41] reported that spiral wave can also be induced in subexcitable media by applying external electric field.

In a neuronal system, information can be transmitted between neurons via synapses, and the electrical activity of a given neuron can be changed by other neurons that synapse onto it. Furthermore, the electrical behavior of some neurons can be heavily affected by the presence of autapses, which is an unusual type of synapse generated by a neuron on itself [42–47]. Some researchers suggested that the effect of autapse on electric activities of neuron can be described by inputting additive loop current with time delay similar to the external forcing current on the neuron [48,49], as a result, a positive feedback with time delay can excite the quiescent neuron to become a continuous oscillator and generate different types of electric activities such as spiking, bursting and chaotic states. Particularly, Wang et al. [50] investigated the transition of activities in Hindmarsh–Rose neurons that had three different types of autapses by using bifurcation analysis and calculating distribution of information entropy. In fact, time delayed-feedback (negative feedback) can suppress the chaotic state in oscillator [51] and spiral wave in the media [52]. For spiking or bursting states in neurons, autapses with negative time delayed-feedback can stabilize the neuron to step into quiescent state, and hence it is interesting to investigate the case when autapses are considered in a local area in the network (only a fraction of neurons are connected to autapse not all neurons are connected to autapses). Some previous results have confirmed that continuous pulse or target wave or spiral wave can be developed in the network when some autapses connected the neurons are in positive time delayed-feedback type [53,54]. As mentioned in Refs. [55], autaptic modulations can shape the bursting behavior of biological neurons, and they confirmed that synaptic delays have no obvious effects in the case of autaptic-excitation, while a subtle effect of synaptic delays was observed in the case of autaptic-inhibition. Ma et al. [56] also confirmed that the squid giant synapse and the calyx of Held can be useful to introduce reagents into their large presynaptic terminals, and one potential function (or role) of the hippocampal neuron autapse could be it changes a protein level by exogenous DNA or RNA. More importantly, Connelly [57] gave some new evidences to detect potential functional role of autapse by extending the Wang–Buzsáki model of gamma oscillations, and it found that autapses increased the synchrony of basket cell membrane potentials across the network during neocortical gamma oscillations as well as allowed the network to oscillate over a broader range of depolarizing drive. As mentioned above, the response of neuronal activity is much dependent on the feedback type (negative or positive) in autapse, and the transition of dynamical properties could be interesting. For an isolated neuron, a switch selection between positive and negative feedback in autapse make neuron become excitable or quiescent. As a result, transition of spatial pattern occurs when collective electrical activities of neuronal network are detected. Therefore, the development and transition of electric activities of neurons in network can be more reliable to give useful clues to understand the neuronal activities and response to external forcing. We argue that negative time delayed-feedback in autapse connected to some neurons in a local area of the network may generate ‘defects’ in the network, and ordered wave can also emit from these defects to regulate the collective behaviors of neurons. For simplicity, the Hindmarsh–Rose neuron model is used to describe the local kinetics of nodes in the neuronal network with nearest-neighbor connection.

2. Model and scheme

The dynamics of the ring network is described by

\[
\begin{align*}
\frac{dx_i(t)}{dt} &= a x_i(t) - x_i(t) + b x_i(t)^3 - z_i(t) + I_{\text{ext}} + I_{\text{aut}} + D (x_{i-1}(t - \tau_1) + x_{i-1}(t - \tau_1) - 2x_i(t)) \\
\frac{dy_i(t)}{dt} &= c - d x_i(t)^2 - y_i(t) \\
\frac{dz_i(t)}{dt} &= r [s(x_i(t) - x_0) - z_i(t)]
\end{align*}
\]

(1)

where the subscript \(i\) denotes the node position in the network in regular type, \(D\) represents the coupling intensity between adjacent neurons, \(\tau_1\) is the time delay when signal propagates from one neuron to another. \(I_{\text{ext}}, I_{\text{aut}}\) defines the external forcing current and autaptic current, respectively. The definition and interpretation for the parameters and variables can be found in detail in Refs. [2,18,50] and references therein. Generally, parameters are often selected as \(a = 1.0, b = 3.0, c = 2.0,\)
where the subscripts \((ij)\) denote the node position in the network, \(x_{ij}, y_{ij}, z_{ij}\) represents the membrane potential, recovery variable, slow adaption current in node \((i,j)\), and \(D\) also defines the coupling intensity between neurons, \(I_{ext}\) still means the external forcing current, the autaptic current due to the effect of autapse (with electric type) is often described as follows

\[
I_{aut} = g\left(\frac{x_{ij}(t)}{C_0} - x_{ij}(t)\right), \quad \text{for two-dimensional array, } (i,j) \text{ is node position}
\]

where \(g\) is the gain for autapse, \(\tau\) is the time delay associated in the loop to the neuron itself, for a view of autapse structure; readers refer to Refs. [42, 48]. For an isolated neuron, negative feedback is imposed on the membrane potential by selecting positive value for gain \(g\); while positive feedback occurs by giving negative value for gain \(g\). As reported in Refs. [48, 50] positive feedback in autapse is effective to exciting quiescent neurons. Herein, we tend to discuss the case for negative feedback in the autapse (with electric type) connected to neuron and a fraction of neurons in the ring network, square array type network. In the case of ring network, 100 neurons are used to construct the network with nearest-neighbor connection type and the pulse propagation along the ring network is investigated. In the two-dimensional space, 200 \(\times \) 200 neurons are placed uniformly on the nodes in a two-dimensional array with regular connection type, and the selection of spiral wave is detected.

3. Numerical results and discussion

In the numerical studies, the fourth order Runge–Kutta algorithm is used for calculating the dynamic equation of single neuron, while Euler forward difference algorithm is used to calculate the dynamics equations for ring network and a two-dimensional array network with no-flux boundary condition being used. The time step is about \(h = 0.01\), the initial values for all neurons are selected as \((3.0, 0.3, 0.1)\) for a quiescent state, the transient period for calculating is about 5000 time units. Differs from the previous works, we investigate the case for negative feedback in the autapse. At first, a numerical example is presented to illustrate the stabilization effect on electrical activity of neuron induced by negative feedback in the autapse, and the results are shown in Fig. 1.

The results in Fig. 1 confirm that a quiescent neuron can be excited and step into spiking state under positive feedback in the autapse, while a negative feedback in the autapse can drive the neuron to become stable state within short transient periods.
period. Furthermore, an active excitable neuron can also be suppressed by negative feedback in autapse. Extensive numerical results confirm that excitable or oscillating neuron can also be stabilized successfully by negative feedback in the autapse with increasing the feedback gain. It is more important to investigate the similar case in the network of neurons.

It is understood that positive feedback in autapses in a local area of a two-dimensional array network can introduce heterogeneity, thus target-like ordered wave can emit from this local area. Furthermore, the local distribution of autapse with positive feedback plays like a pacemaker and can generate continuous traveling wave to regulate the collective behaviors of neurons. However, the negative feedback in autapses connected to neurons in the network used to stabilize the electric activities of neurons, as a result, a fraction of neurons in a local area of the network can be stabilized by negative feedback in the autapse, and thus the stabilized area can regulate the collective behaviors of neurons like defects in the network. For simplicity, we call the area driven by autapses with negative feedback type as 'defect area'. In the case of ring network, 100 neurons are used for numerical studies, and autapse are considered on the neurons for nodes \(55 \leq i \leq 60\) and external forcing current is selected as \(I_{\text{ext}} = 1.0\) for these neurons, while the rest neurons are imposed the same forcing current \(I_{\text{ext}} = 1.3\). As a result, negative feedback in the local area leads to 'death' or sleep of neuron thus a defect area emerges in the ring network to block the wave propagation along the ring network, numerical results are plotted in Fig. 2.

The results in Fig. 2 show that pulses in the ring network are blocked by the 'defect area' and neurons in the 'defect area' keep quiescent greatly when the coupling intensity is low. With increasing the intensity of coupling between adjacent neurons, the size for the 'defect area' becomes large and expands along the ring network, more and more neurons are suppressed to become quiescent states. The transient period for suppressing the excitable neuron can be much shorten by further increasing the intensity of coupling, thus collective electric activities of all neurons in the ring network can be stabilized. To further understand the transition of electric activities of neurons, the series of average membrane potentials of all neurons are calculated, and the results are plotted in Fig. 3.

The results in Fig. 3 confirm that most of the neurons keep spiking and robust to the invasion from the 'defect area' with weak intensity of coupling being used. However, more and more excitable neurons are slow down with the invasion from the 'defect area' due to negative feedback in the autapses, and the ring network become homogeneous greatly. To discern the effect of time delay \(\tau_1\) between adjacent neurons, we investigate the case for \(\tau_1 = 0.0\), and the results presented in Fig. 4 can make a contrast with the results in Fig. 2.

The results in Fig. 4 show that the 'defect area' can be activated and the neurons in the 'defect area' begin to change electric activities simultaneously with increasing the intensity of coupling greatly. As shown in Fig. 4(a) and (b), the 'defect area' keeps intact against the invasion from other excitable neurons in the rest area of the ring network when the coupling intensity is low. Clearly, the interaction between the 'defect area' and the other excitable neurons is much dependent on the time delay \(\tau_1\) and coupling intensity \(D\). That is to say, the switch between 'quiescent' and 'homogeneous' state in the ring network can be particularly dominated by the time delay \(\tau_1\). In fact, the excitable neurons close to the border of the 'defect area' keeps invading the quiescent neurons in the 'defect area' and the continuous pulses propagate along the ring network with high speed under stronger coupling intensity, thus the quiescent state in the 'defect area' can be waken up. Compared the results in Figs. 4 and 2, it confirms the developed states of the 'defect area' and the collective behaviors of ring network mainly are

![Fig. 2](image-url) The wave propagation along the ring network induced by negative feedback in autapses connected to neurons \((55 \leq i \leq 60)\) under different mutual coupling intensities. The gain and time delay in autapse and time delay between adjacent connection neurons is selected as \(g = 0.5\), \(\tau = 50\), \(\tau_1 = 10\), respectively. The external forcing current is selected as \(I_{\text{ext}} = 1.0\) for neurons in the 'defect area' \((55 \leq i \leq 60)\), otherwise, \(I_{\text{ext}} = 1.3\). For coupling intensity \((a) D = 0.5\), \((b) D = 0.8\), \((c) D = 1.0\).
dependent on the time delay $\tau_1$ between neurons, it is due to the time delay $\tau_1$ that the ‘defect area’ can enlarge and slow down the pulse propagation from the other excitable neurons. Furthermore, the effect of gain intensity $g$ and time delay in the autapse with negative feedback type is also investigated, and the results are plotted in Figs. 5 and 6, respectively.

The results in Fig. 5 that the ‘defect area’ keeps robust to the excitable pulses and the pulses are blocked by the ‘defect area’ when the gain intensity is increased, extensive numerical results still confirm that the ‘defect area’ can coexist with the pulses along the ring network even if the time delay in the autapse is changed in a large range.

The results in Fig. 6 show that the pulses can coexist with the ‘defect area’ well under negative feedback even if the time delay in the autapse is increased greatly.

In fact, the neurons in the ‘defect area’ plays like a stable ‘island’, the pulses from the excitable neurons can inundate the quiescent neurons when the pulses are propagated with high speed (stronger coupling intensity $D$) without time delay between neurons being considered. However, when time delay between adjacent neurons is considered, the ‘defect area’ seems to remember its position and the neurons close to the ‘defect area’ keeps invaded by quiescent neurons in the ‘defect area’, and the memory effect from $\tau_1$ makes more excitable neurons are suppressed, thus the ring network can become homogeneous by increasing the coupling intensity as shown in Fig. 2. For further study the effect of distribution of autapse on the collective behaviors of network, the formation mechanism of ‘defects’ is also discussed, and the results are plotted in Fig. 7.
The results in Fig. 7c show that the ring network becomes homogeneous and the neuronal network become quiescent state when all neurons are driven by autapses with negative feedback type. A subthreshold forcing in a local areas as $I_{\text{ext}} = 1.0$ is helpful to design a stable artificial defect, particularly, negative feedback in the autapse in a local area of the ring network can form defects in the ring network even if all neurons are driven by an upthreshold forcing as $I_{\text{ext}} = 1.3$. As a result, negative feedback in the autapse mainly contributes the formation of local defects. We also calculate the time series of average membrane potentials of neurons associated with the case in Fig. 7, and the results are shown in Fig. 8.

The results in Fig. 7c show that the ring network becomes homogeneous and the neuronal network become quiescent state when all neurons are driven by autapses with negative feedback type. A subthreshold forcing in a local areas as $I_{\text{ext}} = 1.0$ is helpful to design a stable artificial defect, particularly, negative feedback in the autapse in a local area of the ring network can form defects in the ring network even if all neurons are driven by an upthreshold forcing as $I_{\text{ext}} = 1.3$. As a result, negative feedback in the autapse mainly contributes the formation of local defects. We also calculate the time series of average membrane potentials of neurons associated with the case in Fig. 7, and the results are shown in Fig. 8.
be associated with the response of neurons to external forcing and this assumption need further verification by experimental evidences, for our study in this paper, the switch between feedback type can be considered in dynamical analysis. In the case of ring network, the propagation of pulse regulate the collective behaviors of neurons in network, it is also interesting to investigate the case in a two-dimensional array network.

Firstly, for simplicity, it supposes that only single autapse is considered in the network, and it is connected to the neuron in node (100, 100), the transition of collective behaviors of neurons is investigated in a two-dimensional array network with no-flux boundary condition being used. The snapshots for the developed states under different coupling intensities at $t = 5000$ time units are plotted in Fig. 9.

The results in Fig. 9 confirm that target wave can be induced in a local area even if the density of autapse is much low (only one autapse is considered), and the stronger coupling intensity enhances the propagation of target wave so that the target wave can grow up quickly. The potential formation mechanism for emergence of target wave is that negative feedback...
in the electric autapse generates stable state as defect, which introduces heterogeneity in a local area and thus target wave can emit from this local area. To observe the development of target wave, it refers to Fig. 10.

![Figure 9](image9.png)

**Fig. 9.** The developed pattern induced by single autapse with negative feedback type at \( t = 5000 \) time units under different coupling intensities. The gain intensity, time delay in autapse is selected as \( g = 0.5, \tau = 20 \), respectively. Autapse is connected to node \((100, 100)\) and forcing current \( I_{\text{ext}} = 1.0 \), other nodes are driven by \( I_{\text{ext}} = 1.3 \). For (a) \( D = 0.01 \), (b) \( D = 0.02 \), (c) \( D = 0.03 \), (d) \( D = 0.04 \), (e) \( D = 0.05 \), (f) \( D = 0.06 \), (g) \( D = 0.07 \), (h) \( D = 0.08 \).

![Figure 10](image10.png)

**Fig. 10.** The development of target wave induced by single autapse with negative feedback type. The gain intensity, time delay in autapse, coupling intensity is selected as \( g = 0.5, \tau = 20, D = 0.04 \), respectively. Autapse is connected to node \((100, 100)\) and forcing current \( I_{\text{ext}} = 1.0 \), other nodes are driven by \( I_{\text{ext}} = 1.3 \). For (a) \( t = 1000 \), (b) \( t = 2000 \), (c) \( t = 4000 \), (d) \( t = 5000 \) time units.

The results in Fig. 10 confirm that target wave emits from the defect node continuously, and the wave front propagates outwardly. During the transient period as Fig. 10(c) and (d), the outward target wave competes with the outside regular square wave, which results from synchronization under mutual coupling, then overcome the outside ordered wave and may occupy the network completely. Furthermore, the density of autapses is increased that more autapses are considered
in the network, it is found that target wave emerges in the network easily with increasing the density of autapses in the network. It is also interesting to investigate the case that autapses are considered in several areas in the network. In numerical studies, four local areas represented the nodes as $(1 \leq i, j \leq 2), (199 \leq i, j \leq 200), (1 \leq i \leq 2, 199 \leq j \leq 200), (199 \leq i \leq 200, 1 \leq j \leq 2)$, which is driven by external forcing current as $I_{ext} = 1.0$, the rests nodes are driven by $I_{ext} = 1.3$. The gain intensity, time delay in autapse is selected as $g = 0.5$, $\tau = 20$, respectively. For (a) $D = 0.03$, (b) $D = 0.04$, (c) $D = 0.05$, (d) $D = 0.06$, (e) $D = 0.07$, (f) $D = 0.08$, (g) $D = 0.09$, (h) $D = 0.1$, (i) $D = 0.2$, (j) $D = 0.3$.

![Fig. 11](image). The developed pattern induced by autapses with negative feedback type at $t = 5000$ time units under different coupling intensities. The autapses are imposed on the nodes as $(1 \leq i, j \leq 2), (1 \leq i \leq 2, 199 \leq j \leq 200), (199 \leq i \leq 200, 1 \leq j \leq 2)$, which is driven by external forcing current as $I_{ext} = 1.0$, the rests nodes are driven by $I_{ext} = 1.3$. The gain intensity, time delay in autapse is selected as $g = 0.5$, $\tau = 20$, respectively. For (a) $D = 0.03$, (b) $D = 0.04$, (c) $D = 0.05$, (d) $D = 0.06$, (e) $D = 0.07$, (f) $D = 0.08$, (g) $D = 0.09$, (h) $D = 0.1$, (i) $D = 0.2$, (j) $D = 0.3$.

![Fig. 12](image). Diagram for the wave propagation from the four defects in the corners of the network included $200 \times 200$ neurons in a two-dimensional array. Four different colors are used to mark the wave front emitted from the four different defects areas.

The results in Fig. 11 confirm that continuous target wave can be induced when autapses with negative feedback type are distributed symmetrically (four corner position in the network) when the coupling intensity is beyond threshold about $D = 0.04$. When autapses with negative feedback type are introduced into the four local areas, four defects areas occur as wave source, the wave fronts from the four wave sources encounters collision with certain transient period and then perfect wave front like target wave is developed when each wave front from the four wave sources pass through the center after collision. When the coupling intensity is low as shown in Fig. 11(a) and (b), the outside wavefront seems like square because of synchronization between these neurons. For a visual view of the target development and formation, it reads in Fig. 12.
to propagate outwardly. Finally, it investigates the case that three local areas are connected with autapses under negative feedback type, and the results are shown in Fig. 13.

The results in Fig. 13 shown three target waves emit from the three defects areas, which induced by imposing autapses with negative feedback type on three different local areas in the network, and these target waves coexist with each other because they hold the same frequency and velocity in the network. As a result, these target waves seldom grow up and occupy the network because each target wave in a local area can be blocked by the other two target waves.

The wave formation mechanism is that pulses (and/or continuous wave fronts) are blocked by the defects induced by negative time delayed-feedback, then the pulses (or wave fronts) propagate forwardly (or outwardly) due to diffraction effect, thus continuous wave fronts are propagated to form stable pulses or target waves. It is different from the case under local positive feedback in autapse that generates stable oscillating source in a local area.

4. Conclusions

The effect of autapse with negative feedback type on the collective electric behaviors of neurons in ring network, two-dimensional array network is investigated, respectively. It is found that (1) Excitable neurons can be suppressed when negative feedback in autapse is imposed on all neurons uniformly, and the network becomes homogeneous. (2) Negative feedback in the autapses connected to some neurons of network generates 'defects' in a local area, and these defects can block the pulses, and even calm down the excitable pulses by increasing the coupling intensity $D$. (3) In the case of two-dimensional array network, target waves also emit from the defects induced by negative feedback in the autapses. (4) A group of target waves can coexist with each other when the 'defects areas' emerge in different local areas in the network because the generated target wave propagate with the same velocity and frequency. (5) Diffraction induced pulses and/or target waves are formed in the network, it is much different from the case under positive feedback type by generating continuous oscillating source in a local area.

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References
