Simple zero singularity analysis in a coupled FitzHugh–Nagumo neural system with delay

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ABSTRACT

A FitzHugh–Nagumo (FHN) model with delayed coupling is considered to investigate the steady state bifurcation due to the coupling strength and delay. The center manifold reduction and normal form method are employed to study the bifurcation from zero singularity. We show that the coupling strength can induce both the pitchfork and transcritical bifurcations and the coupled delay has the most significant impact on these bifurcations. An example is applied to display the above results. Our investigation will contribute to the understanding of the influence of the time delay in signal transmission on the dynamics of coupled neurons.

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1. Introduction

The two-dimensional FitzHugh–Nagumo (FHN) model [1,2] can describe physiological phenomena similar to those corresponding to the Hodgkin–Huxley (HH) model [3]. In [4], a complete topological and qualitative investigation of the FHN equation is done and a rich variety of nonlinear phenomena are observed. In recent years, to understand information processing in the brain, the FHN model is commonly used to study neural firings due to its simplicity. The analysis of the behavior in coupled FHN neural systems has for some time past been the subject of many papers [5–7], in which a rich bifurcation behaviors for equilibrium and limit cycle are observed.

A single FHN neuron is modeled in this paper by the following Eq. [21]

\[
\begin{align*}
\dot{u}_1 &= -u_1(u_1 - 1)(u_1 - a) - u_2 + c \tanh(u_2(t - \tau)) \\
\dot{u}_2 &= b(u_1 - \gamma u_2)
\end{align*}
\]

where \(0 < a < 0.5\), \(b\), \(\gamma\) are positive constants; \(u_1\) represents the membrane potential, \(u_2\) is a recovery variable. If the several neurons modeled in Eq. (1.1) are coupled, the time delay in the coupled system is inevitable due to the finite propagating speed in the signal transmission between the neurons [11,12]. To consider effects of such coupling on dynamics of the coupled system, we here model a coupled FHN neural system as

\[
\begin{align*}
\dot{u}_1 &= -u_1(u_1 - 1)(u_1 - a) - u_2 + c \tanh(u_2(t - \tau)) \\
\dot{u}_2 &= b(u_1 - \gamma u_2) \\
\dot{u}_3 &= -u_3(u_3 - 1)(u_3 - a) - u_4 + c \tanh(u_4(t - \tau)) \\
\dot{u}_4 &= b(u_3 - \gamma u_4)
\end{align*}
\]

where \(c\) measures the coupling strength; \(\tau > 0\) represents the time delay in signal transmission. We consider \(c\) as bifurcation parameter. All the constants used in the paper are listed in Table 1.

The research for coupled FHN systems with delay has attracted many authors’ attention. Nikola et al. [8] investigate Hopf bifurcation of coupled FHN neurons with delayed coupling, they include that the time delay can induce the Hopf bifurcation. In [9], different synchronization states in a coupled FHN system with delay are observed with the variation of the coupling strength and delay. The authors show that the stability and the patterns of exactly synchronous oscillations depend on the type of excitability and type of coupling. Wang et al. [10] report time delay can control the occurrence of some bifurcations in two synaptically coupled FHN neurons. All works mentioned above have promoted greatly a deep understanding for dynamics of coupled FHN systems with delay. Almost all authors only focus on Hopf bifurcation when they deal with the bifurcation analysis problems in coupled FHN neural systems with delay. However, effects of the coupled strength and delay on steady state bifurcations in coupled systems may be an import topic.

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problems in such models have been treated in [25–27] by using LMI approach. Then, the relative problem in the stochastic delayed neural systems will be considered as one of our extensive topics.

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