Effects of multiple cracks on the forced response of centrifugal impellers

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ABSTRACT

The effects of multiple cracks on the forced response of centrifugal impellers are investigated using a finite-element based component mode synthesis method (CMS) in this paper. The main objective is to gain some insights into the response characteristics of multiple cracked impellers and to explore efficient methods for identifying the cracks. First, in order to generate an efficient model for the nonlinear vibration analysis, a novel hybrid interface CMS method is proposed and used to conduct reduced-order modeling for the cracked impeller. Then, a method for multiple cracks modeling is developed to account for the crack breathing effects. Finally, numerical results are presented using a representative impeller with double cracks. The shifts of natural frequencies and the nonlinear forced response due to multiple cracks are of interest. Lengths and relative positions of the cracks are also considered. The results show that the natural frequencies and forced response become complexly depending on the lengths and relative positions of cracks, and the response amplitudes of blades periodically fluctuate versus blade number when an impeller suffers from cracks or mistuning. A potential method for identifying the lengths and relative positions of multiple cracks are also discussed in this paper.

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1. Introduction

Cracks on key equipment are severe security risk in industrial production. The vibration of cracked structures and relevant detection techniques has been the research focus in the past decades [1–3]. Centrifugal compressors are important and widely used equipment in the steel, electric power and petrochemical industry. As centrifugal compressors advance towards high-speed and high-performance, frequent accidents due to cracks in impellers have been serious challenges for industrial applications, threatening the operation safety of the whole compressor sets. This issue has become the main concern for the reliabilities of centrifugal compressors [4–6].

In practical applications, it was found that multiple cracks frequently appeared at the weak weld joints of centrifugal impellers. Multiple cracks in impellers have received a lot of attention in turbomachinery industry. Efficient techniques for detecting the cracks have become an urgent need for the reliable operation of centrifugal compressors. However, the vibration response of multiple cracked impellers is not clear yet. Few works were reported to deal with the vibration of impellers with multiple cracks, much less to develop efficient techniques for crack detection. So, vibration analysis on
multiple cracked impellers is conducted in this paper to gain some insights into the response characteristics and to explore efficient methods for detecting the cracks.

Most of the previous studies on multiple cracks were concerning about the vibration analysis and crack identification for beams [7–11], rotors [12–16] and pipes [17,18], which were reviewed in literature [19]. Chasalevis et al. [7] studied the dynamic behavior of a cracked beam with two transverse surface cracks and combined both response and natural frequency measurements to identify multiple cracks. Mazanoglu and Sabuncu [8] presented the flexural vibration of non-uniform Rayleigh beams with single-edge and double-edge cracks. Maghsoodi et al. [9] used a simple method to localize and quantify multiple cracks in multi-stepped beams with natural frequencies and the estimated mode shapes. Chasalevis et al. [13] investigated the coupled bending vibration of a stationary shaft with two cracks. Lin and Chu [14] studied the dynamic behavior of a rotor system with slant cracks on the shaft. In another paper of Chu [16], parametric instability of a rotor-bearing system with two breathing transverse cracks was discussed. Murigendrappa et al. [17] presented a frequency-based experimental identification method for multiple cracks in straight pipes filled with fluid. Naniwadekar et al. [18] proposed a natural frequency-based technique for crack detection by modeling the cracks in a pipe as rotational springs. However, the structural features of impellers are quite different from beams, rotors and pipes. The conclusions obtained from these structures may not be suitable to centrifugal impellers.

Regarding axial compressors, several investigations [20–25] have been conducted on the vibration response of bladed disks with a single crack using simplified or finite element-based models. Kuang et al. [20] simplified blades as Euler–Bernoulli beams and treated the crack as local disorder of the system to study the effects of local disorder on the mode localization. Hou [21] studied the mechanisms of cracking-induced mistuning in bladed disks using a lumped-mass beams model. Huang [22] used pre-twisted blades to investigate the vibration localization due to crack in a blade system. Saito et al. [24] studied the effects of mistuning and cracking on the forced response of a bladed disk. Liu and Jiang [25] presented a cracked hexahedral finite element method for dynamic analysis of cracked blades. Wang et al. [26] presented an efficient method for nonlinear vibration analysis of mistuned centrifugal impellers with crack damages and investigated the effects of mistuning and cracks on the vibration features of centrifugal impellers. However, the effects of multiple cracks have not yet been reported. Moreover, the geometry of an impeller is more complex than a bladed disk, and no simplified model is available for centrifugal impellers. Besides, the mode shapes of an impeller are mostly cover-dominated and disk-dominated, whereas the mode shapes of interest in a bladed disk are blade-dominated. So, the effects of multiple cracks on the vibration of impellers are studied in this paper.

In this paper, a hybrid interface CMS method is employed and developed to generate an efficient model for the impeller with multiple cracks. The DOFs of nodes on the multiple crack surfaces are retained in the reduced order model to simulate the crack breathing effects. Several other DOFs on the interfaces between different substructures are also regarded as fix-interface coordinates to avoid rigid body motion. The obtained nonlinear dynamic differential equations are solved by the harmonic balance method. Based on the obtained model, an impeller with double cracks is employed as representative during numerical analysis. Natural frequency shift, nonlinear forced response and vibration localization due to multiple cracks are of interest in this paper. The lengths and relative positions of double cracks are also considered to parametrically study their effects on the forced response. Mistuning effects due to manufacturing tolerance and components deterioration are also taken into account. Finally, vibration localization of the double cracked impeller is presented.

This paper is organized as follows. In Section 2, a reduced-order modeling technique for impellers with multiple cracks is presented. Section 3 contains the numerical results of the impeller with multiple cracks. In Section 4, discussions on potential method for identifying the lengths and positions of cracks are presented. Conclusions are given in Section 5.

## 2. Reduced-order modeling for impellers with multiple cracks

In order to accurately predict the vibration response of an impeller, the finite element model with refined mesh grids should be employed. However, the finite element model may have more than a hundred thousand DOFs. Despite of the advances in computer hardware during recent years, it is still too expensive to employ such a large model to predict the forced response directly. Thus, an order reduction method should be used to reduce the orders of the model. In this section, the formulations of the reduced-order modeling method for impellers with multiple cracks are presented. The breathing effects of multiple cracks are also taken into account using a finite-element based contact method.

### 2.1. Equations of motion

In the modeling process of CMS, a structure is partitioned into several substructures. Each substructure is analyzed separately to reduce the number of DOFs. An impeller consists of one cover, one disk and several blades, as is shown in Fig. 1. The finite element model and its basic sector model are shown in the figure. According to practical experience, the positions of multiple cracks are mostly located at the weld toe on the cover side of blades, as is illustrated in Fig. 1.

Due to the numerous interfaces between substructures, the single-sector finite element model is used to generate the model of the whole structure to simplify the preprocessing step. Under the Cartesian coordinate system, the overall mass and stiffness matrices of the cover, blade and disk substructures can be obtained from the matrices of their single-sector
models by rotation transformation. The transformation equations of the substructures are uniformly written as

$$M_\text{s}^R = R_s^T \left( I_N \otimes M_\text{s}^C \right) R_s, \quad K_\text{s}^R = R_s^T \left( I_N \otimes K_\text{s}^C \right) R_s,$$

where subscript “s” is a label representing substructures, and the blade, cover and disk substructures are represented as “b,” “c” and “d” respectively; $M_\text{s}^R$ and $K_\text{s}^R$ are the mass and stiffness matrices after rotation; $M_\text{s}^C$ and $K_\text{s}^C$ are the mass and stiffness matrices of the single-sector model; $I_N$ is an identity matrix of dimension $N$, where $N$ is the number of sectors; the symbol $\otimes$ denotes the Kronecker product; $R_s$ is the rotation transformation matrix, which is a pseudo-block matrix with $R_n$ for $n = 1, 2, ..., N$ along its diagonal blocks, where $R_n$ is the rotation transformation matrix from the nth sector to the basic sector.

For the cover and disk substructures, redundant DOFs on the overlapped boundaries between every two adjacent sectors are eliminated by applying displacement compatibility. The simple flow chart of the modeling approach is shown in Fig. 2. When modeling cracked impeller with hybrid interface CMS method, the DOFs of a substructure are divided into internal, free-interface and fix-interface coordinates. The DOFs on the crack surfaces are regarded as fix-interface coordinates. Some interface DOFs are also regarded as fix-interface coordinates to eliminate rigid body motion. The DOFs of external force nodes are deemed as free-interface coordinates for applying external excitations. Then, the DOFs of the substructures are respectively represented as

$$u_b = \left\{ u_{\text{ins}}^b, u_{\text{lod}}^b, u_{\text{fre}}^b, u_{\text{fix}}^b \right\}, \quad u_c = \left\{ u_{\text{ins}}^c, u_{\text{fre}}^c, u_{\text{fix}}^c \right\}, \quad u_d = \left\{ u_{\text{ins}}^d, u_{\text{fre}}^d, u_{\text{fix}}^d \right\}$$

where the superscript “ins”, “lod”, “fre” and “fix” denote the DOFs in the internal, external load, free-interface and fix-interface sets, respectively. As multiple cracks locate on the interfaces between cover and blades, the crack DOFs exist only in $u_{\text{fix}}^b$ and $u_{\text{fix}}^c$, which can be further written as

$$u_{\text{fix}}^b = \left\{ u_{\text{crack}}^b, u_{\text{fix}}^b, u_{\text{fix}}^b \right\}, \quad u_{\text{fix}}^c = \left\{ u_{\text{crack}}^c, u_{\text{fix}}^c, u_{\text{fix}}^c \right\}$$
where $u_f^{\text{fix}}$ and $u_{bd}^{\text{fix}}$ are the fix-interface coordinates of blade with regard to cover and disk; $u_r^{\text{fix}}$ denotes the fix-interface coordinates of cover versus blade; $u_r^{\text{track}}$ and $u_r^{\text{track}}$ denote the DOFs of nodes on the crack surfaces. For example, if an impeller has two cracks located at the $i$th and $j$th sectors, $u_r^{\text{track}}$ and $u_r^{\text{track}}$ can be represented as

$$
u_r^{\text{track}} = \begin{cases} u_i^{\text{track}} \\ u_j^{\text{track}} \end{cases}, \quad u_r = \begin{cases} u_i^{\text{track}} \\ u_j^{\text{track}} \end{cases}, \quad i \neq j$$

(4)

where the element vectors in $u_r^{\text{track}}$ and $u_r^{\text{track}}$ denote the DOFs on the $i$th and $j$th crack surfaces. The number of element vectors depends on the number of cracks in the impeller. Then, the governing equations of motion of the three substructures can be uniformly written as

$$M \ddot{u}(t) + C \dot{u}(t) + K u(t) = f(t) + f_r(u)$$

(5)

where $u$ is the displacement vector of a certain substructure; $M$, $C$, and $K$ are the mass, damping and stiffness matrices; $b$ is the external force vector; $f_r(u)$ is a force vector of interface reactions. As external forces act only on blades, the vector $b(t)$ is a non-zero vector. So, $b(t)$ is simply written as $b(t)$ in the following parts. The external forces acting on blades are assumed to be travelling-wave excitation, and the force on the $n$th sector is represented as

$$b_n(t) = B \cos(\omega t - \phi_n), \quad n = 1, 2, \ldots, N$$

(6)

where $B$ is the force amplitude vector; $\omega$ is the angular frequency of excitation; and $\phi_n$ denotes the phase angle and is defined as $\phi_n = (n-1)2\pi C / N$, where $C$ is the engine order of excitation.

2.2. Reduced-order modeling

2.2.1. Substructure analysis

The hybrid interface CMS method in literature [27,28] is further developed for analyzing the vibration of impellers with multiple cracks. The DOFs transformations equations of the hybrid interface CMS method can be written as

$$u_s = \begin{bmatrix} u_t^s \\ u_c^s \end{bmatrix} = \begin{bmatrix} \Phi_s^t & \Psi_s^t & \Psi_s^d & \Psi_s^c & p_s^t \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Phi_s \Psi_s^t \\ f_s^t \\ C_s^t \end{bmatrix} = \Phi_s^t p_s$$

(7)

where $u_t^s$ is a displacement vector, and $u_c^s = [u_t^{\text{int}}, u_c^{\text{bd}}, u_c^{\text{fre}}]$; $u_c^s$ is the displacement vector of fix-interface DOFs; $I$ is an identity matrix; $\Phi_s^t$ denotes a truncated set of normal modes; $\Psi_s^t$ denotes the attach modes with regard to external excitation DOFs; $\Psi_s^d$ denotes the attach modes of free-interface DOFs; $\Psi_s^c$ is the constraint modes of fix-interface DOFs; $p_s$, $f_s^t$, $C_s^t$ and $u_c^s$ are the generalized coordinates corresponding to the above modes. For the cover and disk, $\Psi_s^d$ and $f_s^d$ no longer exist because there are no excitation nodes on the two components. Substituting Eq. (7) into Eq. (5), the governing equations can be represented as

$$M_d p_d + C_{dh} p_d + K_{dh} p_d = b_{dh}(t) + f_{dh}(p_d)$$

(8)

where $M_d$, $C_{dh}$ and $K_{dh}$ are the mass, damping and stiffness matrices of components; and

$$M_d = \Phi_s^d M \Phi_s^d, \quad C_{dh} = \Phi_s^d C \Phi_s^d, \quad K_{dh} = \Phi_s^d K \Phi_s^d, \quad b_{dh} = \Phi_s^d b_d(t)$$

2.2.2. Free-interface coordinate synthesis

As substructures have been analyzed, they should be assembled to obtain the whole structure. The assembled mass, damping and stiffness matrices of the system are shown as

$$M = \begin{bmatrix} M_{H} & 0 & 0 \\ 0 & M_{BH} & 0 \\ 0 & 0 & M_{DL} \end{bmatrix}, \quad C = \begin{bmatrix} C_{H} & 0 & 0 \\ 0 & C_{BH} & 0 \\ 0 & 0 & C_{DL} \end{bmatrix}, \quad K = \begin{bmatrix} K_{H} & 0 & 0 \\ 0 & K_{BH} & 0 \\ 0 & 0 & K_{DL} \end{bmatrix}$$

(9)

$$M_d = \Phi_s^d M \Phi_s^d, \quad C_{dh} = \Phi_s^d C \Phi_s^d, \quad K_{dh} = \Phi_s^d K \Phi_s^d, \quad b_{dh} = \Phi_s^d b_d(t)$$

$$f_{dh} = \Phi_s^d f$$

Interface DOFs should be dealt with to synthesize the substructures. First, the free-interface coordinates are synthesized by applying interface displacement compatibility condition and interface reaction equilibrium condition. As the blades are connected to the cover and disk substructures, the interface displacement compatibility condition can be written as

$$u_r^{\text{fre}} = \begin{bmatrix} u_r^{\text{fre}} \\ u_r^{\text{fre}} \end{bmatrix} = 0$$

(10)

The interface reaction equilibrium condition is shown as

$$f_{b}^{d} + \begin{bmatrix} f_{b}^{d} \\ f_{b}^{d} \end{bmatrix} = 0$$

(11)
The displacement vectors $u^{re}_b$, $u^{re}_c$, and $u^{re}_d$ can be obtained from Eq. (7) and represented as:

$$
u^{re}_b = B_b \left( \Phi_b^k p_b^k + \Psi_b^{lod} f_b^{lod} + \Psi_b^{lod} u_b^c \right)$$

$$
u^{re}_c = B_c \left( \Phi_c^k p_c^k + \Psi_c^{f_d} f_d^{lod} + \Psi_c^{f_d} u_c^c \right)$$

$$
u^{re}_d = B_d \left( \Phi_d^k p_d^k + \Psi_d^{f_d} f_d^{lod} + \Psi_d^{f_d} u_d^c \right)$$

(12)

where $B_b$, $B_c$, and $B_d$ are Boolean matrices, and

$$B_b = \begin{bmatrix} 0 & 0 & I_b \end{bmatrix}$$

$$B_c = \begin{bmatrix} 0 & 0 & I_c \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0 & 0 & I_d \end{bmatrix}$$

(13)

where $I_b$, $I_c$, and $I_d$ are identity matrices. Substituting Eq. (12) and Eq. (11) into Eq. (10), an equation can be deduced as:

$$H_0 f_d^b = H_1 \begin{bmatrix} p_b^k & p_d^k \end{bmatrix} + H_2 \begin{bmatrix} u_b^c & u_d^c \end{bmatrix} + H_3 p_b^k + H_4 u_b^c + H_5 f_b^{lod}$$

(14)

where

$$H_0 = \begin{bmatrix} B_b & 0 \\ 0 & B_d \end{bmatrix}$$

$$H_1 = \begin{bmatrix} B_c \Phi_c^k & 0 \\ 0 & B_d \Phi_d^k \end{bmatrix}$$

$$H_2 = \begin{bmatrix} B_c \Psi_c^k & 0 \\ 0 & B_d \Psi_d^k \end{bmatrix}$$

$$H_3 = -B_b \Phi_b^k$$

$$H_4 = -B_b \Psi_b^k$$

$$H_5 = -B_b \Psi_b^{lod}$$

(15)

As can be seen in Eq. (14), the interface reaction vector $f_b^{lod}$ is retained as generalized coordinates in the reduced order model. The elements of stiffness matrix with respect to the vector represent structural flexibility coefficient, which are several orders smaller than the other diagonal elements. This issue makes the stiffness matrix ill-conditioned and leads to inaccurate results. A method to settle this problem is to turn coordinates $f_b^{lod}$ into its displacement vector $u_b^{lod}$. Then, according to Eq. (7), displacement vector $u_b^{lod}$ can be represented as:

$$u_b^{lod} = R_b \left( \Phi_b^k p_b^k + \Psi_b^{lod} f_b^{lod} + \Psi_b^{lod} u_b^c \right)$$

(16)

where $B_b^{lod}$ is Boolean matrix and $B_b^{lod} = [0 I_b 0]$, where $I_b$ is an identity matrix. According to Eq. (16), $f_b^{lod}$ can be represented as:

$$f_b^{lod} = R_1 p_b^k + R_2 f_b^d + R_3 u_b^c + R_4 u_b^{lod}$$

(17)

where

$$R_1 = -B_b^{lod} \left( B_b^{lod} \Phi_b^k \right)^{-1} B_b^{lod} \Phi_b^k$$

$$R_2 = -B_b^{lod} \left( B_b^{lod} \Psi_b^d \right)^{-1} B_b^{lod} \Psi_b^d$$

$$R_3 = -B_b^{lod} \left( B_b^{lod} \Psi_b^c \right)^{-1} B_b^{lod} \Psi_b^c$$

$$R_4 = -B_b^{lod} \left( B_b^{lod} \Psi_b^{lod} \right)^{-1}$$

(18)

Substituting Eq. (17) into Eq. (14), the equation can be written as:

$$f_b^d = H_{inv} \begin{bmatrix} p_b^k & p_d^k \end{bmatrix} + H_2 \begin{bmatrix} u_b^c & u_d^c \end{bmatrix} + (H_3 + H_5 R_3) p_b^k + (H_4 + H_5 R_3) u_b^c + H_5 R_4 f_b^{lod}$$

(19)

where $H_{inv} = (H_0 - H_3 R_3)^{-1}$. Then, substituting Eq. (19) into Eq. (11) and Eq. (17), the free-interface generalized coordinates $f_b^d$, $f_b^{lod}$, $f_c^c$ and $f_d^d$ can be expressed by the retained coordinates $p_b^k$, $p_d^k$, $u_b^{lod}$, $u_c^c$, $u_d^c$ and $u_d^d$.
2.2.3. Fix-interface coordinate synthesis

After synthesizing the free-interface coordinates, fix-interface coordinates should be dealt with subsequently. In the previous modeling process, a number of DOFs are retained in the vectors $\mathbf{u}_{bi}^\text{crack}$ and $\mathbf{u}_{bj}^\text{crack}$. The number of DOFs in the vectors is obtained according to the maximum crack length of interest. When shorter cracks are considered, redundant DOFs on the crack surfaces should be synthesized as fix-interface coordinates. For an impeller with two cracks in the $i$th and $j$th sectors, the crack DOFs are divided as

$$
\mathbf{u}_{bi}^\text{crack} = \begin{bmatrix} \mathbf{u}_{bi}^\text{kep} \\ \mathbf{u}_{bi}^\text{rem} \end{bmatrix}, \quad \mathbf{u}_{bj}^\text{crack} = \begin{bmatrix} \mathbf{u}_{bj}^\text{kep} \\ \mathbf{u}_{bj}^\text{rem} \end{bmatrix}
$$

$$
\mathbf{u}_{ci}^\text{crack} = \begin{bmatrix} \mathbf{u}_{ci}^\text{kep} \\ \mathbf{u}_{ci}^\text{rem} \end{bmatrix}, \quad \mathbf{u}_{cj}^\text{crack} = \begin{bmatrix} \mathbf{u}_{cj}^\text{kep} \\ \mathbf{u}_{cj}^\text{rem} \end{bmatrix}
$$

(20)

where subscripts “kep” and “rem” denote the kept crack DOFs and removed crack DOFs, respectively. The numbers of DOFs in vectors $\mathbf{u}_{bi}^\text{kep}$ and $\mathbf{u}_{bj}^\text{kep}$ are related to the lengths of $i$th crack and $j$th crack. When an impeller has more than two cracks, the crack DOFs can be divided similarly. The removed crack DOFs and other fix-interface coordinates are synthesized using interface displacement compatibility conditions, which are shown as

$$
\mathbf{u}_{bi}^\text{rem} = \mathbf{u}_{ci}^\text{rem}, \quad \mathbf{u}_{bj}^\text{rem} = \mathbf{u}_{cj}^\text{rem}
$$

$$
\mathbf{u}_{fix}^b = \mathbf{u}_{fix}^b, \quad \mathbf{u}_{fix}^c = \mathbf{u}_{fix}^c
$$

(21)

Eq. (21) results in an equation of coordinate transformation, by which the reduced-order model of the impeller with multiple cracks can be obtained. The retained system coordinates $\mathbf{p}$ can be represented as

$$
\mathbf{p} = \begin{bmatrix} \mathbf{u}_{i}^\text{kep}^T \\ \mathbf{u}_{j}^\text{kep}^T \\ \mathbf{u}_{b}^\text{loc}^T \\ \mathbf{p}_l^T \\ \mathbf{p}_r^T \end{bmatrix}
$$

(22)

where

$$
\mathbf{u}_i^\text{kep} = \begin{bmatrix} \mathbf{u}_{bi}^\text{kep} \\ \mathbf{u}_{ci}^\text{kep} \end{bmatrix}, \quad \mathbf{u}_j^\text{kep} = \begin{bmatrix} \mathbf{u}_{bj}^\text{kep} \\ \mathbf{u}_{cj}^\text{kep} \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} \mathbf{p}_l \\ \mathbf{p}_r \end{bmatrix}
$$

(23)

The governing equations of motion of the reduced-order model are written as

$$
\mathbf{M}^r \ddot{\mathbf{p}}(t) + \mathbf{C}^r \dot{\mathbf{p}}(t) + \mathbf{K}^r \mathbf{p}(t) = \mathbf{b}(t) + \mathbf{f}^r(\mathbf{p})
$$

(24)

where $\mathbf{M}^r$, $\mathbf{C}^r$, and $\mathbf{K}^r$ are the mass, damping and stiffness matrices respectively; $\mathbf{b}(t)$ denotes the external force vector; $\mathbf{f}^r(\mathbf{p})$ contains the interface reactions of the kept crack DOFs.

2.3. Multiple cracks modeling

As the substructures have been assembled, a method for modeling multiple cracks is needed to account for the crack breathing effects. Based on the obtained reduced order model, the nonlinear forces and associated coordinate transformations for modeling multiple cracks are presented in this section. A method proposed by Poudou et al. [29] which was used for modeling intermittent contact problem is developed for impellers with multiple cracks in this section. The crack breathing effects are simulated by defining contact pairs on the crack surfaces. As is illustrated in Fig. 3, the $l$th contact pair consists of contact node $C_l$ and target node $T_l$. The contact pairs on crack surfaces are defined in local coordinate systems, and the three perpendicular directions of the local coordinate system at node $C_k$ are defined as $\mathbf{n}_k^1$, $\mathbf{n}_k^2$ and $\mathbf{n}_k^3$ respectively, where $\mathbf{n}_k^1$ is the normal vector defined as

$$
\mathbf{n}_k^1 = \frac{\mathbf{v}_1^l \times \mathbf{v}_2^l}{||\mathbf{v}_1^l \times \mathbf{v}_2^l||}
$$

(25)

where $\mathbf{v}_1^l$ and $\mathbf{v}_2^l$ are vectors that connect node $C_k$ and its adjacent nodes. The direction of the second axis $\mathbf{n}_k^2$ is chosen as $\mathbf{v}_1^l$, and $\mathbf{n}_k^3 = \mathbf{n}_k^1 \times \mathbf{n}_k^2$. The DOFs of nodes on the crack surfaces are expressed in the local coordinate systems. For simplicity, the deductions of the $i$th sector crack are illustrated as representative, and the modeling of other cracks can be treated similarly. The transformation equation is written as

$$
\mathbf{u}_{kep}^i = \begin{bmatrix} \mathbf{u}_{bi}^\text{kep}^i \\ \mathbf{u}_{ci}^\text{kep}^i \end{bmatrix} = \begin{bmatrix} \mathbf{N}_k^i & \mathbf{0} \\ \mathbf{0} & -\mathbf{N}_k^i \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}_{ci} \\ \dot{\mathbf{u}}_{rf} \end{bmatrix}
$$

(26)

where $\mathbf{N}_k^i$ is a block matrix with $\mathbf{N}_k^i$ for $k = 1, 2, \ldots, n$ along its diagonal blocks, where $\mathbf{N}_k^i = (\mathbf{n}_k^1, \mathbf{n}_k^2, \mathbf{n}_k^3)$; $\dot{\mathbf{u}}_{ci}$ and $\dot{\mathbf{u}}_{rf}$ are vectors of coordinates in the local coordinate systems.
The crack breathing effects are considered by introducing nonlinear forces in the contact pairs to penalize the relative normal penetration. The effects of tangential friction forces are ignored. The nonlinear forces of the kth contact pair are defined as

$$ f_{CI}^{k} = -k^* \left( \hat{u}_{CI}^{k1} + \hat{u}_{TI}^{k1} \right), \quad f_{TI}^{k} = f_{CI}^{k} $$

where $k^*$ is the penalty coefficient; $(\cdot)$ denotes the Macaulay bracket defined as $\langle u \rangle = (u + |u|)/2$; $\hat{u}_{CI}^{k1}$ and $\hat{u}_{TI}^{k1}$ are the normal displacements of the contact node and target node of the kth contact pair on the ith crack; The displacements $\hat{u}_{CI}^{k1}$ and $\hat{u}_{TI}^{k1}$ can be further transformed as follows

$$ \begin{bmatrix} \hat{u}_{CI}^{k1} \\ \hat{u}_{TI}^{k1} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \hat{u}_{nl}^k \\ \hat{u}_{l}^k \end{bmatrix} = R_k \begin{bmatrix} \hat{u}_{nl}^k \\ \hat{u}_{l}^k \end{bmatrix} $$

where $\hat{u}_{nl}^k$ and $\hat{u}_{l}^k$ denote the nonlinear and linear coordinates of the kth contact pair. The transformation of Eq. (28) leads to a result that nonlinear contact forces act only on the nonlinear DOFs. Then, a nonlinear coordinate vector $\hat{\mathbf{u}}_{nl}^i$ is defined, and $\hat{\mathbf{q}}_{nl}^i = \{ \hat{\mathbf{u}}_{nl}^1, \ldots, \hat{\mathbf{u}}_{nl}^m \}$. The linear coordinates are collected in vector $\hat{\mathbf{u}}_l^i$. The DOFs of nodes on the jth crack can be dealt with in the same way, and vectors $\hat{\mathbf{u}}_{nl}^j$ and $\hat{\mathbf{u}}_l^j$ denote the nonlinear and linear coordinates. By reordering the coordinates of the system, the generalized coordinates $\mathbf{q}$ can be expressed as

$$ \mathbf{q} = \begin{bmatrix} \mathbf{q}_{nl} \\ \mathbf{q}_l \end{bmatrix} $$

where $\mathbf{q}_{nl}$ represents the nonlinear coordinates, and $\mathbf{q}_{nl}^T = \{ \hat{\mathbf{u}}_{nl}^1, \hat{\mathbf{u}}_{nl}^2, \ldots, \hat{\mathbf{u}}_{nl}^m \}$; $\mathbf{q}_l$ contains all the other coordinates, and $\mathbf{q}_l^j = \{ \hat{\mathbf{u}}_{l}^1, \hat{\mathbf{u}}_{l}^2, \ldots, \hat{\mathbf{u}}_{nl}^j, \hat{\mathbf{p}}^j, \hat{\mathbf{u}}^j \}$. Then, the governing equations of motion can be written as

$$ M\ddot{\mathbf{q}} + C\dot{\mathbf{q}} + K\mathbf{q} = \mathbf{b} + f_{nl}(\mathbf{q}) $$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the reduced-order model respectively; External forces $\mathbf{b}$ act only on the linear DOFs; $f_{nl}(\mathbf{q})$ contains the nonlinear contact forces on the multiple crack surfaces. The nonlinear differential governing equations are solved by the harmonic balance method.

3. Numerical results

In this section, numerical results of the impeller with multiple cracks are presented. The effects of multiple cracks on vibration localization and on the shift of natural frequencies are presented. The finite element model is shown in Fig. 1. The material is an alloy steel with Young’s modulus $E = 214.6$ GPa, density $\rho = 7850$ kg/m$^3$ and Poisson’s ratio $\nu = 0.3006$. The damping model is the Rayleigh damping, $\mathbf{C} = \beta \mathbf{K}$, where $\beta$ is a damping coefficient and $\beta = 1 \times 10^{-7}$. When solving the nonlinear equations with harmonic balance method, the first ten harmonics from 0 to 9 were used and the penalty coefficient was set to be $1 \times 10^9$ N/m. The cracks located at the toe of weld joints with the maximum length of 48 mm. The resulting reduced-order model contained 1146 DOFs, including 150 normal modes for each substructure.

A double-cracked impeller was employed as representative to study the effects of multiple cracks on the vibration of impellers. When an impeller suffers from two cracks, there exist many cases due to different relative crack positions. Due to
the property of cyclic symmetry in an impeller, two cracks in sectors 1 and 11 led to the same natural frequency shift compared with the case of sectors 1 and 2. Then, three typical cases were investigated as representatives, as shown in Fig. 4. In subsequent sections, the three cases are referred to as “case 1”, “case 2” and “case 3”, respectively.

In order to validate the accuracy of the proposed method, comparison between the first twenty natural frequencies from reduced-order model with that from ANSYS was carried out. Relative error of the natural frequencies is shown in Fig. 5, and the maximum relative error is less than 0.1%. The frequency of the 20th natural frequency is 4922 Hz, which is larger than the frequency range of interest. Therefore, the proposed method is accurate enough for vibration analysis of impellers.

### 3.1. Natural frequency shift due to double cracks

First, the effects of double cracks on natural frequencies are investigated. The modes of interest are the one nodal diameter (1ND) mode and two nodal diameters (2ND) mode. The lengths of both cracks range from 0 mm to 48 mm with an
interval of 2 mm. Figs. 6 and 7 present the second and fifth mode shapes of the impeller with two 24 mm cracks, where red rectangular boxes illustrate the positions of cracks. As can be seen in Fig. 6, the nodal lines of the three mode shapes adaptively rotate to different positions to make the loss of each equivalent bending stiffness to a maximum. This phenomenon is also occurred in the 2ND modes as shown in Fig. 7.

The shifts of natural frequencies versus the length of double cracks are shown in Figs. 8–11. Figs. 8 and 9 show the shifts of the second and third natural frequencies, which both belong to the 1ND modes. As can be seen in the Figs, the second and third frequencies of case 2 exhibit quite different variation characteristics from the others. The second natural frequency of case 2 decreases less compared with the frequencies of case 1 and case 3. However, the third natural frequency of case 2 decreases much significantly than the other two cases. This can be explained by the mode shape shown in Fig. 6(b). The second mode shape is a mode with its nodal line across the middle place between the two cracked blades, which results in

Fig. 7. Fifth mode shapes (2ND) of impeller with two 24 mm cracks: (a) case 1 (2663.2 Hz), (b) case 2 (2647.3 Hz) and (c) case 3 (2651.0 Hz). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 8. Shift of the second natural frequency versus the length of cracks: (a) case 1, (b) case 2 and (c) case 3.

Fig. 9. Shift of the third natural frequency versus the length of cracks: (a) case 1, (b) case 2 and (c) case 3.
less loss of equivalent bending stiffness than the other two. The third mode shape of the impeller has a nodal line perpendicular to the nodal line of the second mode shape. So, with respect to the third mode shapes, crack distribution of case 2 leads to much larger decrease of equivalent bending stiffness, resulting in the large decrease of third natural frequency.

With respect to the two 2ND natural frequencies shown in Figs. 10 and 11, relatively small differences between the three cases are observed. This is due to the features of the three 2ND mode shapes shown in Fig. 7. A 2ND mode shape contains two nodal lines, and the structure can be deemed to vibrate along the nodal lines. By properly adjusting the positions of nodal lines, equivalent bending stiffness loss regarding the fifth and sixth modes is approximately identical. Thus, relative small differences in the fifth and sixth natural frequencies occur. Then, indicators based on shifts of the fifth and sixth natural frequencies only are insufficient to distinguish the crack positions. Moreover, when the lengths of cracks exceed 40 mm, the fifth and sixth natural frequencies decrease rapidly. It is because that the 2ND modes become gradually localized to the cracked blades. This phenomenon becomes more and more universal in high order mode shapes.

Shifts of several natural frequencies of the impeller with two 40 mm cracks are listed in Table 1. As can be seen in the table, the third natural frequencies are not sensitive to crack. The fifth frequencies decrease about 5.2% when the crack is 40 mm corresponding to 15.4% crack length ratio. The shifts of 1ND natural frequencies are relatively small, and that of 2ND modes are much larger. This is in accord with the phenomenon that high order natural frequencies are more sensitive to local damage.

Table 1
Natural frequency shifts of the impeller with two 40 mm cracks.

<table>
<thead>
<tr>
<th>Items</th>
<th>2nd (Hz)</th>
<th>3rd (Hz)</th>
<th>5th (Hz)</th>
<th>6th (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy impeller</td>
<td>2113.00</td>
<td>2113.00</td>
<td>2706.60</td>
<td>2706.60</td>
</tr>
<tr>
<td>Case 1</td>
<td>−65.95</td>
<td>−8.33</td>
<td>−133.59</td>
<td>−98.09</td>
</tr>
<tr>
<td>Case 2</td>
<td>−45.23</td>
<td>−28.27</td>
<td>−144.84</td>
<td>−60.70</td>
</tr>
<tr>
<td>Case 3</td>
<td>−63.74</td>
<td>−4.18</td>
<td>−140.75</td>
<td>−63.74</td>
</tr>
</tbody>
</table>
When more than one crack appear in an impeller, the shifts of natural frequencies become more complexly depending upon the depths and relative positions of cracks. Some of the natural frequencies are sensitive to relative crack positions, such as the third natural frequency. Other frequencies may not distinguish the crack positions well. What's worse, various combinations of positions and lengths of multiple cracks can lead to identical changes in natural frequencies. Then, it would be a challenge to quantitatively identify the lengths and positions of multiple cracks.

### 3.2. Effects of double cracks on the forced response

In this section, the effects of double cracks on the forced response of impellers are investigated. The forced response for each mode contains three models. The first model is an impeller without any crack. The second model refers to the impeller with a single 24 mm crack at the 11th sector. The third model corresponds to the impeller with two 24 mm cracks, and the positions of the double crack are shown in Fig. 4. These models are referred to as no crack model, single crack model and double crack model, respectively. The external forces are engine order excitation and the engine order is defined as the ratio of the excitation frequency to the rotating frequency. The forced response of interest is that in the frequency range of the 1ND and 2ND modes. The frequency response of the cracked impeller is obtained from the excitation node on the 11th blade as shown in Figs. 12 and 13.

As can be seen in Fig. 12, the frequency response of healthy impeller in frequency range of the 1ND modes differs from that of single crack model and double crack model, and the sole resonant peak splits into two isolated peaks due to the split of resonant frequencies. The two identical resonant frequencies of the same nodal diameters for a healthy impeller are no longer equal when the impeller suffers from crack damages. Moreover, the results of double crack model are significantly different from that of the single crack model in the frequencies and amplitudes of resonant peaks. Double cracks lead to larger resonant frequency decrease than a single crack, especially for the lower resonant frequencies. The positions of double cracks have remarkable effects on the frequency response. As can be seen in Fig. 12(a)–(c), the maximum resonant amplitudes and resonant frequencies depend on the positions of double cracks. The lower resonant frequency of case 3 has the largest frequency decrease, and case 2 leads to the largest decrease in the upper resonant frequencies.

The frequency response of the 2ND modes shown in Fig. 13 presents a similar phenomenon as that of the 1ND modes. Both a single crack and double cracks lead to the split of resonant peak. The frequency responses of impeller with two cracks are also dependent on the positions of cracks. On the other hand, the results of the 2ND modes also exhibit several distinct features. The model of case 2 leads to the largest decrease in the lower resonant frequencies, and Case 1 leads to the largest decrease in the upper resonant frequencies. The maximum resonant amplitudes of double crack models are not notably increased compared with that of the single crack models. The shifts of resonant frequencies are in some extent consistent with the variation tendencies of natural frequencies shown in Figs. 8–11.

Then, the effects of double cracks on steady state time domain response are studied using the impeller with two 24 mm cracks. The external forces are acted on the excitation nodes, and the force on the nth sector is as follows

$$\mathbf{b}_n(t) = \mathbf{B}_0 \cos (N \times 2\pi ft - \phi_n), \quad n = 1, 2, \ldots, N$$

where $\mathbf{B}_0$ contains the amplitudes of force components, and the amplitude of the resultant force equals to 100 N; $f$ denotes the rotation frequency; $\phi_n$ is the phase angle and is defined as $\phi_n = (n - 1)2\pi C / N$, where $C$ is the engine order of excitation.

The impeller rotates at a speed of 10,000 r/min, and the excitation frequency of external forces equals 1833.3 Hz. The engine orders of interest are the engine order 1 and engine order 2 excitations. The time domain response of excitation node on sector 1 of double crack models is shown in Fig. 14. As can be seen in Fig. 14(a) and (b), due to the nonlinear effects of breathing crack, the time domain response is no longer harmonics but generalized periodic signals. In Table 2, the amplitudes of the first five harmonic components are presented. The nonlinear time domain responses consist of several harmonic components. As can be seen in the table, harmonic 1 plays the most important role in the responses, and the other harmonics can't be neglected. Moreover, crack positions have remarkable effects on the response. The engine orders of excitation also affect the time domain response, and the differences in waveforms mean the different compositions of harmonic components.

The existence of double cracks makes the forced response more complex. The results are obviously depended on the positions of cracks, no matter for the frequency response or the time domain response. From the perspective of diagnosis, this characteristic provides the possibility for identifying the lengths and positions of cracks.

### 3.3. Considerations of mistuning effects

For a real impeller, mistuning is an unavoidable factor due to manufacturing tolerances and property deteriorations during services. When an impeller suffers from multiple crack damage and mistuning at the same time, the response features of such a structure are studied in this section. The mistuning effects are considered by introducing small random variation in the Young's modulus of each blade. The Young's modulus of the nth blade can be represented as

$$E_n = (1 + \delta_n)E_0$$

where $E_0$ is the modulus of each blade. The Young's modulus of the nth blade is represented as

$$E_n = (1 + \delta_n)E_0$$
where $E_0$ is the nominal Young’s modulus; $\delta_n$ is the ratio of Young’s modulus change. The mistuning parameter $\delta_n$ is randomly obtained from a uniform distribution with mean value of 0 and standard deviation of 0.02, which is shown in Table 3.

In order to investigate the combined effects of mistuning and cracks, analysis was carried out on an impeller under two different conditions. The first one was a tuned impeller with double crack, and the other one was its mistuned counterpart. These two models are referred to as “Tuned” and “Mistuned”, respectively. For simplicity, the double crack model of case 3 was employed as representative. The nonlinear frequency response of the tuned and mistuned impeller in frequency range of the 1ND and 2ND modes is shown in Fig. 15(a) and (b).
As can be seen in the figures, mistuning leads to noticeable shift of resonant frequencies in the frequency ranges of 1ND and 2ND modes. Changes in the lower resonant frequencies are more obvious. However in this special case, mistuning does not significantly increase the maximum resonant amplitudes. Moreover, mistuning only changes the resonant frequencies but does not introduce additional resonant peaks. This phenomenon is unique for centrifugal impellers and is determined by the structural characteristics of impellers. An impeller is fabricated by welding cover, blades and disk components, which makes the components be strongly coupled. The characteristics of strong structural coupling can be seen from the modes shown in (Figs. (6) and 7). The mode shapes are not significantly localized to cracked blades unless the cracks are large enough.

**Fig. 13.** Frequency response of the cracked impeller in frequency range of the 2ND modes: (a) case 1, (b) case 2 and (c) case 3.
3.4. Vibration localization due to double cracks and mistuning

For cyclic symmetric structures, such as centrifugal impellers and bladed disks, the destruction of cyclic symmetry caused by mistuning and cracks may lead to mode localization and forced response localization about a few blades. The concentration of vibration energy will result in increases in the maximum response amplitude and stress levels. Then, the effects of multiple cracks and mistuning on vibration localization of a centrifugal impeller are investigated in this section. The double cracked impeller of case 3 is employed as representative, and the mistuning parameters used are listed in Table 3. For comparison, a healthy impeller, an impeller with two 24 mm cracks and a mistuned impeller with double

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**Fig. 14.** Time domain responses of excitation node on sector 1 of double crack model with an excitation frequency of 1833.3 Hz: (a) engine order 1 excitation and (b) engine order 2 excitation.

**Table 2**

<table>
<thead>
<tr>
<th>Item</th>
<th>Cases</th>
<th>0th</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
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</thead>
<tbody>
<tr>
<td>1ND</td>
<td>Case1</td>
<td>7.01E-04</td>
<td>4.14E-03</td>
<td>1.41E-03</td>
<td>1.05E-04</td>
<td>5.27E-05</td>
</tr>
<tr>
<td></td>
<td>Case2</td>
<td>8.22E-05</td>
<td>3.22E-03</td>
<td>3.99E-05</td>
<td>2.37E-04</td>
<td>5.70E-05</td>
</tr>
<tr>
<td></td>
<td>Case3</td>
<td>7.39E-05</td>
<td>3.24E-03</td>
<td>5.30E-05</td>
<td>4.29E-04</td>
<td>2.77E-05</td>
</tr>
<tr>
<td>2ND</td>
<td>Case1</td>
<td>5.94E-04</td>
<td>3.70E-03</td>
<td>1.25E-03</td>
<td>9.57E-05</td>
<td>1.11E-04</td>
</tr>
<tr>
<td></td>
<td>Case2</td>
<td>6.90E-05</td>
<td>2.91E-03</td>
<td>9.31E-05</td>
<td>2.50E-04</td>
<td>3.90E-05</td>
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<tr>
<td></td>
<td>Case3</td>
<td>6.23E-05</td>
<td>2.86E-03</td>
<td>6.31E-05</td>
<td>4.42E-04</td>
<td>6.08E-05</td>
</tr>
</tbody>
</table>

**Fig. 15.** Frequency response of tuned and mistuned impeller with two 24 mm cracks of case 3: (a) the 1ND modes and (b) the 2ND modes.

3.4. Vibration localization due to double cracks and mistuning

For cyclic symmetric structures, such as centrifugal impellers and bladed disks, the destruction of cyclic symmetry caused by mistuning and cracks may lead to mode localization and forced response localization about a few blades. The concentration of vibration energy will result in increases in the maximum response amplitude and stress levels. Then, the effects of multiple cracks and mistuning on vibration localization of a centrifugal impeller are investigated in this section. The double cracked impeller of case 3 is employed as representative, and the mistuning parameters used are listed in Table 3. For comparison, a healthy impeller, an impeller with two 24 mm cracks and a mistuned impeller with double
24 mm cracks are employed. The results of these models are shown in Figs. 16–18. The 1ND mode and 2ND mode are considered. The results of interest are the maximum resonant amplitudes of blades at upper resonant frequencies.

In Fig. 16(a) and (b), the response amplitudes of blades in a healthy and tuned impeller are illustrated. The amplitudes of all the blades are approximately equal with the maximum relative error less than 0.5%, which is caused by the artificial constraints of motion during reduced order modeling. In an ideal tuned impeller, all blades have the same response amplitudes, and localization phenomenon does not occur among the blades.

When the property of cyclic symmetry is destroyed by cracks and other mistuning factors, the response amplitudes of blades are no longer identical, as shown in Figs. 17 and 18. Vibration becomes localized to some blades. However, the maximum amplitude of blades does not increase significantly compared with the amplitudes of ideal cyclic and healthy impeller. This is because that the modes of an impeller are mostly cover-dominated or disk-dominated. The strain energy of such modes tends to be concentrated in the cover and disk components. When fatigue cracks exist at the weld joints between blades and cover, the vibration energy is mostly transferred to the cover and disk components. As cover and disk components have much larger bending stiffness than a blade, the response amplitudes of the components will not increase notably. This is also the reason why an impeller tends to have the property of strong structural coupling. Moreover, vibration energy in cracked blades can be transferred to their adjacent blades via the cover and disk components. So, the sixth blade in Fig. 17(a) which is a cracked blade has smaller response amplitude than its adjacent fifth blade.

Another interesting phenomenon is the fluctuation of response amplitudes versus blade index for the mistuned impeller with double cracks. For the results of 1ND modes shown in Figs. 17(a) and 18(a), there are two obvious troughs and crests. This phenomenon can be explained by the 1ND mode shapes shown in Fig. 6. The 1ND mode shape has a nodal line across the whole impeller, so the modal amplitudes of blades have a tendency of fluctuation with two troughs and crests. For an ideal tuned impeller, vibration energy can be transferred throughout the whole structure, resulting in the identical response amplitudes between blades. When cyclic symmetry is destroyed, part of the energy will be reflected among some sectors, leading to energy

![Fig. 16. Response amplitudes of blades in a healthy impeller at the maximum resonant point: (a) the 1ND modes and (b) the 2ND modes.](image-url)
concentration among the substructure. The results of 2ND modes shown in Figs. 17(b) and 18(b) have similar conclusions. The response amplitudes of blades fluctuate four times, which conform to the mode shapes of 2ND shown in Fig. 7.

Both mistuning and cracks can lead to vibration localization, but the two factors have different effects. Vibration localization of blades is closely relevant to the positions of nodal lines, and the blades near nodal lines tend to have small response amplitudes. The results discussed above may be applicable to impellers with small or medium cracks. When cracks become large enough, many mode shapes will be significantly localized to the cracked blades, and the nodal lines in the mode shapes will be unobvious.

4. Discussions

Although, numerical results in previous sections are with respect to double cracked impellers, the obtained conclusions can be applied to impellers with more than two cracks. Of course, quantitative results of impellers with more cracks remain to be computed. Then, based on the obtained vibration characteristics, effective methods can be developed to detect the cracks. Due to the complexity of multiple cracked impellers, the shifts of natural frequencies and the forced response amplitudes of blades should be combined together to identify the lengths and relative positions of cracks. The outline of a potential method for quantitatively identifying multiple cracks is presented as follows.

First, the shifts of natural frequencies that are sensitive to cracks should be estimated to determine whether cracks exist. If the shifts of natural frequencies are all very small, it can be deemed that there is no crack in the impeller or the crack is too small to detect. Otherwise, the impeller is thought to have cracks, and the subsequent steps need to be carried out.

Second, the forced response amplitudes of blades and covers can be used to determine the positions of cracks. Generally speaking, the response amplitudes of the cracked blades in an impeller are larger than that of healthy blades, especially when the excitation points are placed near the possible locations of cracks, such as the weld toe of cover side of blades. This
phenomenon will be more obvious for blades with medium or large cracks. Then, the positions and the number of cracks can be determined.

Finally, the lengths of cracks can be obtained using the relationships between natural frequencies and crack lengths, such as the shifts of natural frequencies versus crack lengths shown in Figs. 8–11.

Identifying the lengths and positions of multiple cracks in centrifugal impellers is a challenge work. Artificial intelligence algorithms, such as artificial neural network (ANN), genetic algorithm (GA) and support vector machine (SVM), can be employed to develop more effective parameter identification technique. Feasibility and experimental validation of the proposed method is not included in this paper.

5. Conclusions

In this paper, the effects of multiple cracks on the forced response of a centrifugal impeller are presented. Natural frequency shifts, nonlinear forced response and vibration localization due to double cracks are studied using the proposed finite-element based hybrid interface component mode synthesis method. According to the results in this study, some main conclusions are summarized as follows.
(1) The shifts of natural frequencies become more complex depending on the lengths and relative positions of multiple cracks when more than one crack appears in an impeller. Some orders of natural frequencies are sensitive to the lengths and positions of cracks, while others are not.

(2) Both a single crack and double cracks can split the sole resonant peak of a healthy impeller into two isolated resonant peaks. However, double cracks lead to much larger decreases in resonant frequencies, and the decreases in frequencies are closely related to the relative positions of multiple cracks.

(3) Due to the property of strong structural coupling in an impeller, the response amplitudes of cracked blades are not significantly increased, and vibration localization phenomenon about the blades are not notable.

(4) The response amplitudes of blades periodically fluctuate versus blade number when an impeller suffers from cracks or mistuning. Vibration localization of blades is relevant to the positions of nodal lines, and the blades near nodal lines tend to have small response amplitudes.

Multiple cracks have significant effects on the vibration of impellers. From the perspective of crack detection, this characteristic provides the possibilities for identifying the lengths and relative positions of multiple cracks. Although, the topic of identifying multiple cracks is not the focus of this paper, the obtained results can provide theoretical instructions for developing effective techniques for multiple cracks diagnosis.

Acknowledgments

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