A new method for predicting nonlinear structural vibrations induced by ground impact loading

Jun Liu a,*, Yu Zhang b, Bin Yun c

a Key Laboratory of Ministry of Education for Geomechanics and Embankment Engineering, and Institute of Engineering Safety and Disaster Prevention, Hohai University, 1 Xikang Road, Nanjing, Jiangsu 210098, PR China
b Institute of Engineering Safety and Disaster Prevention, Hohai University, 1 Xikang Road, Nanjing, Jiangsu 210098, PR China
c Geotechnical Research Institute, Hohai University, 1 Xikang Road, Nanjing, Jiangsu 210098, PR China

1. Introduction

The structural vibrations induced by impact loading in the vicinity of a structure are an inevitable phenomenon in the construction of civil engineering projects because of processes such as dynamic compaction, structural collapse-induced touchdown in demolition blasting, mine blasting and the impacts of objects to the ground that arise from accidents. Structural vibrations may cause structural damage or even a fracture when the strength of the vibration is severe. Therefore, the search for effective methods for predicting and controlling structural vibrations has aroused extensive interest.

A medium can be divided into two sections according to the propagation path of the seismic wave caused by the impact loading: one section from the impact source to the structural foundation and a second from the structural foundation to a position of interest inside the structure. Then, the structural vibration can be regarded as a response to the vibration of its foundation. The methods for calculating the structural response to a ground vibration are relatively well developed, such as the mode-superposition response spectrum method (MSRSM) and the time-history analysis method (THAM). However,
modeling the wave propagation in the geo-medium between the impact source and the structural foundation is a complex mechanical problem because the mechanical behavior of the geo-medium is difficult to accurately determine by calculations alone. Hence, the key issue in the modeling of structural vibrations is how to describe the vibration of the foundation under impact loading. At present, comparative studies of blast-induced ground vibrations have been motivated by safety requirements and thus have focused on, for instance, developing methods for the site-specific prediction and control of ground vibrations induced by blasting. The methods for such purposes mainly include numerical simulations and statistical analyses as well as predictions of the time history of structural vibrations based on site measurements.

The method of numerical simulation mainly utilizes a business software or program developed by the researchers themselves to simulate the stress wave propagation and mechanical responses of soil and rock media under blasting loads [1–6]. The vibrational characteristics of any position near the impact source can be obtained by applying the numerical simulation method. However, it is not easy to identify the mechanical parameters of soil and rock media. Meanwhile, certain differences are evident between the simulated results and the actual responses because the soil and rock materials are generally regarded as homogeneous, continuous and isotropic media in simulations. In addition, numerical simulations are time-consuming and require the technical staff to have a certain amount of experience. Thus, the application of the numerical simulation method is not very convenient in practical engineering.

Statistical analytical methods mostly include the use of the empirical model based on waveform analysis [7–10], the time series prediction method [11], the neural network forecast model [12–15], the fuzzy modeling approach [16] and the support vector machine [17], among others. For example, based on the analysis of a large amount of vibrational test data, Otuonye [11] forecasted the probable vibrational effects caused by blasting, i.e., he predicted the waveform record of the blasting design by modeling measured data. Based on a large number of blasting tests performed in an open pit, Chakraborty et al. [12] compared and analyzed the forecast results obtained by applying different empirical formulas by comprehensively analyzing parameters such as the borehole diameter, the burden, the single hole charge weight, the charge factor, the largest charge weight per delay period, the measured points distance, and the waveform record, from which a neural network forecast model was established. The general ideas behind the statistical analytical methods are simple. However, a great number of vibration tests are necessary before employing them. Additionally, the methods have poor on-site applicability.

Methods that can predict the time history of a vibration are more highly preferred in practical engineering endeavors because the vibration level and structural response can be easily evaluated by analyzing the predicted results. Anderson et al. [18–20] reported a technique for predicting the site-specific ground vibrations caused by blasting operations. The technique involves measuring the time history of a ground response to a single-charge blast and then repeatedly superposing this time-domain signal, delayed in time, to construct a synthetic multiple-charge response time history. This procedure is repeated for a large range of delay-period intervals. The ranges of the amplitude levels of the predicted multiple-charge signals are then mapped on a delay versus frequency plot. The best delay period, consistent with the currently accepted blasting practice, is then selected for suitably reducing the vibrations at critical ground frequencies. Hinzen [21] utilized the advantages of the single-shot signal superposition in the time domain to predict blast-induced vibrations. In addition to the spectral amplitudes, all of the phase effects from the superposition were included in the synthetic seismogram to optimize the firing times. Ghosh et al. [22] deemed that, owing to dispersion and attenuation, Rayleigh waves usually carried the largest fraction of the vibrational energy when the distances from a near surface blast were greater than 1500 feet. Therefore, a model, which is based on the method proposed by Anderson et al. [18–20], i.e., superposing the Rayleigh waves generated by a single-shot signal in the time domain, has been developed to predict the site-specific vibrations excited by blasting operations using multiple-charges. In addition, methods similar to the superposition approach were used by Aldas et al. [23] and Svinkin et al. [24] to predict machine impact- and blast-induced vibrations. These superposing approaches indicate that the ground response is linear, i.e., has relatively small deflections. However, a linear assumption does not hold in general, especially for large charge weights.

To overcome the limitations of the linear superposition method, two nonlinear models, one based solely on charge-weight scaling and the other based on the distinct notion of the blast damage, were proposed by Blair [25]. In the former method, however, it is difficult to determine how the charge-weight scaling significantly affects the predicted result, and in the latter method, a numerical calculation is required.

Altogether, the prediction of the vibrational effects induced by ground impact loading is extremely complex. There are some limitations to the existing methods, as discussed above. In this work, a hybrid method has been proposed to calculate the theoretical seismograms of structural vibrations. The word “hybrid” denotes a combination of field measurements and computer simulations. The impact loading is simplified as an impulse train. Then, based on nonlinear system theory, a novel method is proposed to predict the signal induced by impact loading. To identify the nonlinear system, field measurements are needed. The proposed method is suitable for predicting the vibrations of a specific measured site, both at a field level and inside of the structure. Hence, the predicted results are more precise than those of the existing methods in which the media are depicted as a linear system. In addition, the best delay period can be obtained by examining a plot of the vibration frequency versus the delay period, which can effectively diminish the structural vibration for a known impulse train.

In the following sections, the linear methods are first introduced. Then, the nonlinear method and the identification of a nonlinear system are presented. Furthermore, two examples, the impulse train induced by either a hammer impact or a blast, are discussed.
2. Prediction method

2.1. Linear method

Based on the research of Anderson et al. [18–20], a linear model, which can predict the site-specific ground vibrations excited by a blasting operation, was developed by Hizen [21]. The blasting vibrations induced by multiple sites (holes) at a specific location can be obtained by the linear superposition of a single shot. Hizen [21] assumed that the shapes of the displacement or velocity signals were identical for all of the individual holes. Thus, the ground movements at a location \( x \) over a certain time domain can be represented as

\[
u(x,t) = u(x,t) \ast m_R
\]  

(1)

where \( u(x,t) \) is the displacement or velocity signal at the location \( x \) from a single shot, which can be obtained by a single-shot in situ experiment, “\( \ast \)” denotes a convolution operation, and \( m_R \) is an impulse train, which can be expressed as

\[
m_R = \sum_{i=1}^{N} a_i \delta(t-t_i)
\]  

(2)

where \( N \) is the number of impulses, \( t_i \) is the occurrence time of impulse \( i \), and \( a_i \) is the scaling factor for the seismic effects of an individual hole, which can be calculated as follows:

\[
a_i = q_i / q_0
\]  

(3)

where \( q_i \) and \( q_0 \) are the charge weights of the \( i \)th hole and the single-shot in situ experiment, respectively.

In this work, we also used the method proposed by Hizen [21] to predict the vibration at a specific location both under the impact loadings induced by an object hitting the ground and by blasting to make a comparison with the nonlinear method proposed in the following section. The charge weight in Eq. (3) should be replaced by impact energy in the case of predicting the vibration induced by the impact loading caused by a hammer.

2.2. Nonlinear method

2.2.1. Volterra functional series

Ample evidence indicates that neither the charge weight nor the impact energy is directly proportional to the amplitude of the vibrational wave in the time domain in the case of the same impact source and specific measurement location, and the same evidence also exists for the frequency domain, i.e., the amplitude of the spectrum is also not proportional to the charge weight or impact energy. The reason for this lies in the fact that the media is not a homogeneous continuum, which results in wave attenuation and dissipation during the wave’s propagation through the media. Hence, a geotechnical medium may present nonlinear characteristics for a wave propagating through it. Therefore, a nonlinear system is required to describe the responses of a geotechnical medium to impact or blasting.

In this work, the Volterra functional series [26], which is based on a nonlinear model, was employed to describe the nonlinear characteristics of the geotechnical medium between the impact or explosion source and the measured location. The Volterra functional series can be expressed as

\[
y(t) = \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, \ldots, \tau_n) \prod_{i=1}^{n} \left[x(t-\tau_i) d\tau_i\right]
\]  

(4)

where \( x(t) \) and \( y(t) \) are the input and output, respectively, and \( h_n(\tau_1, \tau_2, \ldots, \tau_n) \) is the \( n \)th order Volterra kernel. However, the use the whole Volterra model would introduce many difficulties into both identifying and reconstructing a nonlinear system. Because the \( n \)th Volterra kernel is a function of \( n \) variables, the model that represents the system has to contain a large number of coefficients to determine the system. Therefore, a simplified Volterra model was developed (presented in the following section), which significantly reduces the number of coefficients required for a Volterra series representation.

2.2.2. Identification of a nonlinear system

The kernels can be identified from the input and output data. In this study, the \( n \) kernels have been estimated by impact tests for \( n \) times. For each impact test, the output signal was measured at a location of interest that is a certain distance away from the impact source and called a measured location. The impact loading can be generated by a ground impact or a blast. The impact loading was simplified as an impulse for which the amplitude denotes the potential energy of a ground impact or the charge weight of a blast. In this study, the \( n \) impact tests are mutually independent, which means that the interval time between a neighboring impact test is long enough that no interaction would occur with the neighboring measured signals. It is assumed that \( q_1, q_2, \ldots, q_n \) are the potential energies or charge weights corresponding to the \( n \) impact tests, respectively; then, the \( n \) impulses can be expressed as \( a_1 \delta(t), a_2 \delta(t), \ldots, a_n \delta(t) \), where \( \delta(t) \) denotes the Dirac function, and the amplitudes \( a_1, a_2, \ldots, a_n \) of \( n \) impulses are the respective scaling factors for the seismic effects of the
individual impacts

\[
\begin{align*}
    a_1 &= q_1 / q_{\text{max}} \\
    a_2 &= q_2 / q_{\text{max}} \\
        &\vdots \\
    a_n &= q_n / q_{\text{max}}
\end{align*}
\]  

(5)

where \( q_{\text{max}} = \max(q_1, q_2, \ldots, q_n) \). The \( n \) impulses are regarded as inputs and substituted into Eq. (1) as follows:

\[
\begin{align*}
    a_1 h_1(t) + a_2^2 h_2(t, t) + \cdots + a_n^2 h_n(t, t, \ldots, t) &= y_1(t) \\
    a_2 h_1(t) + a_2^2 h_2(t) + \cdots + a_n^2 h_n(t, t, \ldots, t) &= y_2(t) \\
        &\vdots \\
    a_n h_1(t) + a_n^2 h_2(t, t) + \cdots + a_n^2 h_n(t, t, \ldots, t) &= y_n(t)
\end{align*}
\]  

(6)

where \( y_1(t), y_2(t), \ldots, y_n(t) \) are the output signals corresponding to the respective input impulses. To improve the precision of identification, Eq. (6) was solved in the frequency domain. The Fourier transform of Eq. (6) can be expressed as

\[
\begin{align*}
    a_1 H_1(f) + a_2^2 H_2(f, t) + \cdots + a_n^2 H_n(f, t, \ldots, t) &= Y_1(f) \\
    a_2 H_1(f) + a_2^2 H_2(f) + \cdots + a_n^2 H_n(f, t, \ldots, t) &= Y_2(f) \\
        &\vdots \\
    a_n H_1(f) + a_n^2 H_2(f, t) + \cdots + a_n^2 H_n(f, t, \ldots, t) &= Y_n(f)
\end{align*}
\]  

(7)

where \( H_1(f), H_2(f, t), \ldots, H_n(f, t, \ldots, t) \) are the Fourier transforms of the respective \( n \) kernels \( h_1(t), h_2(t, t), \ldots, h_n(t, t, \ldots, t) \), and \( Y_1(f), Y_2(f), \ldots, Y_n(f) \) are the Fourier transforms of the respective \( n \) measured signals \( y_1(t), y_2(t), \ldots, y_n(t) \). Furthermore, Eq. (7) can be expressed in matrix form

\[
\begin{bmatrix}
    a_1 & a_2^2 & \cdots & a_n^2 \\
    a_2 & a_2^2 & \cdots & a_n^2 \\
    \vdots & \vdots & \ddots & \vdots \\
    a_n & a_n^2 & \cdots & a_n^2
\end{bmatrix}
\begin{bmatrix}
    H_1(f) \\
    H_2(f, t) \\
    \vdots \\
    H_n(f, t, \ldots, t)
\end{bmatrix}
= 
\begin{bmatrix}
    Y_1(f) \\
    Y_2(f) \\
    \vdots \\
    Y_n(f)
\end{bmatrix}
\]  

(8)

Let

\[
A = 
\begin{bmatrix}
    a_1 & a_2^2 & \cdots & a_n^2 \\
    a_2 & a_2^2 & \cdots & a_n^2 \\
    \vdots & \vdots & \ddots & \vdots \\
    a_n & a_n^2 & \cdots & a_n^2
\end{bmatrix}
\]  

(9)

The necessary and sufficient condition under which Eq. (8) has a unique solution is that the inverse matrix \((A^{-1})\) of \( A \) exists, i.e., there are no exactly equal amplitudes among the \( n \) impulses in the impact tests. Substituting Eq. (9) into Eq. (8) and multiplying by \( A^{-1} \) gives

\[
\begin{bmatrix}
    H_1(f) \\
    H_2(f, t) \\
    \vdots \\
    H_n(f, t, \ldots, t)
\end{bmatrix}
= A^{-1}
\begin{bmatrix}
    Y_1(f) \\
    Y_2(f) \\
    \vdots \\
    Y_n(f)
\end{bmatrix}
\]  

(10)

Because the \( n \) kernels have been obtained in the frequency domain, the corresponding kernels in the time domain can be calculated by an inverse Fourier transform

\[
\begin{bmatrix}
    h_1(t) \\
    h_2(t, t) \\
    \vdots \\
    h_n(t, t, \ldots, t)
\end{bmatrix}
= F^{-1}
\begin{bmatrix}
    H_1(f) \\
    H_2(f, t) \\
    \vdots \\
    H_n(f, t, \ldots, t)
\end{bmatrix}
\]  

(11)

where \( F^{-1} \) denotes the respective inverse Fourier transforms.

2.2.3. Predicting the output signal of an impulse train

The predicted signal induced by a known complex impact loading at a measured location can be obtained by calculating the Volterra functional series after the nonlinear system has been identified. The known complex impact loading can be simplified by an impulse train

\[
u(t) = \sum_{i=1}^{m} b_i \delta(t - t_i)
\]  

(12)
where $b$ denotes the amplitude of the impulse train, $t_i$ is the delay period of the $i$th impulse, and $m$ is the number of impulses.

In general, the precision of the predicted results is high enough after the first three terms of the Volterra functional series. A Volterra functional series including three terms can be identified by at least three pairs of input and output signals. If the Volterra functional series sum is itemized into the sum of the separate convolutions and only the first three terms are considered, the relationship between the input and the output described in Eq. (1) will become

$$y(t) = y_1(t) + y_2(t) + y_3(t) = \int_{-\infty}^{\infty} h_1(\tau_1)x(t-\tau_1)d\tau_1 + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 \tag{13}$$

where $y_1(t)$, $y_2(t)$ and $y_3(t)$ are the three output signals corresponding to the three Volterra kernels $h_1(\tau_1)$, $h_2(\tau_1,\tau_2)$ and $h_3(\tau_1,\tau_2,\tau_3)$, respectively. Substituting Eq. (12) into Eq. (13) and expressing it by convolution ($\ast$) gives

$$y(t) = h_1(t)u(t) + h_2(t,u(t)u(t)u(t) = h_3(t,u(t)u(t)u(t)u(t)$$

(14)

where $u^n(t) = \sum_{i=1}^{m} b_i^2 \delta(t-t_i)$ ($n=2,3$). Finally, the predicted signal induced by the impulse train at the specific site can be calculated by Eq. (14).

2.2.4. Convergence of this method

In a mathematical sense, the nonlinear model should be convergent as long as the inverse matrix of $A$ (see Eq. (9)) exists, i.e., the amplitudes of the impulses that are used to identify the nonlinear system are all different. However, two similar amplitudes would result in large numerical errors. Therefore, large intervals should be set for the amplitudes of impulses used for in situ experiments to ensure the accuracy of the predicted results. In addition, the amplitudes of the prediction impulse trains should have the similar values as those of the identification impulses. The reason for this lies in the fact that there are large differences in the mechanical responses of a geo-medium between strong and weak impact loadings. A nonlinear system that is identified by a weak impulse behaves very different from that identified by a strong impulse. The waveforms obtained in experiments represent the mechanical characteristics of the geo-medium used. Thus, different impulse amplitudes will lead to different mechanical responses. The waveform obtained in an experiment performed with weak impulse amplitude will not reflect the mechanical properties observed under strong impulse amplitude. As a result, if a nonlinear system that was identified with a weak impulse was used to predict the vibration caused by a strong impulse, the predicted results would differ greatly from those that would actually occur.

3. Applications

In this section, two cases that were studied to verify validity of the method are discussed. In the first case, the impact loading was generated from a hammer impact on the ground, and the second one was generated from an explosion blast.

3.1. Prediction of the ground vibration induced by hammer impact

In the first case, the response to the impact of a hammer on the ground was studied, as shown in Fig. 1. In this work, the ground and the structure were modeled in an integrated way to predict the structural vibration, instead of distinguishing between the ground and the structure. In other words, the medium between the impact source and the structure is modeled by a Volterra functional series. At the same time, the geo-medium between the impact source and structural foundation is modeled by another Volterra functional series to predict the vibration of structural foundation. The objective is to verify the performance of the method proposed in this paper for separately predicting the behavior of the geo-medium as well as the combined geo-medium and structure.

The vibration of a building that was located 7 m away from the impact source was predicted by the proposed method. Two measurement points at which two vibration transducers were placed were selected, one on the foundation and another inside the structure. The medium between the impact source and the measured location on the foundation of the structure was represented by a nonlinear system named System A, and the medium between the impact source and the measured location inside the structure was represented by another nonlinear system named System B. Then, the hammer impact tests were performed three times; the impulse in each test had a different impact energy, named impulses A, B and C, respectively. The delay period between the consecutive impact tests was long enough (more than 5 min for this case) that no interaction occurred between the measured signals. The impact energy, the distance away from the impact source and the scaling factor of the impact tests are listed in Table 1. Simultaneously, the waveforms at the two measured locations were recorded, as shown in Fig. 2. Moreover, the first three order kernels of the Volterra functional series for Systems A and B could be identified by the input signals, which were constructed by the Dirac functions corresponding to the impact energies in the experiments, and the output signals, which were recorded at the two measured locations according to the method introduced in Section 2.2.2, as shown in Fig. 3. After the two nonlinear systems were identified, the vibration of the two measured locations induced by an impulse train could be predicted by the method introduced in Section 2.2.3. The impulse trains are constructed from the continuous impact of a hammer on the ground at an interval of...
3 s, as shown in Fig. 4. Three output components corresponding to the three order kernels for the two systems are also shown in Fig. 5. To verify the validity of the proposed method, we also give the predicted results of the linear method (see Section 2.1) and the measured waveforms induced by the impulse train, as shown in Fig. 6. For comparison purposes, the Fourier spectra of the predicted waveforms by the linear and nonlinear methods and the measured waveforms are also shown in Fig. 7.
Fig. 3 shows that the amplitude of a Volterra series kernel rapidly attenuates as the kernel order number increases. This result indicates that the contribution of the lower order kernel to the predicted results is larger than that of higher order kernel. Figs. 6 and 7 show that the predicted waveforms and spectra obtained using the proposed nonlinear method are closer to the measured waveforms and spectra than those predicted using the linear method.
3.2. Prediction of blasting-induced vibrations

In the second case, the vibration induced by blasting was studied to verify the validity of the proposed method in the case of blasting loading. The vibrations of two measured locations induced by an impulse train corresponding to two detonated charges were predicted by the same operation used for the first case (as shown in Fig. 1), except the charge weight replaced the impact energy. The wavelets of the two measured locations induced by three single charge detonations (at each location) are shown in Fig. 8, and the corresponding impact energies, the distances away from the impact source and the scaling factors are listed in Table 2. The three Volterra functional series kernels for the two systems are shown in Fig. 9. The impulse train, corresponding to the two charges detonated to induce the vibration required for prediction testing, is shown in Fig. 10. The three output components that correspond to the three order kernels for the two systems are shown in Fig. 11. The waves and their Fourier spectra predicted by the linear and nonlinear methods and the measured waves and spectra are shown in Figs. 12 and 13. The same conclusions reached after conducting the first case can be obtained from the second case. The results indicate that the proposed method can also predict the vibration induced by blasting.

3.3. Selection of the best delay period

Anderson et al. [18–20] developed a technique to describe the relationship between delay time and frequency for a multiple-charge blasting, by which the best delay period for suitably reducing vibrations at critical frequencies can be selected. In this work, we extend this technique to find the best delay for an impulse train.

The procedure, then, is as follows: for an input signal described by an impulse train, as shown in Eq. (12), the corresponding output signal can be calculated by applying the Volterra functional series. Then, different impulse trains are
constructed by changing the delay period of the impulse train systematically over a large range of delay period intervals, and the corresponding output signals can be obtained by repeating the procedure discussed previously. The Fourier spectrum for each output signal is computed, and the maximum among all of the Fourier spectra is determined. Furthermore, the Fourier spectra can be plotted with each spectrum placed on a separate line. The amplitudes are related to a gray color scale with the highest amplitudes corresponding to the darkest shading. The gray scale is calculated using the amplitudes for the entire Fourier spectra; thus, each line consists of characters corresponding to the amplitudes of the Fourier spectrum for a given delay time, plotted versus frequency on the horizontal axis. The vertical axis corresponds to the delay time.

Two gray scale images were generated according to the above procedure for the structural vibration results of examples 1 and 2 (shown in Fig. 14); the delay period increases from 0 to 5 s in example 1, and from 0 to 0.5 s in example 2. Fig. 14 shows that the structural vibration is weak when the delay period is between 1.5–2.5 s and 4.5–4.8 s in the case of example 1 and from 0.13 to 0.17 and approximately 0.3 s in example 2.

<table>
<thead>
<tr>
<th>No.</th>
<th>Charge weight (kg)</th>
<th>Distance (m)</th>
<th>Scaling factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.45</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4.95</td>
<td>30</td>
<td>0.77</td>
</tr>
<tr>
<td>3</td>
<td>1.95</td>
<td>30</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Fig. 8. Wavelets recorded at the measurement locations (a) on the structural foundation and (b) inside the structure during the blasting tests.

Fig. 9. The first three order kernels identified by the blasting tests: (a) System A and (b) System B.
Fig. 10. The impulse train corresponding to the two charges detonated (from which the vibration induced by the impulse train at the measured location can be predicted).

Fig. 11. Three output signals corresponding to their 3 respective kernels: (a) System A and (b) System B.

Fig. 12. The measured results and the results predicted by the linear and nonlinear methods for a two charge detonations: (a) by System A and (b) by System B.
4. Conclusions

In this work, a nonlinear method was proposed to predict the vibrational response to impact loading at a specific location. The Volterra functional series was employed to represent a nonlinear system, which can correspond to a geo-medium or the combination of a geo-medium and structure. The kernels of the Volterra series can be identified by the input and output signals obtained from in situ experiments. The input signals are constructed by the Dirac functions and the impact energy or the charge weight. In fact, the second and higher order Volterra kernels cannot be completely identified by the input of individual impulses (Dirac impulses). Only a slice through the axis of the second or higher order Volterra kernel can be determined by the Dirac impulses. Therefore, it is just a simplified nonlinear method. Nonetheless, the predicted results still approach the measured ones. In addition, the best delay period can be selected for suitably reducing vibrations at critical frequencies. The proposed method is suitable for considering impact loadings, such as the impact of a hammer to the ground and that of blasting, and is appropriate for both geo-media and structures.

Acknowledgments

This work was supported by the National Basic Research Program of China (973 Project) (no. 2007CB714104) and grants from the National Natural Science Foundation of China (nos. 51044003 and 51174076). These grants are gratefully acknowledged.
References