Analysis on the frequency dispersion characteristics of seismic wave caused by low frequency sound source in shallow sea

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A B S T R A C T

The elastic wave in seabed caused by low frequency noise radiated from ship is the so-called ship seismic wave which can be used to identify ship target. In order to analyze the wave components and the propagating properties of ship seismic wave, numerical calculation for frequency dispersion curves of seismic wave caused by low frequency sound source in shallow sea has been carried out. According to the numerical example for shallow sea with hard sea bottom, time series of seismic wave at seabed are mostly composed of normal modes and interface waves. Each normal mode has a well defined low frequency cut-off, on the contrary the interface wave doesn’t have. The frequency dispersion of normal mode is obviously when frequency is lower than 100 Hz, while interface wave is dispersive only in the infra-sound frequency range. The time series of seismic wave is dominated by interface wave when the source frequency is less than the minimal cut-off frequency of normal modes. As for shallow sea with soft sea bottom, there is only one real root in the dispersion equation of seismic wave. The root corresponds to the interface wave which propagates along the seabed at a slow speed. More normal modes appear when the depth of sea water increases and more source energy transmits through the wave-guide of sea water rather than along the seabed by the interface wave. Normal modes gradually become the significant carriers of source energy and interface wave weak accordingly.

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1. Introduction

In recent years, conventional sonar is faced with big challenge with the appearance of quiet submarine whose noise level in high frequency range is approaching that of the ambient shallow sea. However, the effect of absorbing vibration in low frequency range of quiet submarine is mostly not obvious, and elastic wave is excited in seabed by the low frequency noise of submarine in shallow sea. This kind of elastic wave is commonly called ship seismic wave which is important for conventional sonar to detect quiet submarine in shallow sea (Lu et al., 2004; Chen and Lu, 2005). Investigation of the potentialities of using seismic sensors for the detection of ships and other naval platforms has been carried out earlier in about 40 years ago (Urick, 1968; Richard, 1976). At present, a lot of theoretical calculations and analysis of seismic wave at seabed caused by low frequency sound source have been carried out with the approaches of normal mode (Yan and Zhou, 2006), fast field program (Lu et al., 2011) and parabolic equations (Zhang et al., 2010). The propagating properties of seismic wave were analyzed by the calculated results of transmission lose curves, frequency response curves and so on. The wave components of seismic wave in far-field are normal modes and interface wave in shallow sea. The normal modes propagate mainly in the wave guide of sea water, and the interface wave along the seabed. The two components of seismic wave are guidewaves whose geometrical spreading losses are relatively small compared with body waves and can propagate to far field. The frequency dispersion curve is an efficient tool to analyse the phase speed and group speed of guidewave (Roth et al., 1998; Shao et al., 2007; Zhu et al., 2013). In this work, we first introduced the numerical calculation for frequency dispersion curves of seismic wave caused by low frequency sound source in shallow sea. Second, the frequency dispersion curves of seismic wave in shallow sea with hard sea bottom, soft sea bottom and different depth of sea water were calculated. Lastly, the wave components and the propagating properties of ship seismic wave were analyzed based on these frequency dispersion curves. This work will provide some basic theoretical results for conventional sonar to detect quiet submarine with seismic wave in shallow sea.

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2. Numerical calculation for frequency dispersion curves

For a source distribution along a vertical axis in a stratified half-space, a cylindrical coordinate system is introduced in Fig. 1. The stratified environment is made by a homogeneous, ideal fluid layer above and a half-space of homogeneous, isotropic elastic solid at the sea bottom.

In Fig. 1, \( c_{p1}, \rho_1 \) are the P-wave speed and density of sea water; \( c_{p2}, c_{s2}, \rho_2 \) the P-wave, S-wave speed and density of rock and soil stratum in seabed; \( d \) denotes the source depth; \( H \) means the thickness of fluid layer; \( \phi_1 \) represents the displacement potential of P-wave in shallow sea, and \( \phi_2, \chi_2, \psi_2 \) the displacement potentials of P-wave, SH-wave, SV-wave in seabed.

The displacement in seabed can be derived from potentials \((\phi_2, \chi_2, \psi_2)\),
\[
\mathbf{u} = \nabla \phi + \nabla (\chi \mathbf{e}_r + \chi \nabla \times \mathbf{u}) + \mu (\mathbf{e}_r \times \mathbf{u}) + \rho \mathbf{f}
\]  
(1)
where \( \mathbf{r} = r \mathbf{e}_r + \phi \mathbf{e}_\theta + \psi \mathbf{e}_z \), \( \mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z \) denote the radial, tangential, axial unit vectors in cylindrical coordinates.

The propagation of stress-waves in the elastic rock and soil stratum are described by the displacement equation of motion,
\[
\frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda + 2\mu) \nabla \nabla \mathbf{u} - \mu \nabla (\nabla \mathbf{u}) + \rho \mathbf{f}
\]  
(2)
where \( \lambda \) and \( \mu \) are the Lamé constants of rock and soil stratum; \( \mathbf{f} \) denotes force vector of unit mass and is zero if the influence of gravity is ignored. We now substitute (1) into (2) to obtain the wave equations of displacement potentials which describe the propagation of P-wave, SH-wave and SV-wave in seabed as follows,
\[
\nabla^2 \phi - \frac{1}{c_p^2} \frac{\partial^2 \phi}{\partial t^2} = \delta(t) \delta(z-d) S(t)
\]  
(3)
\[
\nabla^2 \chi + \frac{\partial^2 \chi}{c_s^2} = 0
\]  
(4)
\[
\nabla^2 \psi - \frac{1}{c_s^2} \frac{\partial^2 \psi}{\partial t^2} = 0
\]  
(5)
where \( c_p = \sqrt{(\lambda + 2\mu)/\rho} \) and \( c_s = \sqrt{\mu/\rho} \). \( \delta(t) \) is the Dirac delta function, \( \delta(t) \delta(z-d) S(t) \) represents a point sound source at \((r, z) = (0, d)\) with function \( S(t) \). With the z-axis passing through the sound source and the rock and soil stratum being homogeneous isotropic elastic solid, the field is independent of azimuthal angle, so the SH-wave is disappeared.

The wave equation of fluid layer can also be derived from the elastic equation of motion [2]. We simply let \( p \) equal to density of seawater \( \rho_1 \), set \( \mu = 0 \) and substitute \( \lambda \) with the bulk modulus \( K = \rho_1 c_{p1} \) of sea water. Thus, the wave equation of fluid layer similar to (3) will be obtained.

The boundary conditions for wave equations above are
\[
p = 0, \ z = 0
\]  
(6)
\[
p = (\sigma_{xz})_2, \ z = H
\]  
(7)
\[
(\sigma_{zt})_2 = 0, \ z = H
\]  
(8)
where \( p \) refers to the acoustic pressure in fluid media, \((\sigma_{xz})_2\) and \((\sigma_{zt})_2\) the normal stress and the tangential stress in solid layer, \( w \) the vertical displacement.

As discussed by Jensen et al. (1993) wavenumber integration technique can be used to transform the wave equations in time-space domain into frequency-wavenumber domain by first applying the Fourier transform to obtain the Helmholtz equations and secondary the Hankel transform to get the depth-separated wave equations which are second order ordinary differential equations,
\[
(d^2/dz^2 + k_1^2) \phi_1(k_z, z) = S_0 \delta(z-d)/(2\pi)
\]  
(9)
\[
(d^2/dz^2 + k_2^2) \phi_2(k_z, z) = 0
\]  
(10)
\[
(d^2/dz^2 + k_2^2) \psi_2(k_z, z) = 0
\]  
(11)
where \( S_0 \) is the source strength at angular frequency \( \omega \), \( k_z \) is the vertical wavenumbers of compressional wave in sea water, \( k_z \) and \( k_2 \) are the vertical wavenumbers corresponding to the compressional and shear waves in seabed, respectively, \( \theta \) denotes the horizontal wavenumber. The general solutions of the second order ordinary differential equations above are given by
\[
\phi_1(k_z, z) = S_0 \exp(\pm ik_z(z-d))/(4\pi i k_z) + A^- \exp(-ik_zz) + A^+ \exp(ik_zz)
\]  
(12)
\[
\phi_2(k_z, z) = A_z^- \exp(-ik_zz) + A_z^+ \exp(ik_zz)
\]  
(13)
\[
\psi_2(k_z, z) = B_z^- \exp(-ik_zz) + B_z^+ \exp(ik_zz)
\]  
(14)
Each general solution is composed of an exponentially decaying and an exponentially growing solution. \( A_z^- \) , \( A_z^+ \) are wavefield amplitudes in the rock and soil stratum corresponding to up-going and down-going P-waves, \( B_z^- \) and \( B_z^+ \) to shear waves, respectively. In fluid media only \( A^- \) and \( A^+ \) exist.

If the boundary conditions of displacement and stress from (6) to (9) were satisfied and expressed with the wavefield amplitudes above, we obtain the following linear system of equations,
\[
\mathbf{D} \mathbf{X} = \mathbf{S}
\]  
(16)
where \( \mathbf{D} \) and \( \mathbf{S} \) are global coefficient matrix, global degree-of-freedom vector and global source-field discontinuity vector, respectively.

According to the normal-mode theory, the dispersion relation for the normal modes can be derived by setting the determinant of global coefficient matrix in (16) to zero (Parra and Xu, 1994).
\[
\det(\mathbf{D}(k_z, \omega)) = 0
\]  
(17)
The real roots of (17) correspond to normal modes propagating without leaking energy away from the waveguide (other than geometrical spreading loss). These normal models can be further classified into interface wave propagating along the seafloor and normal modes propagating in the waveguide of shallow sea. The root-finding method we adopted in this paper is the brute-force search method. Given user specified upper and lower bounds of \( k_z \), one selects a certain \( \Delta k_z \) and simply steps along the \( k_z \)-axis evaluating the determinant at each point. The root lies within the intervals where a sign-change is encountered. The \( \Delta k_z \) required should be very small since it is difficult to know a priori how closely spaced the nearest roots may be. The more conservatively one picks this step, the more robust the method is, but then the efficiency suffers. After finding all the real roots of dispersion relation (17) using the brute-force search method, we define the phase velocity as \( c = \omega/k \) and the group velocity as \( c_g = d\omega/dk_z \). Last, we can plot the dispersion curves of phase velocity and group velocity.
In addition, after solving the global system of (16), we get the amplitudes of the up- and downgoing waves in each layer. The displacement potentials in frequency-wavenumber domain can be readily evaluated at any depth by inserting these amplitudes to (13)–(15). The total field at any angular frequency \( \omega \) is obtained by performing the inverse Hankel transform of (13)–(15) at depth \( z \).

For example, the acoustic pressure in fluid layer can be calculated through the following expression where H Hankel function has been replaced by its asymptotic form,

\[
p(r,z,\omega) \approx \rho_i \omega^2 \sqrt{1/(2\pi r)} \exp(-\pi i/4) \times \int_0^{\infty} \phi_1(k_i,z) \sqrt{k_i} \exp(ikr) dk_i,
\]

where \( N \) is the total number of sample points and is usually an integral power of 2, \( \Delta \omega = \omega_{\text{max}} / (N/2 - 1) \).

We now replace the integral by a discrete sum. However, according to standard sampling theory, the discretization in frequency introduces periodicity of \( T \) in time. The actual response in the selected time window \([t_{\text{min}}, t_{\text{min}} + T]\) then becomes

\[
p(r,z,t_j) = \frac{\Delta \omega}{2\pi} \sum_{l=-(N/2-1)}^{N/2-1} [p(r,z,\omega) \exp(2\pi i N l / T)]_{\omega=\omega_l} \psi(t_j - nT) \sum_{n=0}^{N-1} p(r,z,t_j + nT) \]

The last sum represents the wrap-around or aliasing from the periodic time which gives negligible contribution to the result if the time window is properly chosen as follow,

\[
t_{\text{min}} \leq r/c_{\text{max}} \]

and

\[
T \geq T_{\text{max}} \left[ \frac{1}{u_{\text{min}}} - 1/c_{\text{max}} \right] \]

\[
\Delta t = T / N < \frac{1}{8f_{\text{max}}} \]

where \( c_{\text{max}}, f_{\text{max}} \) and \( u_{\text{min}} \) are maximum speed of media, maximum frequency concerned and minimal group velocity of the time series. The responses of displacement or stress in the selected time window can be simply calculated likewise.

More fields can be obtained by simply substituting the wavenumber kernel in (18) with other field parameter of interest, e.g., displacement or stress component. It has been shown that except for ranges shorter than a few wavelengths and very steep propagation angles, accurate evaluation of the inverse Hankel transform, (18), can be obtained by the so-called FFP (Fast Field Program) integration technique introduced by DiNapoli and Deavenport (1980).

The solution of time-dependent wave Eqs. (3)–(5) can be obtained via a Fourier transform of the frequency-domain solution. For example, the acoustic pressure in fluid layer is

\[
p(r,z,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_n(p(r,z,\omega) \exp(i\omega t)) d\omega
\]

where \( S_n \) is the source parameter of interest, e.g., displacement or stress. The time window can be simply calculated likewise.

More fields can be obtained by simply substituting the wavenumber kernel in (18) with other field parameter of interest, e.g., displacement or stress component. It has been shown that except for ranges shorter than a few wavelengths and very steep propagation angles, accurate evaluation of the inverse Hankel transform, (18), can be obtained by the so-called FFP (Fast Field Program) integration technique introduced by DiNapoli and Deavenport (1980).

3. Numerical examples and frequency dispersion characteristics

The geo-acoustic properties of a typical shallow sea environment are listed in Table 1. The sea water depth is 50 m. Following the approach by Zhang et al. (2010), we defined the sea bottom as hard sea bottom if its velocity of shear wave is greater than the sound speed in water, otherwise, the sea bottom as soft sea bottom.

A point source emits a wavelet given by

\[
f(t) = \left\{ \begin{array}{ll}
\frac{1}{2} & 0 \leq t \leq \frac{t_c}{2}
\cos \left[ \frac{2\pi f_c t}{2} \right] & t > \frac{t_c}{2}
\end{array} \right.
\]

where \( f_c \) is the center frequency, \( t_c \) describes the frequency bandwidth of wavelet. \( t_c \) here in this paper is always set to 0.2 s which guarantees a bandwidth of 20 Hz.

First, the frequency dispersion curves of seismic wave in shallow sea with hard sea bottom were calculated (Test 1). After finding all the real roots of the dispersion relation (17), we have plotted dispersion curves of phase velocity and group velocity shown in Figs. 2 and 3, respectively. There are interface wave and

<table>
<thead>
<tr>
<th>Layer</th>
<th>Density (kg/m$^3$)</th>
<th>P-wave speed (m/s)</th>
<th>S-wave speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea water</td>
<td>1000</td>
<td>1500</td>
<td>0</td>
</tr>
<tr>
<td>Hard sea bottom</td>
<td>1850</td>
<td>3200</td>
<td>1800</td>
</tr>
<tr>
<td>Soft sea bottom</td>
<td>1500</td>
<td>1800</td>
<td>800</td>
</tr>
</tbody>
</table>

**Table 1** Parameters for calculation.

**Fig. 2.** Dispersion curves of phase velocity in shallow sea with hard sea bottom.

**Fig. 3.** Dispersion curves of group velocity in shallow sea with hard sea bottom.
3 normal modes displayed in these two plots when the frequency range is below 100 Hz. There is no low-frequency cutoff for the interface wave, while each normal mode has a well-defined low frequency cutoff. As for normal modes, at high frequencies the phase and group velocities both approach the water sound speed (1500 m/s), whereas at cutoff frequency, phase velocities approach the sea bottom S-wave speed (1800 m/s). Similarly, at high frequencies the phase and group velocities of interface wave both approach a speed of 1340 m/s, whereas, the phase and group velocities approach a speed of 1650 m/s at zero frequency. While the phase velocity of interface wave is monotonically decreasing with frequency, the group velocity has a minimum (1270 m/s) at a certain frequency of about 9 Hz, which in time-domain solutions gives rise to the so-called Airy phase forming the tail of a transient interface wave arrival.

According to the dispersion characteristics shown in Figs. 2 and 3, the first mode only exists above the frequency about 20 Hz in Fig. 2. If the source pulse centered at 10 Hz and with a 20 Hz bandwidth, we immediately see from Fig. 2 that no normal mode exists and only the interface wave is excited. To complement the dispersion characteristics illustrated above, we present a snapshot (Fig. 4) of the pressure and vertical stress distribution in space at 2 s ($f_c = 10$ Hz). Note that the peak amplitude of the wave packet is maximal along the seafloor with exponentially decaying amplitude away from the guiding interface (wave is evanescent in both media). This corresponds exactly to interface wave. Thus, interface wave is the significant carrier of the source energy in this frequency range.

As the center frequency of source increases to 20 Hz (ranges from 10 Hz to 30 Hz), the snapshot of pressure and vertical stress for a source located at 10 m depth is shown in Fig. 5. The wave packets distributing within the radial ranges of 3.0 km correspond to the interface waves because their peak amplitudes are maximal along the seafloor with exponentially decaying amplitudes away from the guiding interface. As for the wave packets covering the radial ranges from 3.0 to 3.8 km, their peak amplitude distributions are evidently different from those of interface wave to consider that acoustic pressure reaches its maximum at the mid-depth of sea water and the vertical displacement (in Fig. 6) reverses in sea water layer which are the typical propagating characteristics of first mode according to the normal mode theory (Jensen et al., 1993; Zai-hua et al., 2013).

In order to analyze the influence of seabed hardness to the dispersion characteristics, we calculated the dispersion curves of seismic wave in shallow sea with soft sea bottom listed in Table 1 (Test 2). According to the dispersion relation (17), there is only one real root corresponding to the interface wave which propagates along the seafloor at very low speed. As shown in Fig. 7, the normal mode without leaking energy away from the waveguide does not appear. The interface wave is dispersive only below 5 Hz, otherwise, it propagates at a constant speed of 670 m/s. To complement the interface wave in Fig. 7, we present the range-stacked synthetic seismogram of vertical displacement received at seafloor shown in Fig. 8. It is obvious that only one wave packet exits in the time series of vertical displacement.

As for the influence of sea water depth to the dispersion characteristics of seismic wave in shallow sea, we analyzed the influence with a numerical example based on Test 1 by only changing the sea water depth to 100 m. The dispersion curves of phase velocity and group velocity were shown in Figs. 9 and 10, respectively. When compared to Figs. 2 and 3, it is clear...
that the interface wave is dispersive at lower frequency range (below 10 Hz) and the cutoff frequency of each normal mode decreases. As a result, more normal modes appear when the depth of sea water increases and more source energy transmits through the wave-guide of sea water rather than along the seafloor by the interface wave. Normal modes gradually become the significant carriers of source energy and interface wave weak accordingly.

To complement the above dispersion characteristics, we plotted the range-stacked synthetic seismogram of acoustic pressure received at seafloor for a source located at 10 m depth in Fig. 11. The last arrival with a group velocity of 1340 m/s is the interface wave whose amplitude is smaller than that of the normal modes which arrived earlier. The more normal modes exist, the more source energy transmits through sea water waveguide. Hence, the source energy which transmits in seafloor with interface wave apparently decreases and the wave packets of interface wave become weak accordingly.

4. Conclusions

In order to analyze the wave components and the propagating properties of ship seismic wave, the numerical calculation for frequency dispersion curves of seismic wave caused by low frequency sound source in shallow sea was carried out in this paper. The influence of hard sea bottom, soft sea bottom and different depth of sea water to the frequency dispersion characteristics of seismic wave in shallow sea were discussed. According to the results of numerical examples,

1) The time series of seismic wave at seafloor are mostly composed of interface wave and normal mode waves in shallow sea with hard sea bottom.

2) Each normal mode has a well defined low frequency cut-off, on the contrary the interface wave doesn’t have. The frequency dispersion of normal mode is obviously when frequency is lower than 100 Hz, while interface wave is dispersive only in the infra-sound frequency range.

3) The time series of seismic wave is dominated by interface wave when the source frequency is less than the minimal cut-off frequency of normal modes.

4) As for shallow sea with soft sea bottom, there is only one real root in the dispersion equation of seismic wave. The root corresponds to the interface wave which propagates along the seafloor at a slow speed.

5) More and more normal modes appear when the depth of sea water increases. More source energy transmits through sea water waveguide rather than in seafloor with interface wave. Normal modes gradually become the significant carriers of source energy and interface wave weak accordingly.

The wave components and the propagation properties of ship seismic wave in shallow sea have been discussed to some extent in this paper. The hydrogeology conditions in shallow sea are complex including shallow water, negative sound-speed gradients, sloping sea bottom and poro-elastic sediments. The ocean sediments are characterized by high energy loss due to internal friction. Therefore, in shallow sea environments with significant bottom interaction, it is crucial for a realistic modeling of the propagation characteristics of seismic wave that bottom attenuation be taken into account. The numerical examples in this work need to be improved to incorporate this effect. The propagation
properties of ship seismic wave in shallow sea need to be analyzed further.

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