A new electrical formation factor model for bimodal carbonates: numerical studies using dual-pore percolation network

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SUMMARY
In this paper, we modelled the electrical transport behaviour of bimodal carbonate rocks from a reservoir in China using dual-pore networks. One basic assumption, generally supported by experimental data and microstructure observations in the reservoir samples, was that the low porosity, monomodal rocks had the same properties and structure as the microporous matrix of the high porosity, bimodal samples. We assumed that the matrix was homogeneous and always interconnected but that the connectivity and the pore size distribution of macropore system was randomly variable. Both pore systems were supposed to act locally as ‘in parallel’ electrical conductors, an approach previously used by Bauer et al. Hence, the effect of matrix properties, macropore size distribution and connectivity on electrical properties of bimodal rocks could be modelled and investigated. We simulated electrical current through 3-D, simple cubic and body-centred cubic networks with different coordination numbers, different pipe radius distributions of macropore system and different matrix properties. The main result was that the formation factor of dual-pore network obeyed a ‘universal’ scaling relationship (i.e. independent of lattice type). Based on this result, we extended the power-law model derived by Bernabé et al. for monomodal porous media. We developed methods for evaluating the scale-invariant pore structure parameters in the model using conventional core analysis and satisfactorily tested the proposed model against experimental data from the Chinese reservoir as well as some other previously published data sets.

Key words: Numerical approximations and analysis; Downhole methods; Electrical properties; Microstructures; Permeability and porosity; Fracture and flow.

1 INTRODUCTION
The electrical properties of carbonate rocks are of great interest in many fields such as geology, geophysics and petroleum engineering. The electrical conductivity of porous rocks (sandstones and carbonates) is influenced primarily by the transport of ions through the bulk-saturating electrolyte, which is related to microstructural properties, such as porosity, pore-throat tortuosity and pore connectivity (Archie 1942; Sahimi 1993; Friedman & Seaton 1998; Sun 2007; Glover 2009; Bernabé et al. 2011; Yue & Tao 2013). For porous media fully saturated by brine, Archie (1942) proposed a formula relating formation factor $F$ and porosity $\phi$, $F = \phi^{-m}$, where $m$ is the cementation exponent. The validity of Archie’s equation was verified in a number of studies (Archie 1942; 1952; Fatt 1956; Wyllie 1960; Sun 2007; Bernabé et al. 2011). However, many measurements showing a poor fit with Archie’s equation have been reported, especially in the case of carbonate rocks (Yue & Tao 2013). Although the physical meaning of Archie’s law for monomodal porous media (sandstones) has been clarified in many research reports (Dullien 1992; Sun 2007; Glover 2009; Kozlov et al. 2012; Yue & Tao 2013), it has not been resolved yet in the case of bimodal porous media.

Many researchers have tried to study the petrophysical transport behaviour using pore network models (Fatt 1956; Sahimi 1993; Suman & Knight 1997; Friedman & Seaton 1998; Tsakiroglou & Fleury 1999; Xu et al. 1999; Békri et al. 2004, 2005; Han et al. 2009; Bernabé et al. 2010, 2011; Zhan et al. 2010). It has been proved that pore network models can reproduce the electrical properties of monomodal porous media. To study the transport properties of bimodal porous media such as carbonate rocks, two kinds of dual-pore network models were developed. Békri et al. (2004, 2005) presented a dual-pore network model, constructed at two different scales. Namely, a small scale network representing the microporous matrix was embedded in a large scale network corresponding to the secondary porosity (vugs, fractures and so forth). Tsakiroglou and Ioannidis (2008) also used this kind of dual-pore network in combination with the critical path analysis (CPA) of percolation theory to study the transport properties of contaminated bimodal soil. Bauer et al. (2011, 2012) developed another type of dual-pore network model based on $\mu$-CT images, in which the
micropore system was segmented as a separated phase with average properties. One major assumption used by these authors was that macro- and micropore systems locally acted in parallel. Since the micropores are unresolved in standard $\mu$-CT images and embedded dual-pore models need very large computing capacity, Bauer et al.’s dual-pore network model is particularly well suited for the investigation of carbonate rocks.

In this paper, we used a similar dual-pore percolation network model to investigate the electrical properties of bimodal carbonate rocks. Our model is different from Bauer et al.’s model in that it does not have the requirement of a connected macropore network. Without a connected macropore network system, the dual-pore percolation network is still connected by the matrix, which is indeed always present in real carbonate rocks. We note that surface electrical conduction can also be accounted for (Johnson always present in real carbonate rocks. We note that surface electrical conduction can also be accounted for (Johnson et al. 1986; Revil et al. 1998; Revil & Cathles 1999; Wildenschild et al. 2000; Zhan et al. 2010). However, in order to simplify our present investigation, we assumed that surface conductance is absent in our networks. With the aid of previous studies on monomodal pore network (Bernabé et al. 2011), an approximate model was developed to estimate the electrical formation factor of bimodal carbonates from the scale-invariant topological parameters of the macropore structure and matrix electrical property. These predicted values of the selected samples’ electrical transport property calculated by the approximate model were compared with measured ones to evaluate the validity of the approach.

2 PORE CHARACTERISTICS OF CARBONATE ROCKS

2.1 Sample selection

To perform this work, several reservoir carbonate rocks in China were collected and examined using a broad set of investigation techniques, such as mercury injection capillary pressure (MICP), porosity and permeability measurements and scanning electronic microscope (SEM), petrographic thin sections and so on. The Chinese samples come from the XiangGuoSi gas reservoir in the Sichuan-Chongqing region, southwest of China. This is a carbonate reservoir located in a carboniferous formation at a depth of 2000–2500 m (Zhong & Lin 1982; Mao & Zhen 2010). The sedimentary and diagenetic processes responsible for the rock microstructure and properties are not of primary interest here. Our focus is on the petrophysical attributes of the reservoir rocks. Broadly speaking, carbonate rocks can be classified into two kinds (Archie 1952): (1) low porosity, low permeability, monomodal carbonate rocks with only primary porosity; (2) bimodal carbonate rocks with secondary porosity (vugs, cracks and so forth). The rock samples studied here were all collected from the same carbonate reservoir in China and have very similar primary porosity structures. It seemed therefore reasonable to assume that the carbonate rocks which only contained primary porosity, had a pore structure equivalent to that of the microporous matrix of carbonate rock samples containing secondary pores. Values of the porosity, gas permeability and formation factor of Chinese reservoir carbonate samples are shown in Table 1. The formation factor was measured after fully saturating the samples with high-salinity brine (100 000 ppm NaCl brine, for eliminating surface conductance; Revil et al. 1998). Several samples, whose formation factor was not measured, were nevertheless included in the pore structure characterization and analysis. In addition, we also examined published experimental data from Bauer et al.’s study (Bauer et al. 2011, 2012; namely, Est, Lav and Res) and Padhy et al.’s study (Padhy et al. 2006; namely, SC16a, SC6a, SC4b, 13P20H, 16BP17H and 16BP8H). These data are also shown in Table 1. Fig. 1 shows the Archie $F–\phi$ relationship of the various samples mentioned above.

2.2 The characteristics of micropore system

The microporous matrix of carbonate rocks is usually microgranular (Moctezuma et al. 2003). As shown in Table 1, the porosity of reservoir carbonate samples without secondary pores has values lower than 10 per cent. SEM images (Fig. 2) show that the matrix pore size is less than 1 $\mu$m, unresolved in conventional $\mu$-CT images.

Table 1. The data set of carbonate samples.

<table>
<thead>
<tr>
<th>Sample number</th>
<th>Porosity (per cent)</th>
<th>(Gas) permeability (mD)</th>
<th>Formation factor</th>
<th>Archie cementation exponent, $m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C3</td>
<td>10.1</td>
<td>0.659</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>C4</td>
<td>10.9</td>
<td>4.160</td>
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<tr>
<td>C5</td>
<td>11.6</td>
<td>16.182</td>
<td>54.315</td>
<td>1.85</td>
</tr>
<tr>
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<td>6.0</td>
<td>2.926</td>
<td>112.498</td>
<td>1.67</td>
</tr>
<tr>
<td>C12</td>
<td>9.5</td>
<td>2.223</td>
<td>87.826</td>
<td>1.89</td>
</tr>
<tr>
<td>Chinese reservoir samples</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C20</td>
<td>9.0</td>
<td>2.720</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>C21</td>
<td>9.3</td>
<td>0.794</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>C23</td>
<td>11.9</td>
<td>5.870</td>
<td>70.985</td>
<td>2.00</td>
</tr>
<tr>
<td>C26</td>
<td>5.9</td>
<td>4.484</td>
<td>102.450</td>
<td>1.63</td>
</tr>
<tr>
<td>C29</td>
<td>5.3</td>
<td>0.206</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>C43</td>
<td>7.2</td>
<td>13.237</td>
<td>78.970</td>
<td>1.66</td>
</tr>
<tr>
<td>C44</td>
<td>7.9</td>
<td>0.284</td>
<td>56.104</td>
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<td>Bauer carbonates</td>
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</tr>
<tr>
<td>Est</td>
<td>25.0</td>
<td>273.0</td>
<td>24.00</td>
<td>2.29</td>
</tr>
<tr>
<td>Lav</td>
<td>29.0</td>
<td>90.0</td>
<td>13.00</td>
<td>2.07</td>
</tr>
<tr>
<td>Res</td>
<td>20.6</td>
<td>23.0</td>
<td>50.00</td>
<td>2.47</td>
</tr>
<tr>
<td>Padhy carbonates</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SC16a</td>
<td>31.9</td>
<td>2406.0</td>
<td>21.372</td>
<td>2.68</td>
</tr>
<tr>
<td>SC6a</td>
<td>16.2</td>
<td>1040.0</td>
<td>22.071</td>
<td>1.69</td>
</tr>
<tr>
<td>SC4b</td>
<td>46.8</td>
<td>4740.0</td>
<td>3.053</td>
<td>1.47</td>
</tr>
<tr>
<td>13P20H</td>
<td>15.5</td>
<td>115.0</td>
<td>33.906</td>
<td>1.89</td>
</tr>
<tr>
<td>16BP17H</td>
<td>14.4</td>
<td>72.0</td>
<td>51.112</td>
<td>2.03</td>
</tr>
<tr>
<td>16BP8H</td>
<td>14.3</td>
<td>40.0</td>
<td>47.960</td>
<td>1.99</td>
</tr>
</tbody>
</table>
2.3 The characteristics of macropore system

The macropore system, often called secondary pores, includes vugs and cracks. As shown in Fig. 4, the secondary pores are overprinted on the carbonate matrix and enhance the overall pore connectivity. The structure of macropore system is highly variable and depends on the degree of development of the secondary pores. In these rock samples the overall pore size distribution determined from MICP measurements is approximately bimodal (Fig. 5), suggesting that the macropores in the Chinese reservoir samples were effectively interconnected. The secondary pores were disorderly distributed in the matrix (Fig. 4) and their spatial distribution was heterogeneous. In general, the connectivity of macropore system is observed to vary in space (Bauer et al. 2011). On the basis of vugs interconnection, Lucia (1999) identified two types of vugs, namely, separate and touching vugs. In next section, we will discuss network modelling of carbonate rocks with these two types of vugs.

3 NETWORK SIMULATION OF ELECTRICAL CONDUCTION THROUGH BIMODAL POROUS MEDIA

3.1 Network modelling of dual-pore porous media

Simple cubic (SC) and body-centre cubic (BCC) networks of circular cylindrical pipes are used in this paper. In 3-D SC network model, six pipes connect one node to the six adjoining nodes, yielding a coordination number \( z = 6 \) when fully connected. A coordination number of 8 characterizes 3-D, fully connected, BCC networks. Different values of the coordination number can be obtained by randomly removing pipes from SC and BCC networks (i.e. \( z < 6 \) and 8, respectively). In a previous network simulation study, Bernabè et al. (2011) observed ‘universal’ power laws,
media with permeable matrix, we first superposed the SC model of the macropores on a homogeneous and totally interconnected matrix, as shown in Fig. 6. As mentioned earlier and as illustrated in Fig. 6(b), we consider the matrix as a continuous, uniform phase with essentially identical average properties in all samples. Electrical conduction in the continuous matrix can be simulated using finite difference equations formally identical to the network Kirchoff equations if the nodes of the finite difference mesh coincide exactly with the network nodes. In that case the macropores and the matrix volume elements act in parallel and a single set of equations applies to both pore systems, a concept used in Bauer et al. ’s dual-pore network method (Bauer et al. 2011, 2012). Here, we built a similar dual-pore network model by segmenting the matrix into equal blocks. For the purpose of simulating electrical conductance, the shape of the blocks does not need to be specified. We only have to adjust the conductance of the blocks so as to yield the correct macroscale conductivity for a dual-pore network containing only matrix elements (no macropores). Thus, we transform the porous media model shown in Fig. 6(b) into the equivalent dual-pore percolation network model as shown in Fig. 6(c). The blue and red lines represent the in parallel networks of macropores (Fig. 6d) and matrix elements (Fig. 6e), respectively. As in classic network percolation studies (Sahimi 1993), variations in coordination number of the macropores network (Fig. 6d) were produced by randomly selecting a number of pipes according to a probability $1 - p$ (where $p$ is the occupancy probability), and assigning them a radius equal to zero. It is important to notice that the matrix is always interconnected in our model. When the macropores are disconnected with each other, the dual-pore percolation network is still connected by the matrix, in agreement with the microstructure observations (Fig. 4). Hence, a percolation threshold is absent.

The porosity and formation factor of the matrix are denoted $\phi_m$, and $F_m$, respectively. To study the effect of $F_m$ on rock electrical properties, we set $F_m = 50, 100, 200, 500$ and 1000 in the network realizations. The pore size variability of macropore system was controlled using the same techniques and parameters values as in Bernabé et al. (2010) and Bernabé et al. (2011). The pipe radii were selected according to the log-uniform distribution with five different values of the normalized standard deviation $\sigma / \langle r \rangle$ (namely, 0.05, 0.3, 0.55, 0.8 and 1.05). $\sigma / \langle r \rangle$ is a parameter, which can be measured in real rock using 2-D or 3-D microstructure quantitative analysis [such as, MICP or nuclear magnetic resonance (NMR)]. The pipe radii distributions were adjusted to produce a constant hydraulic radius value of macropore system ($r_{H} = 40$ $\mu$m) in all cases. The pipe length $l$ of macropore was taken equal to 300 $\mu$m in all network realizations.

Furthermore, past studies have shown that the results of network simulations are more likely to be applicable to real rocks if they can be shown to be independent of the type of lattice used. This is similar to the concept of ‘universality’ introduced in percolation theory. Here, we verified the ‘universality’ of our results by running additional simulations using BCC networks. The microstructure parameters $r_{H}$, $l$ and $\sigma / \langle r \rangle$ were identically assigned in BCC and SC networks. Moreover, in order to make the BCC and

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**Table 2.** Some pore characteristic parameters of matrix samples.

<table>
<thead>
<tr>
<th>Sample number</th>
<th>$r_{\text{max}}$ ($\mu$m)</th>
<th>$\langle r \rangle$ (nD)</th>
<th>$\sigma / \langle r \rangle$</th>
<th>$\phi_m$ (per cent)</th>
<th>$F_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C8</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>5.99</td>
<td>112.498</td>
</tr>
<tr>
<td>C26</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>5.94</td>
<td>102.450</td>
</tr>
<tr>
<td>C29</td>
<td>8.906</td>
<td>0.526</td>
<td>1.62</td>
<td>5.30</td>
<td>–</td>
</tr>
<tr>
<td>SC6a</td>
<td>27.5</td>
<td>12.06</td>
<td>0.316</td>
<td>16.20</td>
<td>22.071</td>
</tr>
</tbody>
</table>
SC simulations directly comparable, we randomly removed 25 per cent of the BCC network bonds (each bond including both a pipe and an \textit{in parallel} matrix element) to obtain a coordination number of 6. The removal of bonds should not be interpreted as a physical removal of matrix elements but rather as a transformation of the finite difference mesh used to calculate conduction in the matrix from a regular SC mesh to a somewhat irregular, incomplete BCC mesh. Starting from these SC and BCC networks with \( z = 6 \), we were able to further decrease the macropore coordination number by randomly removing pipes (i.e. setting the pipe radius to zero) while keeping their associated matrix elements. In order to explore the very high macropore connectivity range, we also constructed

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{The SEM images of Chinese reservoir carbonates with secondary pores or vugs (C3, C4, C20 and C21).}
\end{figure}
BCC networks with $z$ greater than 6 by re-inserting pipes that had previously been removed to prepare the $z = 6$ networks described above (note that values of $z$ greater than 7 have experimentally been observed, Gharbi & Blunt 2012). Strictly speaking, these networks have an unphysical geometry but they always produced results completely consistent with the rest of the study. Our simulations results did show a satisfactory degree of ‘universality’ as will be discussed later.

### 3.2 Simulation of electrical conduction in dual-pore network

Simulation of electrical current transport in networks has been extensively discussed in the past (e.g. Suman & Knight 1997; Friedman & Seaton 1998; Tsakirogolou & Fleury 1999; Bernabé et al. 2010 and references therein). The voltage values at the network nodes are calculated by solving Kirchoff laws with periodic
by definition, the local hydraulic radius is equal to the pipe radius \( r_i \), \( l \) is the length of each pipe. In the case of pipes with elliptical cross-sections, this expression becomes:

\[
g_i = \sigma_w \frac{\pi a_i b_i}{l},
\]

where \( a_i \) and \( b_i \) are the major and minor dimensions of each pipe, respectively. In this case, the local hydraulic radius can be obtained from the cross-section area \( A_i = \pi a_i b_i \) and perimeter \( P_i = \pi [3(a_i + b_i) - (3a_i + 10a_i b_i + 3b_i)]^{1/2} \). Ramanujan formula. Then, the equation above can be rewritten as (Bernabé et al. 2011):

\[
g_i = \sigma_w \frac{\pi r_i^2 [3(1 + \varepsilon) - \sqrt{3\varepsilon^2 + 10\varepsilon + 3}]^2}{4\varepsilon},
\]

where \( \varepsilon = b_i/a_i \) is the cross-section aspect ratio. Electrical conductivity of circular pipes network multiplied by the function \([3(1 + \varepsilon) - (3\varepsilon^2 + 10\varepsilon + 3)]^{1/2}/4\varepsilon\) can be applied to elliptical ones provided the aspect ratio does not vary spatially in the porous medium. Note that when \( \varepsilon \) is small enough (<0.01), the shape of elliptic cross-section will look like a cramp, implying that elliptic micro-cracks can also be included in our pore network model. We can also consider pores with another shape, namely, thin flat slits. For a slit-shaped pore, the electrical conductance is given by (Friedman & Seaton 1998):

\[
g_i = \sigma_w \frac{bw}{l},
\]

where \( b \) and \( w \) are the width and aperture, respectively. Concerning the matrix, the conductivity \( g_m \) of each matrix segment is given by:

\[
g_m = \frac{\sigma_w A_m}{F_m l_m},
\]

where \( F_m \) is the formation factor of matrix, \( l_m \) and \( A_m \) are the length and cross section area of the matrix segments. It is important to notice that there is no percolation threshold in our dual-pore percolation networks (i.e. \( p_c \) or \( z_c \) are equal to zero). In our network realizations, we used \( 25 \times 25 \times 25 \) and \( 22 \times 22 \times 22 \) networks for SC and BCC, respectively, also only considered pipe with circular cross-section (i.e. \( \varepsilon = 1 \)) and the matrix cubic with configurations \( (l_m = l \) and \( A_m = F) \). In the next section, the numerical results are reported and we also demonstrate that we used realistic values for the network parameters.

### 3.3 Analysis of simulation results and inferred model

In cases with \( z = 0 \), that is, when there is no macropores but only the matrix in the networks, the simulated formation factors of SC and BCC networks were equal to 48, 95, 190, 476 and 950 for different values of \( F_m \), only slightly 5 per cent lower than the target values \( (F_m = 50, 100, 200, 500 \) and 1000). This small 5 per cent underestimation was caused by our choice of the rectangular matrix segments shown in Fig. 6, which have a slightly higher conductance than needed. Since the discrepancy was so small, we retained the same conductance values for the rest of the study. In Fig. 7, the results obtained using SC and BCC lattices (circular and square dots, respectively) are intermixed, showing 'universal' behaviour in all cases considered. For the sake of completeness, the previously established 'universal' power laws and the corresponding simulated data for monomodal networks (Bernabé et al. 2011), equivalent to dual-pore networks with \( F_m = +\infty \), are also shown in Fig. 7. For \( F_m = 50, 100, 200, 500 \) and 1000 cases, the simulated results of our dual-pore network models (SC and BCC) were plotted with
Formation factor model for bimodal carbonates

Figure 6. The sketch of networks from classic network model to the equivalent dual-pore network. (a) The classic percolation network. (b) The matrix–vug (macropore system) bimodal porous media model, which looks like the Warren-Root model (Warren & Root 1963). The blocks represent the matrix, the blue parts represent macropore system. (c) The equivalent dual-pore network model, which is constituted by the macropore network (d) and micropore network (e), both networks act electrically in parallel.

network model of Bernabé et al. (2011) for $1/F$, eq. (7) can be changed into the following model:

$$
\frac{1}{F} = CF \left( \frac{r_H}{l} \right)^2 (z - z_c)^\gamma + \frac{1}{F_{z=1.5}},
$$

(8)

where $z$ is the coordination number of the macropore system, $l$ is the average pore-throat length of the macropore system ($\mu$m), and $r_H = 2V_p/S_p$ (where $V_p$ and $S_p$ is the pore-throat volume and surface area of macropore system, respectively) ($\mu$m), $\gamma$ and $C_F$ are functions of the normalized standard deviation $\sigma/\langle r \rangle$ of the macropore radius distribution. The approximate expressions for $\gamma$ and $C_F$ are as follow:

$$
\gamma = 1.2903 + 0.045527\sigma/\langle r \rangle + 0.8239(\sigma/\langle r \rangle)^2,
$$

(10)

where $\varepsilon$ is the pore-throat cross-section aspect ratio [see Bernabé et al. (2011) for more details]. Assuming a linear relationship between $F_m$ and $F_{z=1.5}$ (i.e. $F_{z=1.5} = aF_m$, where $a$ is the proportional coefficient), the model can be written as:

$$
\frac{1}{F} = CF \left( \frac{r_H}{l} \right)^2 (z - z_c)^\gamma + \frac{1}{aF_m}.
$$

(11)
Figure 7. Illustrations of the ‘universal’ scaling relationship between the inverse formation factor $1/F$ and $z^{-1.5}$ of monomodal network with different $\sigma/\langle r \rangle$ (red dots and lines, see Bernabé et al. 2011 for details) and dual-pore network (dots with various colours except red) with different $\sigma/\langle r \rangle$ and different $F_m$ for SC and BCC lattices. Results obtained using SC and BCC lattices are represented with circular and square dots, respectively, and different colours for each value of $F_m$. The predicted values of $1/F$ calculated by eq. (11) are also drawn in this diagram (solid lines with appropriate colours except red).
Table 3. The values of $a$ for different $F_m$ and $\sigma/r$.  

<table>
<thead>
<tr>
<th>$\sigma/r$</th>
<th>$F_m = 50$</th>
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<th>$F_m = 500$</th>
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<tbody>
<tr>
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<td>0.3</td>
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</tbody>
</table>

We calculated the proportional coefficient $a$ and found that it ranged between 0.3 and 0.75. The values of $a$ for different $F_m$ and $\sigma/r$ are shown in Table 3. On the basis of these values in Table 3, we found that $a$ followed an approximate power-law relationship with $F_m$ (i.e. $a = F_m^{-n}$, where $n$ equal to 0.2–0.3). In order to test the model for $z > z_c$, we simplified eq. (11) by setting $a$ constant, equal to 0.6, the average of the simulated values corresponding to $F_m = 50, 100$ and 200 cases. For $F_m \leq 200$, we compared the predictions of eq. (11) with constant $a$ (solid line in Fig. 7) for different network configurations to the corresponding simulated results in Fig. 7 (circle and square dots) and generally observed a good fit when $z - z_c$ is greater than some finite value depending on the heterogeneity of the networks (0.3–0.9 for $\sigma/r$ varying from 0.05 to 1.05, respectively). Eq. (11) with $a = 0.6$ becomes more and more invalid as the percolation threshold of the macro pores is approached, due to an increasing number of dead-end and isolated macro pores. Hence, we hereafter limited the use of eq. (11) with $a = 0.6$ to the range $z \geq 2.5$ and $F_m \leq 200$. For $F_m > 200$ cases, the values of $a$ in Table 3 were used for the comparison between predictions of eq. (11) (lines) and the corresponding simulated results (dots) in Fig. 7.

The simulated results for $z \leq z_c$ are shown in Fig. 8. We see that the behaviour remained ‘universal’ in this range of $z$. In order to model the simulated data of Fig. 8, we tried to use binary mixing theory. We considered two phases, namely, the microporous matrix as the low-conductivity phase and the macropore system at maximum

![Figure 8](http://gji.oxfordjournals.org/)  

**Figure 8.** Comparison of the simulated results (circular and square dots corresponding to SC and BCC networks, respectively) for $z < z_c$ and the predictions of eq. (12) (lines). The different colours indicate the value of $F_m$ used in each case.
connectivity as the high-conductivity phase (i.e. here with a coordination number \(Z = 6\)). The volume fraction of the high-conductivity phase is equal to the occupancy probability \(p\) (Sahimi 1993), itself approximately equal to \(2/Z\) (see fig. 1 in Bernabé et al. 2010). We tested various binary mixing models (Bernabé et al. 2004) and found that the best fit was obtained with geometric averaging as expressed below.

\[
\frac{1}{F} = \left[ C_r \left( \frac{r_h}{l} \right)^2 (Z - z)^y \right]^2 \left( \frac{1}{F_m} \right)^{1-2} .
\]  \hspace{1cm} \text{(12)}

As illustrated in Fig. 8, the values of \(1/F\) predicted by eq. (12) (solid lines) and the simulated data for different network configurations (i.e. \(\sigma/(r) = 0.3, \sigma/(r) = 0.55\) and \(\sigma/(r) = 0.8\)) agreed with each other well, particularly for relatively high matrix resistivity (i.e. \(F_m > 100\)). We see also that, as expected, eq. (12) increasingly underestimates \(1/F\) as the macropore percolation threshold is approached and passed.

4 TESTING THE MODEL: COMPARISON TO EXPERIMENTAL DATA

In order to apply our model to the three groups of samples described in a previous section, we simply set the value of \(a\) to 0.6 in the following studies. Furthermore, the coordination number, \(z\), hydraulic radius, \(r_h\), pore-throat length, \(l\), normalized standard deviation, \(\sigma/(r)\), and cross-section aspect ratio, \(\varepsilon\), are also required. We must therefore devise methods with sufficient, independent information to estimate all the parameters. For example, capillary pressure curves are used for estimating \(r_{th}\) and \(\sigma/(r)\). But there may still be some very poorly constrained parameters, such as aspect ratio \(\varepsilon\) and the coordination number \(z\). 2-D images can be used to determine the average value of \(\varepsilon\). In order to assess the quality of the proposed model, we tried to determine the input values producing the best fit to the experimental data and verify that these optimized values were not unrealistic. For the most uncertain parameters, we were only able to give a range of reasonable values that could be used for a sensitivity analysis. For the sake of consistency, we used the same misfit measures (i.e. \(E_i = |\ln(F_{\text{predicted}}) - \ln(F_{\text{measured}})|\) and \(E_\xi = \exp[\xi]\), see Bernabé et al. 2011) in all cases. We will also discuss the values of the relative error (i.e. \(E_i = |F_{\text{predicted}} - F_{\text{measured}}|/F_{\text{measured}}\).

4.1 Bauer carbonate rocks

We first considered Bauer et al. (2011) and Bauer et al.’s (2012) carbonate rocks. Some measurement data of three carbonate samples (two outcrops and one reservoir carbonate) were given in Bauer et al. (2011) and Bauer et al. (2012), such as permeability, formation factor, porosity, capillary pressure curve and so on. According to the \(\mu\)-CT images of Bauer et al., the limit between micro- and macropores is given by the \(\mu\)-CT resolution (3 \(\mu\)m). Based on the published evidence, we estimated the six parameters of our model as follows:

(1) 3-D images of the three samples provide significant information about the coordination number \(z\) of the macropore system. Bauer et al. (2012) reported that the range of the coordination number is 3–4 (lower than six, which is the value expected for unconsolidated granular media).

(2) We estimated \(r_{th}\) and \(\sigma/(r)\) using the capillary pressure curves. According to the definition provided below, we simply estimated \(r_{th}\) using pore size frequency distribution, which can be obtained from the capillary pressure curves of the three samples:

\[
r_{th} = \frac{2V_p}{5p} = \frac{2\sum \pi r_i^2 p_i}{\sum 2\pi r_i p_i},
\]  \hspace{1cm} \text{(13)}

where \(p_i\) is the frequency of pore radius \(r_i\).

(3) It is reasonable to assume that the pore aspect ratio \(\varepsilon\) is equal to one.

(4) Since the porosity of the microporous matrix was given for the three carbonate rocks in Bauer et al. (2012), \(F_m\) can be estimated using Archie’s law, \(F_m = \phi_m^{\alpha - m}\). Here, we set \(F_m\) to a single average value (100) for all Bauer et al.’s samples.

(5) Based on the \(\mu\)-CT images of the three samples, the pore-throat length \(l\) and the length \(l_m\) of the microporous cuboid lining the macropore along the flow direction were determined by Bauer et al. (2011). It is important to notice that \(l\) should be identical to \(l_m\) in our dual-pore network simulations. Bauer et al. (2011) reported that the values of \(l_m\) of the samples ‘Lav’ and ‘Est’ were 40–80 \(\mu\)m and 56.25 \(\mu\)m, respectively, while \(l\) was about 11 \(\mu\)m for sample ‘Est’. After trying various values, we ended up setting \(l = 40 \mu\)m for sample ‘Lav’ and \(l = 56.25 \mu\)m for sample ‘Est’. There was no direct information about \(l\) for sample ‘Res’ in Bauer et al. (2011), but we reasoned that, since the porosity of sample ‘Res’ was 20.6 per cent (less than 24.7 and 28.7 per cent for ‘Est’ and ‘Lav’, respectively), \(l\) of sample ‘Res’ may be equal to the maximum value of \(l\) in Bauer’s carbonate samples (i.e. 80 \(\mu\)m). Furthermore, as part of a sensitivity analysis on \(l\), we tested \(l = 90 \mu\)m in all three samples.

All input values of six parameters, the predicted values of formation factor by eq. (11) and the fit quality measures (i.e. \(E_i\) and \(E_\xi\)) were given in Table 4. The results are shown in Fig. 9, illustrating that, although some of the input parameters, especially the estimated value of pore-throat length, are uncertain, we generally obtained a good fit.

4.2 Padhy carbonate rocks

Padhy et al. (2006) performed electrical resistivity measurements on five vuggy carbonate cores (namely, SC16a, SC4b, 13P20H, 16BP17H and 16P8H) and a non-vuggy synthetic sample (SC6a). Some experimental data of these samples were reported, such as total porosity, matrix porosity, permeability, MICP and so on. Fortunately, the matrix porosity and vug porosity were reported, the limit pore size between the macropore and micropore system can be estimated according to these data. Then, the six model parameters (\(z\), \(r_{th}\), \(\sigma/(r)\), \(\varepsilon\), \(F_m\) and \(l\)) of these five vuggy carbonate samples were estimated as follow:

(1) Since matrix porosity \(\phi_m\) and matrix cementation exponent \(m\) of the five samples were given in Padhy et al. (2006), \(F_m\) can be estimated from Archie’s law, \(F_m = \phi_m^{-m}\).

(2) According to capillary pressure curve and the limit pore size between the macropore and micropore system, \(r_{th}\) and \(\sigma/(r)\) can be estimated by eq. (13).

(3) According to thin section images of these samples in Padhy et al. (2006), it is reasonable to assume that the pore aspect ratio \(\varepsilon\) is equal to 0.8.

(4) Since there is no precise information about the throat-length \(l\) in Padhy et al. (2006), we estimated \(l\) based on the published thin section images of the samples (see Table 4).

(5) Coordination numbers were not reported in Padhy et al. (2006) and there were no \(\mu\)-CT images of these samples. Since
Table 4. The values of six model parameters of Bauer Carbonates, Padhy Carbonates and Chinese reservoir samples and the predicted results.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( F_m )</th>
<th>( r_H (\mu m) )</th>
<th>( l (\mu m) )</th>
<th>( \sigma/(r) )</th>
<th>( z )</th>
<th>( \epsilon )</th>
<th>( C_F )</th>
<th>( y )</th>
<th>( F ) predicted</th>
<th>( \xi_F )</th>
<th>( E_r ) (per cent)</th>
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<tbody>
<tr>
<td>Bauer carbonates</td>
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<tr>
<td>Est</td>
<td>100</td>
<td>11.8</td>
<td>56.25</td>
<td>0.50</td>
<td>3.88</td>
<td>1.0</td>
<td>0.185</td>
<td>1.519</td>
<td>15.88</td>
<td>1.51</td>
<td>33</td>
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<td>Lav</td>
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<td>13.0</td>
<td>40</td>
<td>0.74</td>
<td>3.47</td>
<td>1.0</td>
<td>0.075</td>
<td>1.775</td>
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<td>1.62</td>
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<tr>
<td>Res</td>
<td>100</td>
<td>12.5</td>
<td>80</td>
<td>0.80</td>
<td>3.52</td>
<td>1.0</td>
<td>0.057</td>
<td>1.854</td>
<td>44.82</td>
<td>1.12</td>
<td>10</td>
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<tr>
<td>Padhy carbonates</td>
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<td></td>
<td></td>
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<tr>
<td>SC16a</td>
<td>185.9</td>
<td>70.1</td>
<td>300</td>
<td>0.15</td>
<td>3.0</td>
<td>0.8</td>
<td>0.415</td>
<td>1.315</td>
<td>21.58</td>
<td>1.01</td>
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<td>48.4</td>
<td>100</td>
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<td>3.0</td>
<td>0.8</td>
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<td>1.353</td>
<td>1.65</td>
<td>1.84</td>
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<td>13P20H</td>
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<td>30.0</td>
<td>200</td>
<td>0.23</td>
<td>3.0</td>
<td>0.8</td>
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<td>1.344</td>
<td>36.11</td>
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<td>150</td>
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<td>0.8</td>
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<td>0.8</td>
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<td>C5</td>
<td>100</td>
<td>15.2</td>
<td>100</td>
<td>1.66</td>
<td>2.5</td>
<td>0.8</td>
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<td>3.641</td>
<td>59.98</td>
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<td>100</td>
<td>12.4</td>
<td>100</td>
<td>0.80</td>
<td>2.5</td>
<td>0.8</td>
<td>0.057</td>
<td>1.853</td>
<td>56.93</td>
<td>1.52</td>
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<tr>
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<td>100</td>
<td>15.6</td>
<td>100</td>
<td>1.83</td>
<td>2.5</td>
<td>0.8</td>
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<td>59.99</td>
<td>1.18</td>
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<td>7.87</td>
<td>100</td>
<td>0.15</td>
<td>2.5</td>
<td>0.8</td>
<td>0.414</td>
<td>1.315</td>
<td>51.98</td>
<td>1.51</td>
<td>34</td>
</tr>
</tbody>
</table>

Figure 9. Comparison between predicted and measured formation factor \( F \) of Bauer et al.’s, Padhy et al.’s and Chinese reservoir samples. The predicted values of \( F \) were calculated using eq. (11) with the values of the input parameters given in Table 4. The uncertainty range is indicated by the dash lines, corresponding to \( \xi_F = \pm 1.5 \). Predicted formation factors obtained using different input values of \( l \) for Bauer et al.’s samples (open triangle dots) and \( z \) for Padhy et al.’s samples (open circle dots) were also plotted to illustrate the sensitivity of the model to these parameters. In the case of anomalous sample ‘16BP17H’, we also tested eq. (12) with \( z = 1.5 \) (black triangle dot).

Finally, we tested our model against a group of experimental data from reservoir carbonate rocks in China. Several methods were developed to estimate the six parameters needed:

1. Based on the experimental data, we set \( F_m \) equal to 100 for all samples from Chinese reservoir carbonate rocks.
2. According to Fig. 5, the limit between micro- and macropores is about 7 \( \mu m \), as defined by the minimal pore access radius of the macropores. We then estimate \( r_H \) and \( \sigma/(r) \) of macropore system using the macropore part of the capillary pressure curves. We also used eq. (13) to estimate \( r_H \) for each sample. The pore size distributions measured from the capillary pressure curves were used for the estimation of \( \sigma/(r) \). Since all these samples come from the same reservoir, the relationship between secondary porosity \( \phi_H \) and \( r_H \), secondary porosity \( \phi_m \) and \( r_m \) were well fitted by \( r_H = 53.98 (\phi_H/m_{DF})^{0.4386}, \sigma/(r) = 167.76(\phi_H/m_{DF})^{1.5978} \), where \( \phi_m = 0.6 \) per cent.
3. We estimated the pore-throat length \( l \) of the macropores by observation of 2-D images, as shown in Fig. 4. We found that values of \( l \) varying from 50 to 250 \( \mu m \), nevertheless these 2-D values were somewhat imprecise. Since the average value of \( l \) was close to 100 \( \mu m \), we simply set the input value of \( l \) as 100 \( \mu m \).
4. Having no 3-D images of these reservoir samples, we could not estimate the coordination number of the macropores directly. Since the coordination number of the Bauer carbonate rocks is 3.5–3.8 and the porosity of the Chinese reservoir carbonate rocks was less than the porosity of the Bauer carbonates and Padhy 2005), which leads to homogeneous macropores size distribution. As shown in Fig. 9, the predicted values of \( F \) did fit relatively well the experimental values, except sample ‘16BP17H’, for which the large discrepancy was most likely caused by an inaccurate estimate of the coordination number. The SEM and MICP observations for sample ‘16BP17H’ suggested a relatively weak development of the secondary-pores consistent with a low value of \( z \). The predicted resistivity of this sample using eq. (11) and the input parameters listed in Table 4 were lower than the experimental value by nearly one order of magnitude, indicating that the coordination number \( z \) of this sample was strongly overestimated. Decreasing the coordination number \( z \) of this sample to 1.5 [implying the use of eq. (12)] indeed produced a very good fit (black triangle in Fig. 9) but it is not possible to ascertain from Padhy et al. (2005) that the macropore system of sample ‘16BP17H’ was non-percolating.

4.3 Chinese reservoir carbonate rocks
carbonates, it is reasonable to assign a coordination number lower than 3 to the Chinese reservoir carbonate rocks. In order to simplify the comparison, we set the coordination number to 2.5 in all Chinese reservoir carbonate samples.

(5) According to SEM images (as shown in Fig. 4), the value of the $\varepsilon$ of macropore is 0.5–0.8. Here, we simply set $\varepsilon$ to 0.8 for all samples.

Thus, four parameters (i.e. $F_m$, $\varepsilon$, $r_m$ and $\sigma(r)$) were relatively precisely determined, while the remaining ones (i.e. $l$ and $z$) were roughly estimated. The values of each parameter are shown in Table 4. The comparison between the experimentally measured values of $F$ and the predicted $F$ is shown in Fig. 9.

5 DISCUSSION

A general methodology to build dual-pore percolation networks for estimating the petrophysical transport behaviour of carbonates characterized by bimodal pore size distribution is proposed here. The method is based on the classic percolation network approach with a matrix network in parallel, that is, the micropore system is modelled as a parallel circuit located along the throat of macropore system, thus forming an equivalent dual-pore network. The differences and similarities between our dual-pore network and the classic monomodal network have been analysed here. For example, if $F_m \rightarrow +\infty$, which means that the matrix is nearly impermeable, then the dual-pore network model described above must reduce to a classic monomodal network. The corresponding transformations from eq. (11) to the ‘universal’ power law model of Bernabé et al. (2011) were illustrated in Fig. 7. The transformations have clear physical meanings because an infinitely large $F_m$ corresponds to the matrix porosity $\phi_m \rightarrow 0$ (Glover 2009) and the bimodal porous media becomes monomodal in this case. The upper bound value of $F_m$ in eq. (11) can be as high as positive infinity, but the lower bound cannot be close to 1. Indeed, for $F_m \rightarrow 1$ we can derive from Archie’s law (Archie 1942) that the porosity of matrix will be close to 100 per cent (Glover 2009), implying that the pore size of matrix is larger than the macropores in contradiction with the basic definitions. Here, we considered that the lower bound value of $F_m$ was $F^*$. That is to say, when the microstructure of micropore system is similar to that of macropore system, the bimodal porous media then become equivalent to monomodal porous media. According to the description above, eq. (11) is valid when the values of $F_m$ are in the range $(F^*, +\infty)$, and $z \geq 2.5$, corresponding to an insignificant number of dead-end and isolated macropores. For $z \leq z_c$, eq. (12), that is, geometric averaging of two phases, namely, the maximally connected macropore system and the microporous matrix, was used (Fig. 8).

The need to invoke two different models, eqs (11) and (12), in different ranges of macropore connectivity also shows that the transport behaviour of bimodal porous media is different and more complex than that of monomodal porous media (sandstone). The electrical conductivity of bimodal porous media is not a simple sum of the conductivities of the matrix and macropore system. An obvious characteristic of the models is that the total porosity does not explicitly appear. It is mostly because that a ‘universal’ relationship about the porosity in percolation network is not found (Bernabé et al. 2011). Thus, the physical meaning of Archie cementation exponent $m$ is more complicated in bimodal porous media, since it is affected, among many other factors, by two different scales of connectivity (Glover 2009; Bernabé et al. 2011). The ‘non-Archie’ behaviour of bimodal carbonate rocks is caused by their complex microstructure (Yue & Tao 2013) and could be analysed by our proposed model.

It is assumed here that the micro- and macropore systems can be clearly distinguished. To determine the limit between micro- and macropore is not of the main interest in this paper, although many geological and geophysical methods (Archie 1952; Tsakiroglou & Ioannidis 2008; Tiab et al. 2012) can be used for determining it. In this paper, we consider the low porosity, low permeability samples collected in one well as equivalent to the matrix of the rocks in the same reservoir. The pore structure parameters were therefore estimated on the basis of this approximation. Following the general study in petrophysical field, scale-invariant parameters were introduced to model the electrical behaviour in this study. Two different macropore systems, circular- or ellipse-pore and crack-pore, were included in our data set. For three different groups of experimental data set, we determined the input values of six parameters by different methods. From the description in Section 4, due to lack of direct measurement, the estimated values of coordination number $z$ of macropore system were more imprecise than other parameters. The estimation of coordination number $z$ in porous media is a difficult, but crucial problem, previously investigated in many research papers (Doyen 1988; Bernabé et al. 2010, 2011). Many studies indicated that the coordination number of porous media is positively related to the porosity of porous media (Lindquist et al. 2000; Bernabé et al. 2010). Since the porosity of Chinese reservoir samples is less than that of Bauer et al.’s and Padhy et al.’s samples, it is reasonable to assume that coordination number $z$ of Chinese reservoir samples is less than 3. On the basis of simple assumptions in Section 4, the fitting results also look well. While the parameters, $F_m$, $\varepsilon$, $r_m$ and $\sigma(r)$ can be relatively accurately determined by conventional core analyses (such as, SEM, MICP), the estimation of $l$ and $z$ was very difficult. We performed simplified sensitivity analyses on $l$ for Bauer et al.’s samples and $z$ for Padhy et al.’s samples. As shown in Fig. 9 excluding sample ‘16BP17H’, it seems that the simulations were relatively insensitive to changes in $l$ and $z$ (with a slightly larger sensitivity for $l$). Sample ‘16BP17H’ is an outlier with a microstructure that probably cannot be treated as bimodal.

Tsakiroglou & Ioannidis (2008) believed that in dual-pore network model, the permeability is controlled by the throat sizes of the network of large pore, while the small pore system contributes to porosity and affects the capillary pressure curves over the high-pressure range. Although we do not study the permeability of our dual-pore network directly, the formation factor and permeability are all single phase transport properties, and have similar transport mechanisms based on Darcy’s and Ohm’s law (Yue & Tao 2013). Our simulations show that the micropore system plays a very important role when the macropores are not internally connected and may also affects the single phase transport properties of dual-pore porous media even when a fully macropore system is present. The effects of micropore system on two phase flow were qualitatively reported in previous studies (Bauer et al. 2012; Gharbi & Blunt 2012). In the future, our dual-pore percolation network could help quantifying the effect of a microporous matrix on two phase flow in dual-pore rocks.

6 CONCLUSIONS

(1) We developed a dual-pore percolation network model for studying the petrophysical properties of bimodal carbonates. Building the dual-pore model was primarily motivated by the study of Bauer et al. (2011, 2012) based on $\mu$-CT images and dual-pore
network modelling. The new dual-pore percolation network model provides some improvements for studying the petrophysical transport properties of bimodal porous media. For example, disconnected macropores can be included. In the future, other complexities, such as a heterogeneous distribution of matrix properties, can be introduced. When the matrix is impermeable, the dual-pore percolation network model reduces to a monomodal network model, allowing comparison with previously published results.

(2) ‘Universal’ relationships for high coordination number (i.e. $z \geq 2.5$) and low coordination number (i.e. $z \leq 1.5$) were found on the basis of the simulation results of dual-pore network with SC and BCC lattices. Our proposed high macropore connectivity model (eq. 11) was validated with application to three different groups (13 samples) of experimental data set over two orders of magnitude, for which the six model parameters ($F_{\mu}, \sigma, r_{1l}, l, z$ and $z/r_{1l}$) were either actually measured or could be reliably estimated. Simplified sensitivity analysis on $l$ and $z$ were performed, and the results show that the sensitivity of the model (eq. 11) to the poorly constrained parameters was not too high.

(3) Several methods were developed for estimating the pore structure parameters (i.e. $r_{1l}, \sigma, z/r_{1l}$, etc.) with conventional core analyses (such as, SEM, MICP). These methods may be particularly useful when $\mu$-CT images are not available.

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