Durability Life Prediction of a Reducer System Based on Low-amplitude Load Strengthening and Damage

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Durability life prediction is one of the key technologies that affect industrial development of electric vehicles. Load spectrum conversion is discussed in terms of standard road driving cycle and electric vehicle parameters, and a reducer system working load is calculated. A bending stress S–N curve mathematical model with 99% reliability is established based on the Haywood model and Basquin’s empirical equation. Extreme load usually shows the impact load of a gear in a reducer system. Extreme load is extrapolated and extended to 3000 km based on Shanghai’s standard road driving cycle. Reducer system durability test criteria are developed based on the extrapolated load spectrum. The load spectrum of a reducer system is completed based on equal damage. The fatigue life of a reducer system is estimated after considering the material strength under low-amplitude load strengthening and damage. This method also provides a reference to develop a durability test standard for reducer systems. This study provides a technical basis to develop reliable and durable electric vehicles that can be operated normally under urban road conditions.

Keywords: driving cycle, reducer system, load spectrum, durability life prediction

Highlights:
- Load spectrum conversion of Automotive key parts is discussed based on standard road driving cycle and electric vehicle parameters.
- Extreme load is extrapolated and extended to 3000 km based on Shanghai’s standard road driving cycle.
- Reducer system durability test criteria are developed according to the extrapolated load spectrum.
- The load spectrum and the fatigue life of a reducer system are completed based on equal damage.
- The fatigue life of a reducer system is estimated after considering the material strength under low-amplitude load strengthening and damage.

0 INTRODUCTION

Given the contradiction between strict automobile safety regulations and existing fatigue failure, engineers should focus on the reliability and durability of vehicles and vehicular components [1-6]. Wöhler proposed the S-N fatigue life curve and fatigue limit concept by systematically investigating the fatigue phenomenon in 1847, which established the classical fatigue strength theory foundation [7]. Wöhler designed the first fatigue test machine and researched the fatigue test of the locomotive wheel axle in 1850 [8]. Bauchinger introduced the stress strain hysteresis loop concept in 1886 according to the cyclic softening phenomenon [9]. Bairstow established the cyclic hardening and softening concept based on the stress–strain relationship of cyclic load and researched the multistage fatigue test in 1911 [10]. Palmgren proposed linear damage criteria initially applied to ball bearing products in Sweden in 1924 [11]. Gough published a comprehensive book on metal fatigue in 1926 based on multi-axis fatigue and on related studies [12]. Gassner proposed eight-stage program loading spectra of the variable amplitude fatigue test in 1941 according to load level-crossing counting results [13]. The Palmgren–Miner linear cumulative damage regulations were formed by Miner in 1945 according to numerous experimental studies [14, 15]. The Manson–Coffin equation proposed by Manson and Coffin in 1952 based on extensive experimental data is the basis for developing low-cycle fatigue [16]. Corten and Dolan presented nonlinear damage theory of the interaction between load time course and damage in 1956 [17]. Paris proposed Paris’s formula of the crack propagation law in 1963,
which provides a new estimation method for crack propagation life [18]. The partial stress-strain fatigue analysis method was proposed by Wetzel in 1971 based on the Manson–Coffin equation [19]. In addition, the partial stress–strain method was applied to estimate parts life [20–21]. Kachanov published a monograph on damage mechanics in 1986 [22]. Fatigue damage theory has been developed and applied to engineering together with fatigue damage mechanism [23–26].

Research on Chinese automobile durability started late, with Changchun Automotive Institute performing both road and bench tests on the key parts of a vehicle [27]. Dongfeng Automobile Company and Germany LFB Research Institute studied the fatigue life of different parts, such as truck wheels, front axle and frame girder [28]. Wang Dejun and Xu Hao studied the load order effect of two-dimensional distribution, load rejection criterion, and load spectrum enhancement, and summarised the fatigue load spectrum code [29]. The basis is the long-term in-depth study of aircraft structures, which established a solid foundation for the fatigue reliability design of Chinese mechanical structure [30]. The strain-life method in the framework of the Finite Element method has been used to determine the number of stress cycles required for fatigue crack initiation [31]. Method of solving the fuzzy finite element equations is discussed in monosource fuzzy numbers [32]. The fatigue life prediction model of random spectral load was established by considering the exercise effect of low-amplitude load according to the test results of different strength grades of automotive parts [33].

The working load of a reducer system is discussed under standard road cycling condition based on the working load characteristics of parts. The life prediction of a reducer system is studied whilst considering low-amplitude load strengthening and damage according to fatigue damage theory and load spectrum acceleration method. This work provides technical support for the lightweight design of a new Chinese energy automotive industry. The rest of this paper is organized as follows: Section 1 analyses the working load characteristics of a reducer system. Section 2 discusses the durability test criteria developed based on extrapolated load spectrum, and the load spectrum of a reducer system is completed based on equal damage. Section 3 shows the completion of the life prediction of a reducer system. Section 4 presents the discussion and conclusion.

1 REDUCER SYSTEM WORKING LOAD

A hub reducer system is the key part of a wheel drive electric vehicle; the reliability and durability of this system directly affect the operating status and performance of electric vehicles. Measuring the load spectrum of parts such as gears, shafts and bearings is difficult. Key parts are then designed according to the estimated load.

1.1 Hub Reducer System

The hub reducer electric-driven system in Fig.1 includes a motor system and a hub reducer. The hub reducer is discussed in this study. Forces from different road directions are transmitted from the tires to the hubs and then transmitted from the hub bearings to the reducer system. Finally, loads are transmitted to the frame through the suspension. The brake torque is transmitted to the reducer shell through the brake plate, and lateral force is transmitted to the reducer shell through the hub and hub bearing. Brake torque and lateral force are transmitted to the frame through the suspension. Therefore, the reducer shell should sustain the brake torque, longitudinal force, vertical force, lateral force and reaction force of the motor. The force conditions are comparatively complex. The electric vehicle parameters are shown in Table 1, whilst the drive motor parameters are shown in Table 2.

![Fig.1. Electric drive system](image)
Table 1 Electric vehicle parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Kerb mass/kg</td>
<td>1180</td>
</tr>
<tr>
<td>Mechanical efficiency</td>
<td>0.92</td>
</tr>
<tr>
<td>Rolling resistance coefficient</td>
<td>0.015</td>
</tr>
<tr>
<td>Frontal area/m²</td>
<td>2.0</td>
</tr>
<tr>
<td>Reducer system transmission ratio</td>
<td>6.6</td>
</tr>
<tr>
<td>First stage reducer system inertia/kg·m²</td>
<td>0.01</td>
</tr>
<tr>
<td>Air drag coefficient</td>
<td>0.38</td>
</tr>
<tr>
<td>Tire roll radius/m</td>
<td>0.3</td>
</tr>
<tr>
<td>Second stage reducer system inertia/kg·m²</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 2 Parameters of drive motor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent magnet synchronous motor Rated power</td>
<td>15</td>
</tr>
<tr>
<td>Peak power</td>
<td>25</td>
</tr>
<tr>
<td>Rated speed</td>
<td>4000</td>
</tr>
<tr>
<td>Peak speed</td>
<td>9500</td>
</tr>
<tr>
<td>Rated Torque</td>
<td>35.8</td>
</tr>
<tr>
<td>Peak Torque</td>
<td>90</td>
</tr>
</tbody>
</table>

1.2 Working Load Sample

An electric vehicle is studied according to Shanghai’s standard road driving cycle, which is composed of four urban road cycles and a suburban elevated road cycle [34]. According to Shanghai’s standard road driving cycle (Fig. 2), the road is continuously flat, the elevated road gradient is small and the average speed is low. Stepping on the accelerator, slamming the brakes and other bad driving habits occur. Concurrently, passenger and baggage over loadings also take place.

Fig. 2. Shanghai’s standard road driving cycle

According to the stress–strain law of metal materials, the stress–strain hysteretic effect is an inelastic phenomenon in the elastic range (Fig. 3). In a vehicle, the speed of both lift and drop change, and the gear load of the reducer system differs. The conversion coefficient of the rotating mass is corrected based on the stress–strain hysteresis effect in the material to accurately describe the load of the drive system.

The conversion coefficient of the rotating mass δ is corrected according to electric vehicle parameters when dealing with such vehicles. The correction result is shown in Fig. 4.

Fig. 3. Stress–strain hysteresis loop

Fig. 4. Correction result of the conversion coefficient of rotating mass

The drive motor output torque is obtained according to the rotary mass conversion coefficient and the parameters of the drive motor drive system, as shown in Fig. 5.
1.3 Data Processing of Working Load

The time course of the working load for an electric drive system is non-stationary and random. Moreover, the structure fails because of fatigue damage. The power between the drive motor and the reducer system separates when the vehicle is idling. The reducer system parts are not loaded with the load, and idling is then eliminated. The input torque amplitude–frequency histogram of the reducer system after eliminating idling is shown in Fig. 6.

2 LOAD SPECTRUM OF THE REDUCER SYSTEM

2.1 Load Extrapolated

A real load sample is limited to obtain all conditions, such as impact loads, when a vehicle is running on rough roads. The load sample range is enlarged to simulate more accurate results after extrapolation. Extreme loads usually exhibit the impact load of the reducer system gear. Extreme load is extrapolated using statistical theory [35, 36]. The load sample size is insufficiently large to show all conditions, especially for greatly infrequent loads.

After obtaining the torque probability distribution of the load sample based on statistical analysis, extreme load is generated through extrapolation of cumulative frequency distribution. A super value of $10^6$ cumulative cycles or 3000 km road cycles includes all loads and an unusually great load [37]. The load spectra are extrapolated to higher amplitudes and are smoother compared with the observed ones. The extrapolation method is based on random simulation of high maxima and low minima [38]. The input torque cartogram of the reducer system based on the rain flow method is shown in Fig. 7. The input torque cartogram of the reducer system after extrapolation is shown in Fig. 8.

The turning points of the time signal are extracted. In addition, the signal should be rainflow-filtered to remove small oscillations that do not contribute to fatigue damage. The bending fatigue strength of the second reduction gear pinion is the lowest in the reducer system whilst engaging the gears. Considering the transmission ratio and efficiency, the load super value cumulative frequency of the second reduction gear pinion is extended to $10^6$ and the maximum torque $T_{\text{max}}$ ranges from 155 Nm to 206 Nm after extrapolation, based on Fig. 9. The contrasts of the cumulative frequency distributions before and after extrapolation are shown in Fig. 9.
2.2 Gear Load Spectrum

The reducer gears are made of 20CrMnTiH. Table 3 presents the bending fatigue and contact fatigue limit of the prototype, as proven by the gear factory under 99% reliability confidence level. According to Table 3, the bending fatigue stress is 500 MPa when the bending fatigue cycle time is $3 \times 10^6$. During high-cycle fatigue, the fatigue strength logarithm linearly depends on the fatigue life logarithm. Meanwhile, the cycle times of the bending fatigue are $3 \times 10^6$, and the bending fatigue stress is 567 MPa through the linear interpolation method.

Table 3 Bending fatigue and contact fatigue limit of the prototype under 99% reliability confidence level

<table>
<thead>
<tr>
<th>Bending fatigue</th>
<th>Contact fatigue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle times/times</td>
<td>Stress/MPa</td>
</tr>
<tr>
<td>1000</td>
<td>1250</td>
</tr>
<tr>
<td>$3 \times 10^6$</td>
<td>500</td>
</tr>
<tr>
<td>$1 \times 10^9$</td>
<td>425</td>
</tr>
</tbody>
</table>

where $S_{1000}$ is the fatigue strength at a mean life of 1000 cycles; $S_{be}$ is the fatigue limit of the prototype; $b$ is the fatigue strength exponent; and $S_1$ and $N_1$ are the stress amplitude and fatigue cycle life of any one in the high cycle fatigue of the S–N curve, respectively. $S_2$ and $N_2$ are the stress amplitude of the unspecified number and the equivalent fatigue cycle life in the high cycle fatigue of the S–N curve, respectively. $f_a$ is the fatigue strength coefficient. $S_a$ is the stress amplitude. $N_f$ is the fatigue life.

According to Reference [35], the load type correction coefficient $C_L$ is 1.0 because the tooth root of the gear is subjected to the bending cyclic load of the pulsation cycle. The surface roughness $R_a$ is 0.8, and the surface quality correction coefficient $C_S$ is 0.91. Considering the fatigue data dispersion, the reliability correction coefficient $C_D$ is 0.814 under 99% reliability confidence level.

The bending fatigue limit $S_{be}$ is a constant value of 700 MPa according to the estimation principle of the bending fatigue limit.

$$ S_{1000} = \frac{S_{1000} C_R}{K_f'} $$

where the fatigue strength at a mean life of 1000 cycles $S_{1000}$ is 1250 MPa, the reliability level coefficient $C_R$ is 0.814 and the fatigue strength conversion coefficient $K_f'$ is 1.453.

According to Eq. (4), the bending fatigue limit $S_{be}$ is 747 MPa.
\[ \begin{align*}
\lg(S_{1000}') &= \lg(S'_f) + b \cdot \lg(2000) \\
\lg(S_f) &= \lg(S'_f) + b \cdot \lg(2 \times 10^6)
\end{align*} \] (5)

where the bending fatigue strength of the gear with a confidence level of 0.90, and the reliability 0.99 \( S_e \) is 300 MPa. \( S'_f \) is 2039 MPa and \( b \) is \(-0.1321\) after calculation.

The Haywood model is applied to evaluate the fatigue characteristics of the material strength limitation according to the fatigue data acquired from clients. The bending fatigue S–N curve can be expressed as follows under 99% reliability:

\[ S_n = S'_f \left(2N_f\right)^b = 2039\left(2N_f\right)^{-0.1321} \] (6)

The relationship between the bending stress and the cumulative cycle time for the second reduction gear pinion is shown in Fig. 10. Fig. 10 illustrates the cumulative frequency of the damage when the bending stress reaches a certain extent. The abscissa refers to the logarithmic coordinates. For example, the cumulative cycle frequency would be \(3+1=4\) times when the bending stress is larger than 413.5 MPa. Meanwhile, the cumulative cycle frequency would be \(4+3+1=8\) times if the bending stress is larger than 403.0 MPa. The maximum is 1.854 times the fatigue limit, and the number of cycles that exceed the fatigue limit is 39.51% of the total cycle number.

### Table 4: Bending stress of the tooth root and cumulative cycle time after extrapolation

<table>
<thead>
<tr>
<th>Input Torque/Nm</th>
<th>2.696</th>
<th>8.088</th>
<th>13.479</th>
<th>18.871</th>
<th>24.263</th>
<th>29.654</th>
<th>35.047</th>
</tr>
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<tbody>
<tr>
<td>Bending stress of tooth root/MPa</td>
<td>5.2</td>
<td>15.7</td>
<td>26.2</td>
<td>36.6</td>
<td>47.1</td>
<td>57.6</td>
<td>68.0</td>
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<tr>
<td>Frequency/#</td>
<td>106959</td>
<td>138257</td>
<td>42840</td>
<td>16599</td>
<td>9259</td>
<td>6834</td>
<td>3924</td>
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<tr>
<td>Input Torque/Nm</td>
<td>40.438</td>
<td>45.830</td>
<td>51.221</td>
<td>56.613</td>
<td>62.005</td>
<td>67.397</td>
<td>72.789</td>
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<td>Bending stress of tooth root/MPa</td>
<td>78.5</td>
<td>89.0</td>
<td>99.4</td>
<td>109.9</td>
<td>120.4</td>
<td>130.8</td>
<td>141.3</td>
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<tr>
<td>Frequency/#</td>
<td>1893</td>
<td>1051</td>
<td>581</td>
<td>294</td>
<td>163</td>
<td>138</td>
<td>184</td>
</tr>
<tr>
<td>Input Torque/Nm</td>
<td>78.181</td>
<td>83.572</td>
<td>88.964</td>
<td>94.356</td>
<td>99.748</td>
<td>105.140</td>
<td>110.531</td>
</tr>
<tr>
<td>Bending stress of tooth root/MPa</td>
<td>151.8</td>
<td>162.3</td>
<td>172.7</td>
<td>183.2</td>
<td>193.7</td>
<td>204.1</td>
<td>214.6</td>
</tr>
<tr>
<td>Frequency/#</td>
<td>277</td>
<td>356</td>
<td>425</td>
<td>444</td>
<td>402</td>
<td>331</td>
<td>244</td>
</tr>
<tr>
<td>Input Torque/Nm</td>
<td>115.924</td>
<td>121.315</td>
<td>126.707</td>
<td>132.098</td>
<td>137.490</td>
<td>142.882</td>
<td>148.273</td>
</tr>
<tr>
<td>Bending stress of tooth root/MPa</td>
<td>225.1</td>
<td>235.5</td>
<td>246</td>
<td>256.5</td>
<td>266.9</td>
<td>277.4</td>
<td>287.9</td>
</tr>
<tr>
<td>Frequency/#</td>
<td>206</td>
<td>346</td>
<td>544</td>
<td>706</td>
<td>705</td>
<td>524</td>
<td>287</td>
</tr>
<tr>
<td>Input Torque/Nm</td>
<td>153.666</td>
<td>159.057</td>
<td>164.449</td>
<td>169.841</td>
<td>175.232</td>
<td>180.624</td>
<td>186.017</td>
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<tr>
<td>Bending stress of tooth root/MPa</td>
<td>298.3</td>
<td>308.8</td>
<td>319.3</td>
<td>329.7</td>
<td>340.2</td>
<td>350.7</td>
<td>361.1</td>
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<tr>
<td>Frequency/#</td>
<td>132</td>
<td>60</td>
<td>56</td>
<td>44</td>
<td>49</td>
<td>31</td>
<td>27</td>
</tr>
<tr>
<td>Input Torque/Nm</td>
<td>191.408</td>
<td>196.800</td>
<td>202.191</td>
<td>207.583</td>
<td>212.975</td>
<td>229.149</td>
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<tr>
<td>Bending stress of tooth root/MPa</td>
<td>371.6</td>
<td>382.1</td>
<td>392.5</td>
<td>403.0</td>
<td>413.5</td>
<td>444.9</td>
<td></td>
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<tr>
<td>Frequency/#</td>
<td>17</td>
<td>16</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

The life test principle can be determined as follows after extrapolating the load spectrum. Firstly, load cycles below the material fatigue limit are disregarded; 50% of the fatigue limit for a standard test piece is considered an admissible filtering threshold to disregard small load cycles during notch testing. Secondly, 15% of the maximum load is chosen as the threshold if the fatigue limit is unknown, for which non-invasive cycles can be disregarded [35].

The program spectrum is used to simulate the loading experiment instead of the continuous work load spectrum. Whilst all load levels are partitioned using the interval method, the step load spectrum is generated through equal damage of the program and working load spectra. Given
that the small load with tiny, even unappreciable damage to the parts should be deleted, stresses at 45.8 MPa and under would be deleted in this spectrum. Such damages would be divided into nine intervals to obtain nine load levels in the test spectrum.

The load that is less than 50% of the fatigue limitation is disregarded based on the compilation criteria of the load spectrum. Tenth-grade load damage is ignored during the test because the damage of such load is significantly smaller than others. The load spectrum of the second reduction gear pinion is indicated in Table 5.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Torque/Nm</th>
<th>Bending stress of tooth root/MPa</th>
<th>Frequency /#</th>
<th>Fatigue life logarithmic of S-N curve</th>
<th>Fatigue life of S-N curve/#</th>
<th>Damage of each grade load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>229.1</td>
<td>444.9</td>
<td>1</td>
<td>5.00310916</td>
<td>50592</td>
<td>1.97661E-05</td>
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<tr>
<td>2</td>
<td>196.8</td>
<td>382.1</td>
<td>36</td>
<td>5.80544685</td>
<td>160109</td>
<td>0.000225061</td>
</tr>
<tr>
<td>3</td>
<td>175.2</td>
<td>340.2</td>
<td>164</td>
<td>5.88705896</td>
<td>385504</td>
<td>0.000424371</td>
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<td>4</td>
<td>153.7</td>
<td>298.3</td>
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<td>6.31883229</td>
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<td>0.000381375</td>
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<td>132.1</td>
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<td>6.68430397</td>
<td>2418098</td>
<td>0.001007385</td>
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<tr>
<td>6</td>
<td>110.5</td>
<td>246.0</td>
<td>985</td>
<td>6.9502407</td>
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<td>0.000219455</td>
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<td>7</td>
<td>89.0</td>
<td>204.1</td>
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<td>4.97449E-05</td>
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<td>8</td>
<td>67.4</td>
<td>162.3</td>
<td>615</td>
<td>8.32120802</td>
<td>104755788</td>
<td>5.87019E-06</td>
</tr>
<tr>
<td>9</td>
<td>45.8</td>
<td>120.4</td>
<td>554</td>
<td>9.3025776</td>
<td>100377811</td>
<td>5.52149E-07</td>
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<td>10</td>
<td>24.3</td>
<td>78.5</td>
<td>4068</td>
<td>10.7078077</td>
<td>2551395303</td>
<td>1.59444E-07</td>
</tr>
</tbody>
</table>

3 REDUCER SYSTEM LIFE PREDICTION

3.1 Disregarding Low-amplitude Load Strengthening and Damage

According to the gear system characteristics and experiments, if all gears in the gear system are made of the same material with a similar fatigue strength, then a smaller gear means an earlier possibility to fail. The highest bending stress is in the second reduction gear after analysis; thus, this gear should have the shortest life amongst any of the gears in the reducer. If the durability mileage of this pinion exceeds 150000 km, then this mileage indicates the durability of the electric drive system. Other gears do not fail during the ordinary operation of the vehicle. The gear damage after 3000 km can be obtained according to the load spectrum and the S-N curve at 99% reliability for the part after treatment with a zero mean. Based on Miner linear cumulative damage theory [39, 40], the part suffers bending fatigue failure when the damage value is greater than or equal to 1.

According to Miner linear fatigue damage rules, the damage to the gear is as follows with a durability mileage of 3000 km:

\[ D = \sum d_i = 0.002333739 \]

When there are 50 commutation cycles and the durability mileage attains 150000 km, the fatigue damage to the gear is expressed as

\[ D = 50D_i = 0.116686959 \]

The damage value at this point is less than unity. The gear then meets the durability requirements based on the research hypothesis of the Miner linear commutation damage. The durability mileage of the reducer system is expressed as follows when the damage value is 1:

\[ \frac{1}{0.116686959} \times 150000 = 1285500 \text{ km}. \]

Hence, the durability mileage of the reducer system is 1285500 km.

3.2 Considering Low-amplitude Load Strengthening and Damage

After low-amplitude load strengthening, the fatigue strength and the fatigue life of the part both improve under the same load. The low-amplitude load strengthening force model is shown in Fig. 11. After strengthening, the horizontal section of S–N curve is improved and the slope of S–N curve is maintained [41, 42].
The fatigue life of the original structure is $N_0$ (Point A) with stress $S_0$. After low load strengthening, the fatigue life of the structure increases to $N_1$ (Point D) with stress $S_0$. Before low load strengthening, the stress of the structure is $S_1$ with a fatigue life of $N_1$ (Point B). After low load strengthening, the structural stress increases to $S_0$ with fatigue life $N_1$ (Point D). $\sigma_{-1,0}$ and $\sigma_{-1,1}$ represent the fatigue strength or limit before and after low load strengthening, respectively. E and C are the turning points of the S-N curves.

Supposed that s grades of bending stress of root teeth exist in the reducer system. The high load is from 1 to j grade load, whilst the low load is from j+1 to s grade load. The fatigue life of each grade stress is $N_i$ and the cycle number is $n_i$.

The optimal strengthening cycles is $Y$ from j+1 to s grade load. In addition, the spectrum cycle blocks $M_1$ are as follows:

$$M_1 = \frac{Y}{\sum_{i=j+1}^{s} n_i}.$$  

(7)

During the new S–N curve II, the cumulative damage $D_1$ of a single cycle for 1 to j high amplitude load is calculated as follows:

$$D_1 = \sum_{i=1}^{j} \frac{n_i}{N_i}.$$  

(8)

The load spectrum from 1 to s grade is loaded to the parts. The test is finished when the parts fail. Suppose that the spectrum cycle blocks is $M_2$, the cumulative damage $D_2$ of a single cycle is calculated as follows:

$$D_2 = \sum_{i=j+1}^{s} \frac{n_i}{N_i}.$$  

(9)

The reducer gear strength improves after $M_1$. The S–N curve is then changed to II. According to Miner theory, the total cumulative damage with the S–N curve is 1, as follows:

$$M_1 \times D_1 + M_2 \times D_2 = 1.$$  

(10)

According to Eq. (10), the spectrum cycle blocks $M_2$ can be shown as

$$M_2 = \frac{(1-M_1 \times D_1)}{D_2}.$$  

(11)

The total cycle blocks $M_0$ is expressed as

$$M_0 = M_1 + M_2.$$  

(12)

The durability mileage $L$ is calculated by

$$L = 3000 \times M_0.$$  

(13)

Eq. (13) is the life prediction model based on the strength change characteristic. The model synthetically considers low-amplitude load strengthening and damage effect. The result of fatigue life prediction is a more practical situation.

The bending fatigue limit of the gear root is $\sigma_{-1}=300MPa$. According to the test data of low-amplitude load, the load strengthening zone of the gear is $(0.75,0.95) \sigma_{-1}$, and the optimal strengthening load is $0.85 \sigma_{-1}$. The optimal strengthening cycle is 300000, and the improvement ratio of the fatigue strength for the part is 4.2%.

The strengthening load zone of the reducer gear after calculation is 225285 MPa, whilst the optimal strengthening load is 255 MPa. The fifth and sixth grade loads are the strengthening loads in Table 5. The bending stress of the gear root and cycle numbers for the fifth load are 266.9 MPa and 2436, respectively. Moreover, 2436 cycles of the fifth load are equivalent to 895 cycles of the optimal strengthening load. The total cycles of the optimal strengthening load are 3070. After modifying the load spectrum, the optimal strengthening load is the fifth load. The seventh to tenth loads are undamaged and are deleted.

The fatigue life of the reducer system is calculated based on low-amplitude load strengthening and damage model. The fatigue life prediction flowchart is shown in Fig. 12.

Fifth stress is applied 300000 times to obtain the best strengthening effect. The spectrum cycle blocks of the strengthening effect can be calculated according to Eq. (7).  

$$M_1 = \frac{Y}{n_s} = \frac{300000}{3070} = 97.71.$$  

After the optimal strengthening cycle, the improvement ratio of the fatigue strength for the part is 4.2%. Given that the bending stress of the fifth load spectrum is less than the low-amplitude load of the fatigue limit, the fatigue life of the reducer system cannot change with the fifth load spectrum after strengthening. However, the
fatigue life of the other stress grades is shown in Table 6 with a new S–N curve.

![Diagram](image)

**Fig. 12. Fatigue life prediction model based on low-amplitude load strengthening and damage**

**Table 6 Fatigue life and damage of the reducer system with a new S-N curve after strengthening**

<table>
<thead>
<tr>
<th>Grade</th>
<th>Bending stress of tooth root/MPa</th>
<th>Frequency/#</th>
<th>Fatigue life of new S-N curve/#</th>
<th>Damage of each grade load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>444.9</td>
<td>1</td>
<td>69078</td>
<td>1.44764E-05</td>
</tr>
<tr>
<td>2</td>
<td>382.1</td>
<td>36</td>
<td>218612</td>
<td>0.000164675</td>
</tr>
<tr>
<td>3</td>
<td>340.2</td>
<td>164</td>
<td>526365</td>
<td>0.00031157</td>
</tr>
<tr>
<td>4</td>
<td>298.3</td>
<td>397</td>
<td>1422528</td>
<td>0.000279081</td>
</tr>
<tr>
<td>5*</td>
<td>255</td>
<td>3070</td>
<td>3415216</td>
<td>0.000898918</td>
</tr>
</tbody>
</table>
High amplitude loads from 1 to 4 are applied for 97.71 spectrum cycle blocks. According to Eq. (8), the cumulative damage is calculated as follows:

$$M_1 \times D_1 = M_2 \sum_{i=1}^{4} \frac{N_i}{N_i} = 0.075213874.$$ 

According to Eq. (9), the cumulative damage of the load spectrum from 1 to 5* is calculated by

$$D_2 = \sum_{i=1}^{5} \frac{N_i}{N_i} = 0.001668721.$$ 

According to the Eq. (11), $M_2$ is 554.19. According to Eq. (12), the total cycle blocks $M_0$ are 651.89. According to Eq. (13), the durability mileage of the reducer system $L$ can be calculated by

$$L = 3000 \times 651.89 = 1955700 \text{ km}.$$ 

The estimated fatigue life of the reducer system is 1955700 km given the low-amplitude load strengthening and damage. The fatigue life of the reducer system improves by 52.14% compared with that when disregarding the effect of low-amplitude load strengthening and damage.

4 RESULTS AND DISCUSSION

Load spectrum conversion is discussed through a vehicle travelling equation under standard road driving cycles and according to electric vehicle parameters. The working load of the reducer system is calculated. The sample size of load is insufficiently large to show all the conditions, especially for the infrequent great load. Extreme load is extrapolated and extended to 3000 km based on the Shanghai standard road driving cycle. The reducer system durability test criteria are developed based on the extrapolated load spectrum. The Haywood model is applied to evaluate the fatigue characteristics of the material strength limitation according to the fatigue data acquired from clients. The load spectrum of the reducer system is completed according to Miner linear fatigue damage rules.

The fatigue life of the reducer system is calculated based on low-amplitude load strengthening and damage. This method provides a reference to develop a durability test standard for reducer systems. This study also provides a technical basis to develop reliable and durable electric vehicles that can be operated normally under urban road conditions.

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**Durability Life Prediction of a Reducer System Based on Low-amplitude Load Strengthening and Damage**

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