Different configurations of Bus Rapid Transit (BRT) system may cause different additional riderships. In this paper, in terms of network traffic equilibrium assignment principle, the additional riderships estimation model based on Variational Inequality (VI) model is presented. The bus frequency is related to variables including the travel time, the residence time in terminals, and the dwelling time at the stops. The additional riderships are translated into network additional traffic flow firstly. Given the bus frequency, VI model can be turned into Stochastic User Equilibrium (SUE) model to calculate the other variables. The similarity diagonalization method is used to calculate the elastic bus frequency and finally the network additional traffic flow can be computed. The additional riderships under different configurations of BRT system are compared in the numerical test. The results show that the additional riderships under different configurations have large differences and occupy a high percentage of the total ridership.

1. Introduction

Although the foundation ridership can be generated by the increase of the population and the growth of the economy, additional ridership can be generated by Bus Rapid Transit (BRT) system. Different configurations of BRT system induce the diverse scale of additional riderships. Good configurations of BRT system cause the increase of traffic demand and make additional ridership account for a great percentage of the total ridership. The additional riderships induced by different configurations of BRT system are proposed in this paper, which should be considered in the ridership estimation.

The different configurations of BRT system have been attempted by lots of researchers. Laporte et al. [1] reviewed the main optimization methods for the BRT planning. Abdelghany et al. [2] developed a modeling framework for the planning of BRT services in urban transportation network. Aiming at minimizing the total travel time of passengers, Li et al. [3] presented an optimization model for the BRT planning. Schmid [4] proposed a hybrid metaheuristic approach based on large neighborhood search and liner programming to solve the bus rapid transit design problem. In order to capture the effects of BRT services on urban transportation, various methods have been developed, and a majority of those methods are based on simulation. Salem et al. [5] used a CORSIM model to conduct the benefit/cost analysis of the BRT service. Yagi and Mohammadian [6] simulated the BRT development to study the variation of modal split between automobiles and bus rapid transit ridership. Gunawan et al. [7] presented a numerical simulation based on discrete-event approach to identify variables which affects the performance of BRT system. The microscopic simulation techniques are also widely used in the research field of BRT. Yu et al. [8] applied GPS data in the VISSIM-based simulation for BRT system in Beijing. Godavarthi et al. [9] used a microsimulation to find the optimum volume/capacity ratio on BRT routes. Cervero and Kang [10] measured the impacts of implementing BRT on adjacent land usage and land value, by using multilevel models. Based on six characters of BRT system including infrastructure, transport capacity, service level, economic results, safety and emergency management, and energy saving and emission reduction [11], established an evaluation methodology for BRT operation. To study the
modal shifts to BRT from other modes like automobiles, normal buses, Wang et al. [12] developed a binary logistic analysis approach. Falbel et al. [13] studied the impacts of implementation of BRT on the traffic improvement over the urban transport network, including the additional ridership to the transit system. Currie and Delbos [14] used several regression models to explore the relationship between BRT design features and increment of ridership. Sun et al. [15] established a numerical model for the headway optimization as well as scheduling combination of BRT vehicles to increase ridership and improve operation performance of BRT. There are few previous literatures that studied the additional ridership under different configurations of BRT system. This paper developed a numerical test based on Variational Inequality (VI) model for additional ridership estimation under different configurations of BRT system. The paper is organized as follows. Section 2 is the problem description. Section 3 is the model development and algorithm for the proposed model. A numerical test is given in Section 4. Section 5 presents the conclusions.

2. Different Configurations of BRT System Description

In transportation system, there are many literatures about traffic assignment. Some equilibrium analysis for urban transportation networks were proposed in literature [16, 17]. A traffic equilibrium assignment principle was presented, in the principle, passenger’s selection behavior of a transit note was considered. The obtained results showed that the method was effective for network traffic equilibrium assignment [18]. In this paper, the additional ridership estimation under different configurations of BRT system is based on the network traffic equilibrium assignment principle. In terms of the network, the addition ridership account for less than 25% of the total ridership which is the main restriction in the model [19].
The bus frequency which can reflect the travel time has a function relation with the traffic flow on the road link [20]. It is the main variable in the model, which can represent the requirements of the different hardware configurations and the standards of operational service.

The traffic flow on the road link of BRT system is a function of the bus frequency, which is determined by the bus operation cycle and the bus number of each line. \( t_m^l \), denotes the travel time on the road link \( m \) of the bus route \( l \), and the bus operation cycle \( T_l \) is determined by the traffic conditions of network and the dwelling time at the stop \( d_n^l \):

\[
T_l = \rho t_0^l + \sum_{m \in l} t_m^l + \sum_{n \in l} d_n^l
\]  

(1)

where \( m \in l \) and \( n \in l \) represent that the road link \( m \) and transfer stop \( n \) belong to the bus route \( l \), respectively. \( t_0^l \) represents the residence time in the terminal. \( \rho \) denotes the number of terminals and \( \rho = 1 \) indicates there is only a terminal, while \( \rho = 2 \) indicates there are two terminals.

\( N_l \) denotes the bus number of the route \( l \); then bus frequency of the route \( l \) can be calculated with the following equation:

\[
f_l = \frac{N_l}{\rho t_0^l + \sum_{m \in l} t_m^l + \sum_{n \in l} d_n^l}.
\]  

(2)

The travel time \( t_m^l \) has a relationship with adopting signal priority control and the configurations of route. If the signal priority control is adopted, the intersection delay and the travel time will be shortened. When the route is a part of BRT system, the security and the speed of driving will increase. According to the travel time survey in Changzhou BRT, for 1 kilometer section, the variation range of the travel time \( t_m^l \) of different configurations is as shown in Table 1.

If the bus frequency is held constant, the residence time in the terminal \( t_0^l \) is related to the number of buses on the route. And the existence of more buses means longer residence time in the terminal. In terms of the BRT system under different configurations, the bus capacity varies with the bus model. For the convenience of calculation and comparison, the different BRT bus models are replaced by a standard BRT bus model, the size of which is 12 m, and the conversion coefficient is the ratio of bus capacity.

The dwelling time at a stop \( d_n^l \) is consisted of the time of deceleration and acceleration, the time of switching door, and the time of boarding and alighting. When the bus model, the platform, and the lane are constant, the dwelling time at a stop is determined by the number of the passengers boarding and alighting. The boarding and alighting time are determined by passenger number and the average time of per boarding and alighting passenger, respectively. Wirasinghe and Szplett [21] presented that if the given road link traffic flow is \( v \), the number of passengers alighting and boarding at the stop \( n \) on the bus route \( l \) can be yielded as follows:

\[
A_l^l_n = \sum_{s \in S} \delta_{ns}^s \xi_{n_s} \gamma_s v_s,
\]

\[
B_l^l_n = \sum_{s \in S} \delta_{ns}^s \xi_{n_s} \gamma_s v_s,
\]

(3)

where \( A_l^l_n \) and \( B_l^l_n \) denote the number of passengers alighting and boarding at the stop \( n \) on the bus route \( l \), respectively. \( d_{on} \) and \( d_{off} \) denote the average time of per boarding and alighting passenger, respectively. Thus, \( d_{on} A_l^l_n \) denotes the boarding time and \( d_{off} A_l^l_n \) denotes the alighting time at the stop \( n \). \( \xi_{n_s} \) denotes the percentage of ridership who choose route \( l \) on the road link \( s \), and \( v_s \) denotes the ridership on the road link \( s \). \( \delta_{ns}^s \), \( \delta_{nt}^s \), and \( \xi_{n_s} \) are defined as exponential functions, where the superscripts with plus sign and minus sign indicate the departure terminal and the destination terminal on the road link \( s \). The dwelling time at the stop \( n \) can be yielded as follows:

\[
d_n^l = d_{dc}^l + w_{off} A_l^l_n + w_{on} B_l^l_n + d_{ac}^l + d_{door}^l
\]

(4)

where \( d_{dc}^l \) denotes the time of deceleration, which means the time from vehicle slowing down to a full stop. \( d_{ac}^l \) denotes the time of acceleration, which means the time from starting acceleration to moving in a steady speed. \( d_{door}^l \) denotes the time of opening and closing door. Passengers alighting are assumed to be completed before passengers boarding at the stop. The main factors influencing the dwelling time at a stop \( d_n^l \) are the average time of per boarding and alighting passenger \( d_{on} \), \( d_{off} \), respectively. The average time of per boarding and alighting passenger is influenced by fare collection method, the density of passengers inside and outside the vehicle, width of the bus door, whether to keep the platform and the vehicle floor in the same horizontal plane, and other uncertain conditions like weather.

Thus, the bus frequency varies with the different configurations of BRT system, which reflects the hardware configuration and the operation state significantly. When the vehicle model, the running ways form, and the platform style are fixed, the dwelling time of a vehicle can be presented as a function of the traffic flow \( v \) as shown in formula (5). \( v \) denotes the traffic flow on the road link in the public transportation network, which can be transformed to ridership based on the average capacity and the vehicle conversion coefficient

\[
d_n^l = d_n^l(v).
\]

(5)

When the configuration is determined, bus frequency is also a function of the traffic flow \( v \); the elastic bus frequency can be yielded as follows:

\[
f_l = \frac{N_l}{f_l(v)},
\]

(6)

where \( N_l \) denotes the bus number of the route \( l \) and \( f_l(v) \) is the operation cycle related to the traffic flow on bus line \( l \). \( v \) is
the traffic flow in the transit network. The traffic flow can be converted to ridership by bus capacity and vehicle conversion coefficient.

3. The Additional Riderships Estimation Based on VI Model

In the model, the additional ridership is translated into additional traffic flow in the network $\sum \sigma_i$. The relationship between traffic generation and traffic distribution can be represented as follows:

$$\sum_{j \in J} d_{ij} = o_i, \quad i \in I,$$

(7)

where $d_{ij}$ denotes the additional traffic distribution. The relationship between the traffic distribution and the path flow can be formulated as follows:

$$\sum_{r \in R_{ij}} h_{ij}^r = d_{ij}, \quad i \in I, \, j \in J,$$

(8)

where $h_{ij}^r$ denotes the route flow between $i$ and $j$ on route $r$. The relationship between the traffic flow $v$ in the network and the path flow can be defined as follows:

$$v = A (\bar{h} + h),$$

(9)

where $A$ denotes an incidence matrix regarding road link-path. If the road link $s$ is on path $r$, it is equal to 1; otherwise it equals 0. $\bar{h}$ denotes the path flow driven by the current traffic demand, while $h$ denotes the additional path flow driven by the additional traffic demand. Thus,

$$f_l = \frac{N_l}{T_l (A (\bar{h} + h))} = \frac{N_l}{T_l (\bar{h} + h)},$$

(10)

where $N_l$ represents the number of buses on the route $l$, which is often held constant:

$$f_l = f_l (\bar{h}, h).$$

(11)

The passenger flow at the stop is a function of the bus frequency. It makes it hard to obtain the Karush-Kuhn-Tucker (KKT) condition of the Stochastic User Equilibrium (SUE) model [22] and leads to the difficulty of solving the model. When a variable (such as bus frequency) is held constant, the VI model is used to avoid this problem. The VI model can be transformed to the UE optimization model, which has a unique solution.

3.1. VI Model Formulation. It has been proved that the network equilibrium model and the VI model are equivalent. Assuming that the traffic cost $t_{ij}^o > 0$ and the additional traffic demand $d_{ij} \geq 0$, let $x = (\bar{h}, h, d, f^v)^T$, where $x \in R^a$ is a vector of parameters; $\bar{h}$ denotes the path flow driven by the current traffic demand; $h$ denotes the additional path flow driven by the additional traffic demand; $d$ denotes the additional traffic distribution; $f^v$ is the bus frequency vector of route. According to the SUE model, a continuous function $F$ of each variable can be referred to as formula (12) by the letters $A, B, C,$ and $D$ in vector:

$$F \left( h^{n_i} \right) = \left( F_1^i (\bar{h}^{n_i}), F_2^i (\bar{h}^{n_i}), \ldots, F_n^i (\bar{h}^{n_i}) \right)^T,$$

$$= \left( \frac{1}{\theta_1} \ln h^{n_i} + \sum_{s \in S} (t_s + u_s (f^v))^1 \delta^i_{s1}, \frac{1}{\theta_1} \ln h^{n_i} + \sum_{s \in S} (t_s + u_s (f^v))^2 \delta^i_{s2}, \ldots, \frac{1}{\theta_1} \ln h^{n_i} + \sum_{s \in S} (t_s + u_s (f^v))^n \delta^i_{sn} \right)^T = A,$$

$$F \left( h^{n_i} \right) = \left( F_1^i (h^{n_i}), F_2^i (h^{n_i}), \ldots, F_n^i (h^{n_i}) \right)^T,$$

$$= \left( \frac{1}{\theta_2} \ln h^{n_i} + \sum_{s \in S} (t_s + u_s (f^v))^1 \delta^i_{s1}, \frac{1}{\theta_2} \ln h^{n_i} + \sum_{s \in S} (t_s + u_s (f^v))^2 \delta^i_{s2}, \ldots, \frac{1}{\theta_2} \ln h^{n_i} + \sum_{s \in S} (t_s + u_s (f^v))^n \delta^i_{sn} \right)^T = A,$$

$$F \left( d^{n_i} \right) = \left( F_1^i (d^{n_i}), F_2^i (d^{n_i}), \ldots, F_n^i (d^{n_i}) \right)^T,$$

$$= \left( \frac{1}{\theta_2} \ln d^{n_i} + c_j, \frac{1}{\theta_2} \ln d^{n_i} + c_j, \ldots, \frac{1}{\theta_2} \ln d^{n_i} + c_j \right)^T = C,$$

$$F (f^v) = (0, 0, \ldots, 0)^T = D,$$

(12)

where $\theta_1, \theta_2 > 0, \quad i \in I, \quad j \in J.$

The VI model can be expressed as follows:

$$F \left( x^v \right)^T (x^v - x^v) \geq 0,$$

(13)

where there is a constraint $\Omega = \{G(x) \geq 0, H(x) = 0\}$. In formula (12), $t_i$ represents the travel cost on the road link $s$; $u_s^i$ denotes the user waiting time on the road link $s$ and $c_j$ denotes the destination attractive function. The passenger waiting time on the road link $s$ is determined as $u_s = \alpha / f_s$, $f_s$ denotes
The bus frequency on the road link $s$ and is equal to the bus frequency of route $l$, $f_j$. The parameter $\alpha$ is determined by the distribution value of bus headway, where $\alpha = 1$ represents an exponential distribution and $\alpha = 0.5$ represents a uniform distribution [23]. $\alpha = 0.5$ is commonly used in the practical analysis.

Assuming that $x^*$ is a solution of the VI model (13) and the linear independence condition is satisfied at $(x^*)$, then the GKKT (General Karush-Kuhn-Tucker) conditions are satisfied.

The constraint $H(x) = 0$ can be yielded as follows:

$$
\sum_{j \in J} d_{ij} = o_i, \quad i \in I,
$$

$$
\sum_{r \in R_{ij}} h_{ij}^r = \bar{d}_{ij}, \quad i \in I, \quad j \in J,
$$

$$
\sum_{r \in R_{ij}} \bar{h}_{ij}^r = \bar{d}_{ij}, \quad i \in I, \quad j \in J,
$$

and the constraint $G(x) \geq 0$ can be yielded as follows:

$$
0 \leq \frac{\sum o_i}{(\sum d_i + \sum o_i)} \leq \gamma,
$$

$$
\bar{h}_{ij}^r \geq 0, \quad i \in I, \quad j \in J, \quad r \in R_{ij},
$$

$$
\bar{h}_{ij}^r \geq 0, \quad i \in I, \quad j \in J, \quad r \in R_{ij},
$$

$$
d_{ij} \geq 0, \quad i \in I, \quad j \in J,
$$

$$
0 \leq f \leq 60.
$$

Formula (17) is based on the condition that the addition ridership accounts for less than a percent of the total ridership. According to the TCRP118, the percentage was suggested as 25%.

In formula (21), the part $^f \leq 60$ is based on the practices. Usually the time interval of two consecutive buses is in the range of 1 min to 6 min; thus $f_{\text{max}} = 1/(1/60) = 60$ veh/h. The GKKT conditions are defined as follows:

$$
\frac{1}{\theta_1} \ln h_{ij}^* + \sum_j (t_s + u_j (f^*)) s_{ij}^r - v_{ij}^s = 0,
$$

$$
\frac{1}{\theta_2} \ln h_{ij}^* + \sum_j (t_s + u_j (f^*)) s_{ij}^r - v_{ij}^s = 0,
$$

$$
\frac{1}{\theta_2} \ln d_{ij} - v_{ij}^s + v_{ij}^s + c_j = 0,
$$

where, $i \in I$, $j \in J$, $r \in R$.

By invoking (14), (16) is solved as follows:

$$
d_{ij}^* = \frac{\exp[-\theta_2 (v_{ij}^s + c_j)]}{\sum_{j \in J} \exp[-\theta_2 (v_{ij}^s + c_j)]}.
$$

Let $t_{ij}^r = \sum_j t_{ij}^r \delta_{ij}^r$ present the travel cost on route $r$ and $u_j^r = \sum_i u_i j^r \delta_{ij}^r$ present the user waiting time on route $r$. Equations (15) and (16) can be derived as follows:

$$
\bar{h}_{ij} = \frac{\exp[-\theta_1 (t_{ij}^r + u_j^r)]}{\sum_{k \in R_{ij}} \exp[-\theta_1 (t_{ij}^r + u_j^k)]},
$$

$$
\bar{h}_{ij} = \frac{\exp[-\theta_2 (t_{ij}^r + u_j^r)]}{\sum_{k \in R_{ij}} \exp[-\theta_2 (t_{ij}^r + u_j^k)]},
$$

where $P_{ij}^r = \exp[-\theta_1 (t_{ij}^r + u_j^r)] / \sum_{k \in R_{ij}} \exp[-\theta_1 (t_{ij}^r + u_j^k)]$ is the logit model based on the path selection probability.

3.2. Solution Algorithm. When $f$ is given, let $F = V g$; the objective function $g$ can be written as follows:

$$
\min g (\bar{h}_{ij}, d_{ij}) = \frac{\sum_{s \in S} h_{ij}^s (t_s + u_s) d_{ij} \cdot \exp[-\theta_1 (t_{ij}^r + u_j^r)]}{\sum_{r \in R_{ij}} \exp[-\theta_1 (t_{ij}^r + u_j^k)]} + \sum_{j \in J} \left[ \int_0^{\gamma} c_j (y) dy \right]
$$

$$
+ \frac{1}{\theta_1} \sum_{i \in I} \sum_{j \in J} \sum_{r \in R} \bar{h}_{ij} (\ln h_{ij}^* + 1)
$$

$$
+ \frac{1}{\theta_2} \sum_{i \in I} \sum_{j \in J} d_{ij} (\ln d_{ij} - 1)
$$

which subjects to constraints (14)–(21). This model is a SUE assignment problem and satisfies the KKT conditions.
In public transport problems, the passenger travel time is independent of the traffic flow [24–27]. Once the bus frequency is certain, the expected passenger travel time \( C_{ij}^r \) is independent of the path flow and the bus frequency:

\[
C_{ij}^r = t_{ij}^r + u_{ij}^r + e_{ij}^r, \quad i \in I, \ j \in J, \ r \in R_{ij},
\]

(26)

t_{ij}^r \) denotes the travel time at the road link \( r \), \( u_{ij}^r \) indicates the waiting time at the stop, and \( e_{ij}^r \) is the random error. As long as \( f \) is fixed, the optimization problem (25) can be solved without iteration. Meanwhile, the VI model can be transformed into the optimization SUE model to solve the other two variables \( h \) and \( d \). This characteristic can be used in the similarity diagonalization method to iterate and solve the VI model based on the elastic frequency.

4. Numerical Test

4.1. Network Basic Information. In order to demonstrate the performance of the proposed model and solution algorithms, numerical tests were carried out on a real transit network in Changzhou city (see Figure 1). In this study, Wujin district (node 1) and New North district (node 2) are considered as the origins, while Jin Ling road (node 3) and River road (node 4) are considered as the destinations. \( L_1^1 \) (from node 1 to node 5), \( L_1^2 \) (from node 5 to node 6), and \( L_1^3 \) (from node 6 to node 3) are road links of route \( L_1 \), while \( L_2^1 \) and \( L_2^2 \) are the road links of route \( L_2 \).

The road link information of the transit network shown in Figure 1 is given in Table 2. \( L_2 \), \( L_3 \), and \( L_6 \) are BRT routes. The traffic demand is shown in Table 3.

The dwelling time at a stop under different configurations can be yielded as follows:

- primary configuration:
  \[
  d_n^i = \frac{12 + 2.58 (A_{i n}^l + B_{i n}^l)}{3600},
  \]
  (27)

- intermediate configuration:
  \[
  d_n^i = \frac{12.33 \times 0.9 + 12 \times 0.1 + 1.51 (A_{i n}^l + B_{i n}^l)}{3600},
  \]
  (28)

- advanced configuration:
  \[
  d_n^i = \frac{13 \times 0.9 + 12 \times 0.1 + 0.72 (A_{i n}^l + B_{i n}^l)}{3600},
  \]
  (29)

The dwelling time for a regular bus:

\[
  d_n^i = \frac{10 + 2.58 (A_{i n}^l + B_{i n}^l)}{3600}.
\]

4.2. Results. Through the numerical test, it is concluded that the bus frequency under different configurations of BRT line 1 (\( L_2 \) in the network) increases with the traffic flow. The increase of the bus frequency under primary configuration is bigger than that under advanced configuration (Figure 2).

Figure 3 shows the change of the bus frequency on each route with the increase of traffic flow, where the BRT system is under intermediate configuration. The bus frequency variation of BRT lines (\( L_2 \), \( L_3 \), and \( L_6 \)) is more uniform than that of common ones (\( L_1^1 \), \( L_1^2 \), \( L_1^3 \), and \( L_7 \)). The bus frequency of \( L_1 \) is the most sensitive to the additional traffic flow, while the frequency of \( L_3 \) is lower than others because of the shorter travel distance. In addition, Figure 4 shows the additional traffic flow occupy. From Figure 4, it can be found that the total traffic flow increases obviously and the percentages of total traffic flow under different configurations are 11.9%, 18.8% and 22.8%, respectively compared with the original traffic flow (5000/h). The additional traffic flow occupies a percentage of total traffic flow that is 11.9%, 18.8%, and 22.8%, respectively.

5. Conclusions

In this study, the additional ridership prediction method based on VI model under different BRT system configuration are presented, as well as the solution algorithm. The VI model is turned into the SUE model, in order to avoid the solving difficulty resulted from the elastic frequency. A transform between the VI model and the UE model is proposed in the solution, where the similarity diagonalization method can be used. The additional traffic flow under different configurations of BRT system is computed in the numerical test. The results show that the potential ridership of different configurations are obviously different, and the bus frequency under primary configuration has the largest increase with the rise of network traffic flow. It also can be seen that the potential ridership have a big proportion in the total ridership.

Considering those differences in the construction of BRT, different elements of the system should be chosen in different development stages to achieve the objective of TOD, in addition to saving the cost of investments.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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