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Analysis of High Shocking Resistance of an Improved Node-Plane Supporting Vibration Beam Gyroscope

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Abstract
The characteristics of high shock resistance of a novel vibration beam structure gyroscope, namely Node-Plane Supporting Vibration Beam Gyroscopes (NPSG) were researched in this paper. NPSG is a novel rotating rate sensor, which used supporting planes located at node points instead of the supporting pins to support the vibration beam. In this paper, the sensing principle of the NPSG is presented and two types of NPSG are introduced. The dynamic equations of these two sensors are derived and the values of two kinds of gyroscopes' shocking resistance are obtained based on the mechanics vibration principle. The research results show that the shocking resistance of the traditional Node Supporting gyroscope is below $2 \times 10^{-5}$ g, while the improved Node-Plane Supporting one could reach $6 \times 10^{-5}$ g. The experiments for the novel node supporting vibration beam gyroscope proved the presented principle. The designed sensor shows larger stiffness and robust resistance to shaking and striking. This paper provides the theoretical analysis and experimenting foundation for realizing the shocking resistance ability of this novel NPSG.

Keywords: gyroscope; vibration beam, shock resistance, Node-Plane-Supporting

1. Introduction

Vibrating structure gyroscopes (angular rate sensors) have become more and more popular in the industrial and automotive fields [1][2]. Vibrating structure gyroscope is known as a Coriolis vibratory gyroscope because as the plane of oscillation is rotated, the response detected by the transducer results from the Coriolis term in its equations of motion. With the increasing need for low cost angular rate sensor for the application in emerging markets such as consumer electronics etc, the study of vibration beam gyroscope has been of great interesting to engineers and researchers [3][4].

Node supporting vibration beam gyroscope includes two vibration modes. The beam will vibrate while the driving force is generated. Then the linear momentum of vibration beam along the driver plane can be achieved. When an angular velocity of the beam along rotating body is inputted, Coriolis force will excite the vibration of sensing mode, which is vertical with the vector of driving and angular velocity separately. The vibration amplitude of the sensing mode is proportional to the generated linear momentum and the angular velocity [5].

In 1960s, researchers proposed a kind of solid vibration beam gyroscope named node supporting vibration beam gyroscope [1]. It was found that there are many advantages of this kind of gyroscope, such as inexpensive, small-volume, etc. But shock resistance ability of the solid vibration is not strong enough while used in high g-environment such as the measurement while drilling. In recent years, Jalili and his research team presented on a vibration gyroscope systems experiencing coupled flexural / torsional vibration [6], which can improve the ability of shock resistance. But the presented gyroscope is not node supporting vibration structure one, so it can not have the advantage of node supporting vibration structure sensor and can’t utilize the existing research results in recent years. In this paper, a novel kind of Node-Plane Vibration beam gyroscope was proposed, which used the plane to be located
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in the node point to support the vibration beam [7]. The model and resistant moment of two kinds of the node supported vibration beam gyroscope is analyzed through computation and experiment.

In this paper, the working principle of node supporting vibration beam gyroscope is studied. Two kinds of models of the node supporting vibration beam gyroscope are introduced. Then the model and moment of shock resistance are computed. The test result of the prototype of node supporting vibration beam gyroscope verified the vibration analysis. The Node-Plane style can also be used in high g-environment.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_1$</td>
<td>The distance between nodal and end</td>
</tr>
<tr>
<td>$l_2$</td>
<td>The distance between two nodal</td>
</tr>
<tr>
<td>$l_3$</td>
<td>The distance between nodal and end</td>
</tr>
<tr>
<td>$D^+$, $D^-$</td>
<td>Driver transducer</td>
</tr>
<tr>
<td>$v$</td>
<td>The velocity of the mass</td>
</tr>
<tr>
<td>$\omega$</td>
<td>The nation frequency of vibration rate</td>
</tr>
<tr>
<td>$x_o$</td>
<td>the maximum amplitude of vibration</td>
</tr>
<tr>
<td>$\omega_e$</td>
<td>the nation frequency of output rate</td>
</tr>
<tr>
<td>$\dot{x}$, $\dot{y}$</td>
<td>Acceleration of mass</td>
</tr>
<tr>
<td>$f$</td>
<td>The force of beam</td>
</tr>
<tr>
<td>$C, C_e$</td>
<td>The damping of beam under force</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The acceleration of the beam suffer</td>
</tr>
<tr>
<td>$h$</td>
<td>The height of the beam</td>
</tr>
<tr>
<td>$W_s$</td>
<td>the bending coefficient of the beam</td>
</tr>
<tr>
<td>$p$</td>
<td>The density of the beam</td>
</tr>
<tr>
<td>$h_1$</td>
<td>The hight of the beam</td>
</tr>
<tr>
<td>$h_2$</td>
<td>The thickness of the piezoelectric patches</td>
</tr>
<tr>
<td>$b$</td>
<td>The width of the beam</td>
</tr>
<tr>
<td>$R_1$, $R_2$</td>
<td>Readout transducer</td>
</tr>
<tr>
<td>$P$</td>
<td>Mass</td>
</tr>
<tr>
<td>$m$</td>
<td>The acceleration</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Angle Velocity along z axis</td>
</tr>
<tr>
<td>$F$</td>
<td>force</td>
</tr>
<tr>
<td>$Q$, $Q_e$</td>
<td>the Q of beam</td>
</tr>
<tr>
<td>$K_s$, $K_e$</td>
<td>the stiffness of beam</td>
</tr>
<tr>
<td>$l$</td>
<td>The arm of the force</td>
</tr>
<tr>
<td>$b$</td>
<td>The width of the beam</td>
</tr>
<tr>
<td>$L$</td>
<td>length of the beam</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>the stress of the beam</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Surface density of the beam</td>
</tr>
</tbody>
</table>

2. Principle of operation

The node supporting vibration beam gyroscope consists of a vibration beam attached to foundation bed in the node point of beam as shown in Figure 1. In order to actuating and sensing the vibrations in the beam, the piezoelectric actuators named driver transducers $D^+$ and $D^-$ are attached to the upper-surface and lower-surface, and the piezoelectric sensors named readout transducers $R_1$ and $R_2$ are attached to the left-surface and right-surface.

![Figure 1](image1.png)

**Figure 1** Model 1 of the node supporting vibration beam gyroscope

![Figure 2](image2.png)

**Figure 2** Model 2 of the node supporting vibration beam gyroscope

The node supporting structural was used to support the vibration of beam located the node point of the beam in model 1, the Node-Plane structural was used to support the vibration of beam located in the node point of the beam model 2, as shown in Figure 2. In this kind of supporting structure, the
generated linear momentum of the whole beam is zero in any case [8], the pressure between the vibration beam and the base is zero, and the interaction between beam and the external environment is zero [8]. Hence, mechanical isolation between vibration beam and foundation bed has better performance than other supporting structure.

The driving vibration is actuated through supplying a sinusoidal voltage to the piezoelectric patches which is the driving actuator stuck on the upper-surface and lower-surface of the beam body. The beam will vibrate as shown in Figure 3. As the beam is mass continuous, so generated linear momentum of beam will produce in each part of beam separated by the node, while the whole generate linear momentum will keep zero. When the sensor rotating with an angular velocity along the input angular rate (Z axis), shown in Figure 3. The other vibration of the beam is induced by the Coriolis force. The direction of this vibration is vertical with the driving plane and the inputting angular rate, named as the readout plane (Y axis) shown in Figure 3. The amplitude of this bending vibration is proportional to the amount of angular velocity. This second bending vibration, which can be measured by readout transducer stuck on the left-surface and right-surface of the beam. So the angular velocity of the beam can be measured [9]. The two kinds of node supporting vibration beam both on the same principle, therefore, the NPSG has the advantages of all nodes supporting vibration beam gyroscope. What’s more, it can overcome the disadvantage of traditional node supporting gyroscope in the fields of high shock resistance.

3. Mathematical modeling

The structure of node supporting vibration beam gyroscope is shown in Figure 3, and the structure of Node-Plane supporting vibration beam gyroscope as shown in Figure 4. The x-plane is driving plane of both kinds of the gyroscope, y-plane is reading plane, and z-plane is sensing plane. Assuming the mass of the beam is P, P will vibrate in the x-plane when the driving voltage is applied at the condition of the beam rotating along the z-axis.

The force of the mass P:

\[
v = \frac{dr}{dt} = \frac{\partial r}{\partial t} + \omega \times r
\]

\[
a = \frac{dv}{dt} = \frac{\partial^2 r}{\partial t^2} + 2\omega \times \frac{\partial r}{\partial t} + \frac{\partial \omega \times r}{\partial t} + \omega \times (\omega \times r)
\]

From Equation (2), \(\frac{dr}{dt}\) is the absolute rate of P correlated to inertial frame; \(\frac{\partial r}{\partial t}\), \(\frac{\partial^2 r}{\partial t^2}\) is absolute velocity and absolute acceleration correlated to moving frame; \(\omega \times r\) is the convection velocity due to the relatively movement of moving coordinate system to the inertial coordinate system; \(\frac{\partial \omega \times r}{\partial t}\) is tangent acceleration and centripetal acceleration arising from moving the rotating coordinate system implications. \(2\omega \times \frac{\partial r}{\partial t}\) is Coriolis acceleration, which is in relation to the relative motion \(\frac{\partial r}{\partial t}\) and the angular velocity of moving frame. There are two reasons of acceleration \(\omega \times \frac{\partial r}{\partial t}\) : one is the time-varying, \(\omega \times r\) changes with time, the other is the direction of relative velocity changes with the moving coordinate system rotation. So the Coriolis acceleration is \(2\omega \times \frac{\partial r}{\partial t}\).

If the mass of P is m, the force effort on the P is F, then, the movement is:

\[
a = \frac{1}{m} \mathbf{F} = \frac{\partial^2 r}{\partial t^2} + 2\omega \times \frac{\partial r}{\partial t} + \frac{\partial \omega \times r}{\partial t} + \omega \times (\omega \times r)
\]

The equation can be written as a matrix form:

\[
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{pmatrix}
= \begin{pmatrix}
0 & -2\omega_z & 2\omega_y \\
2\omega_z & 0 & -2\omega_x \\
-2\omega_y & 2\omega_x & 0
\end{pmatrix}
\begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix}
+ \begin{pmatrix}
-\omega_x^2 & -\omega_z^2 & 2\omega_x \omega_z \\
\omega_x \omega_y - \omega_z & \omega_x \omega_z + \omega_y & 0 \\
\omega_y \omega_z - \omega_x & \omega_y \omega_x + \omega_z - \omega_x^2 - \omega_y^2
\end{pmatrix}
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{pmatrix}
\]

\[
\begin{pmatrix}
F_x \\
F_y \\
F_z
\end{pmatrix}
= \begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{pmatrix}
\begin{pmatrix}
1/M \\
1/M \\
1/M
\end{pmatrix}
\]

\[
\text{321}
\]
Figures 3 and 4 Working principles of the node supporting vibration beam gyroscope and the nodal-plane support vibration beam gyroscope.

If only considering the movement of the mass in the x-y plane and input angular rate \( \omega_z \), it can be simplified as:

\[
\frac{1}{m} \begin{pmatrix} F_x \\ F_y \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + \begin{pmatrix} 0 & -2\omega_z \\ 2\omega_z & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + \begin{pmatrix} -\omega_z^2 & -\dot{\omega}_z \\ \dot{\omega}_z & -\omega_z^2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{5}
\]

The force \( F_x \) and \( F_y \) depend on the restraining force and damping force of material and the out-force caused by the mechanical transducer. When vibration is small, the restraining force and damping force are unrelated.

\[
\begin{pmatrix} F_x \\ F_y \end{pmatrix} = \begin{pmatrix} f_x \\ f_y \end{pmatrix} - \begin{pmatrix} C_x \dot{x} \\ C_y \dot{y} \end{pmatrix} - \begin{pmatrix} K_x x \\ K_y y \end{pmatrix}
\tag{6}
\]

The equation of the moment can be obtained from Equation (5) and Equation (6):

\[
\frac{1}{m} \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + \begin{pmatrix} \omega_o^2 \omega_o \omega_o & -2\omega_z \\ 2\omega_z & \omega_o \omega_o \omega_o \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + \begin{pmatrix} \omega_o^2 - \omega_o^2 \\ \omega_o - \omega_o \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{7}
\]

where

\[
\begin{align*}
C_x &= \frac{\omega_o}{Q_o}, & C_y &= \frac{\omega_o}{Q_o}, & K_x &= \frac{\omega_o^2}{m}, & K_y &= \frac{\omega_o^2}{m}
\end{align*}
\]

Equation (7) is the essential equation of beam vibratory angular rate sensor named vibration rate, y axis named outputting rate, z axis named inputting rate respectively

\( \omega_o, Q_o \) — the natural frequency of vibration rate, output rate;

\( \omega_o, Q_o \) — the Q of vibration rate, output rate;

\( C_x, C_y \) — the damp of vibration rate, output rate;

\( K_x, K_y \) — the stiffness of vibration rate, output rate.

Equation (7) can be changed into

\[
\frac{f_y}{m} = \ddot{y} + 2\omega_z \dot{y} + \frac{\omega_o}{Q_o} \cdot \ddot{y} + \omega_o \cdot \dot{y} + \omega_o \cdot \ddot{x} + (\omega_o^2 - \omega_o^2) \cdot y \tag{8}
\]

The amplitude of vibration in driving plane is much bigger than in the sensing plane, so the Coriolis and the centrifugal force in the x axis caused by the vibration in the y axis can be approximated to zero.
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\[ x(t) \gg y(t), \quad \dot{x}(t) \gg \dot{y}(t), \quad \omega_c^2 \gg \omega_i^2, \quad \omega_0^2 \gg \omega_i^2 \]

The approximated Equation (8) is:

\[ \frac{f_x}{m} = \ddot{y} + \frac{\omega_c}{Q_c} \cdot \dot{y} + \omega_0^2 \cdot \dot{y} \]

(9)

The beam is in a vibration state under the driving of the mechanical transducer generated force \( f_x \).

The position of mass is

\[ x = x_0 \sin(\omega_c t) \]

(10)

Where \( x_0 \) is the maximum amplitude of vibration, \( \omega_c \) is rate of driver angular. So the velocity \( \dot{x} \) of the mass is

\[ \dot{x} = x_0 \omega_c \cos(\omega_c t) \]

(11)

In this case the rate of moving coordinate system is \( \omega_c \), there Coriolis acceleration in the \( y \)-plane, the size is \( 2\omega_c \dot{x} = 2\omega_c \omega_c x_0 \cos(\omega_c t) \) and the Coriolis is

\[ F_y = 2m\omega_c \omega_c x_0 \cos(\omega_c t) \]

(12)

For the moment equation of beam, substituting Equation (12) into Equation (9)

\[ \frac{f_x}{m} = \ddot{y} + \frac{\omega_c}{Q_c} \cdot \dot{y} + \omega_0^2 \cdot \dot{y} = 2\omega_c \omega_c x_0 \cos(\omega_c t) \]

(13)

Figure 5 Size of designed vibration beam

4. Calculation of the shocking resistance

The shocking resistance, which induced the amplitude of beam vibration equals to the maximum elastic deformation of the beam, is the maximum momentum of the vibration beam gyroscope. The maximum momentum of the node supporting vibration beam gyroscope is the value of shocking, which is too big to cause the structure of gyroscope distorted. Both the node point and beam of gyroscope will be distorted when the shock is applied. Obviously, the node point is the weakest part of the gyroscope to the shocking. So, the value of the shocking resistance is the shocking which made the node supporting point distorted. The structure led to the node supporting vibration beam can’t bear high shocking. While the plane supporting structure will strengthen the supporting structure compared with pin supporting. In this case the vibration beam just as a beam across the plane located in the nodal of beam, so the beam can be seen as a cantilever beam. The deformation of beam was just shown in Figure 6 when the shocking is applied.

When shocking is applied on the beam, the beam will bent by an undergoing force. And the bending moments of beam under force is:

\[ M = f \cdot l \]

(14)

Since the beam’s mass is continuous, assuming the moment of every bit of mass in the beam is \( dM \), the bending moments of the beam is \( M \), which is the effort of the acceleration to the beam:
\[ M = \rho a \int_0^l \frac{l-x}{l} x \, dx = \frac{\rho al^2}{6} \] \hspace{1cm} (15)

Where \( M \) is flexural moment of the beam under the acceleration, the \( \rho \) is the surface density of the beam, \( a \) is the beam bear acceleration, \( l \) is the length of the beam, \( x \) is the direction of mass form the node.

The limitation of the flexural moment depends on the maximum stress of the material and the bending coefficient of the beam, whose cross-section is rectangular. The relationship of them is given as following:

\[ \sigma_{\text{max}} = \frac{M}{W} \] \hspace{1cm} (16)

Where \( \sigma_{\text{max}} \) is the maximum stress of the beam, \( W \) is the bending coefficient of the beam.

The bending coefficient of the beam is determined by the width of the beam and the height of the beam.

\[ W = \frac{bh^3}{6} \] \hspace{1cm} (17)

Where, \( B \) and \( H \) are the width and height of the beam, respectively.

**Figure 6** Deformation of the shocked beam

In order to derive the maximum value of acceleration can be born by beam in this section. Taking the flexural moment of the beam from the Equation (15), the bending coefficient of the beam from the Equation (17) and substitute the \( W \) and \( M \) into Equation (16), Equation (18) will be obtained.

\[ a = \frac{\sigma BH^2}{\rho l^2} \] \hspace{1cm} (18)

The bending coefficient of the cylinder nodal was determined by the diameter of the cylinder support node.

\[ W_{\text{cylinder}} = \frac{\pi D^3}{32} \] \hspace{1cm} (19)

Where, \( D \) is the diameter of the cylinder.

In order to derive the maximum value of acceleration can be input by cylinder support nodal. Taking the flexural moment of the beam from the Equation (14), the bending coefficient of the beam from the Equation (19) and substitute the \( W_{\text{cylinder}} \) and \( M \) into Equation (16), the result is

\[ a = \frac{4 \cdot \sigma_{\text{max}} \pi D^3}{bhL \rho d^3} \] \hspace{1cm} (20)

Where, \( d \) is the distance from the center of the beam to the support.

**5. Characteristics analyzing**

In this section, the shocking value of the two kinds of the vibration beam gyroscope was calculated with the numerical values in the Table 1. The shocking value of gyroscope can be obtained by the above theory and numerical values shown in the Table 1.

The shocking value of the middle part of the beam can be born calculated by Equation (18) with the parameters as Table 1. The structure of both parts of the part 2 is the same, so the shocking value of both parts of part 2 equal to each other. The shocking value of part 2 of the beam can be born solved by Equation (18) with the parameters as Table 1 too.
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Table 1 Physical parameters of the beam

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Numerical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_1$</td>
<td>The distance between nodal and end</td>
<td>13.50mm</td>
</tr>
<tr>
<td>$l_2$</td>
<td>The distance between two nodal</td>
<td>33.00mm</td>
</tr>
<tr>
<td>$l_3$</td>
<td>The distance between nodal and end</td>
<td>13.50mm</td>
</tr>
<tr>
<td>$b$</td>
<td>The width of the beam</td>
<td>4.12mm</td>
</tr>
<tr>
<td>$h_1$</td>
<td>The height of the beam</td>
<td>4.10mm</td>
</tr>
<tr>
<td>$h_2$</td>
<td>The thickness of the piezoelectric patches</td>
<td>0.05mm</td>
</tr>
<tr>
<td>$\rho$</td>
<td>The density of the beam</td>
<td>7900kg*m$^{-3}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>The maximum stress of the beam</td>
<td>1310MPa</td>
</tr>
<tr>
<td>$d$</td>
<td>The distance between edge of beam and point which support the beam</td>
<td>2mm</td>
</tr>
<tr>
<td>$r$</td>
<td>The diameter of the cylinder support node</td>
<td>0.5mm</td>
</tr>
</tbody>
</table>

\[
a = 0.06Bh^2 = \frac{1310 \times 10^6 \times 4.14 \times 10^{-3} \times 4.1^2 \times 10^{-6}}{7900 \times 4.14 \times 4.1 \times 10^{-6} \times 13.5^2 \times 10^{-6}} = 3.8065755 \times 10^2 g
\]

The shocking resistance value of the middle part of the beam is
\[
a = \frac{1310 \times 10^6 \times 4.14 \times 10^{-3} \times 4.1^2 \times 10^{-6}}{7900 \times 4.14 \times 4.1 \times 10^{-6} \times 33^2 \times 10^{-6}} = 3.73044399 \times 10^9 = 3.73 \times 10^9 g
\]

As the Figure 1 shows, the value of the node supporting beam is depending on the value of the node and beam can bear.

The shocking of the nodal can bear
\[
a = \frac{4 \cdot \sigma_{max} \pi D^2}{bhLpd32} = \frac{4.13 \times 10^6 \times 3.14 \cdot (0.5 \times 10^{-3})^3}{4.1 \times 10^{-3} \times 4.14 \times 10^{-3} \times 60 \times 10^{-3} \times 7900 \times 2 \times 10^{-3} \times 32} = 4.075 g
\]

The structure of the node supporting vibration beam is shown in Figure 1. The cylinder support and vibration beam of the gyroscope will be broken when it undergoes an overloading shocking, therefore the shocking value of the gyroscope is that the beam and cylinder support can be born. From the Equation (21) and Equation (22) we can know the value shock of the beam is 63705g. From Equation (23), it can be obtained that the value shocking of the support structure is 407.5g. And the gyroscopes’ value shocking resistance is decided by the support structure. Hence shocking value of the node supporting vibration beam gyroscope is 407.5g.

The structure of the Node-Plane supporting vibration beam is shown in Figure 2. The support structure of this kind of the gyroscope is a plane located in the nodal of the beam, so it was attached to a moving base as a cantilever beam. Therefore the vibration beam is the only vulnerable part of the gyroscope. The shocking of the beam will be calculated from the Equation (21) and Equation (22). The shocking of nodal-plane support vibration beam gyroscope equals to the minimum value of the two parts of the beam is 63705g.

6. Experiments

Figure 7 shows the scheme of vibration beam gyroscope. The digital signal processor (DSP, TMS320F642) sends the control words to the Direct Digital Synthesizer (DDS). According to the controlling words, the DDS outputs the sine signal at the expected working frequency. The signal
produced by the DDS is improved and sent to two fix-frequency signals, one is the driver signal and the other is the driver signal. Two fix-frequency signals are used to drive the vibration beam gyroscope. The resolution of the frequency obtained through the DDS can reach a high level, about 0.1Hz for AD9833, and which is accurately enough for obtaining and tracking the working resonance of the vibration beam gyroscope.

Figure 8 shows the experimental setup and a close-up view of the vibration beam gyroscope. Figure 9 shows the shock testing machine, which used to supply an expected shock to the experiment gyroscope. The important parameter of the gyroscope will change after withstood the shock. Here, four parameters of the gyroscope were tested after shock, which is scattered, as shown in Figure 10 to Figure 13. In those figures, the results of two kinds of vibration beam are compared.

In Figure 10, the short-term bias stability of two vibration beam gyroscopes were given. The figure shows the short-term bias stability of the node supporting vibration beam gyroscope has been very small until the gyroscope suffered a 210 g. The nodal-plane support vibration beam gyroscope keeps good performance when the shocking of the gyroscope is less than 10000 g. Hence, the short-term bias stability is one of the important parameters. If short-term can’t keep at a low level it will not work.

Figure 11 showed the error of the scale factor of output signal of the two kinds of the vibration beam gyroscope after suffering the shocking. The error is stable and small when the shocking is 200 g to the node supporting vibration beam and 10000 g to the nodal-plane support vibration beam.

Figure 7 Driving and sensor scheme of the vibration beam gyroscope

Figure 8 picture of the experimental setup and a close-up view of the vibration beam gyroscope

Figure 9 the shock testing machine

Figure 12 showed the error of the output of the two kinds of the vibration beam gyroscope. The error of output changed at the value of 210 g and 10000 g of the two kinds of the vibration beam.
gyroscope. The value of the shocking is less than 210 g and 11000 g the error of the output keeps small which is less than 0.02 or 0.03.

Figure 13 showed the nonlinear of the both kinds of gyrooscope after different shocking. The nonlinear of the node supporting vibration beam gyrooscope is small. When the gyrooscope suffers the shocking less than 220 g, the nonlinearity will change into chaos significantly. And the 250 g can’t be accepted by the system. It is clear that the NPSG has a better ability against shocking. The nonlinear of the node-plane support vibration beam gyrooscope has no changes after the gyrooscope has been shocked. The nonlinear output of the node-plane support vibration beam gyrooscope is less than 0.005 until the shocking of the gyrooscope reaches 1100 g. When the shocking is bigger than 11000 g the nonlinear of the output also become bigger and the performance of the gyrooscope will get worse.

![Figure 10](image1.png)  
Figure 10 the Short-term bias stability change of the both kinds of gyrooscope

![Figure 11](image2.png)  
Figure 11 the error of the scale factor of the both kind of gyrooscope

Experiment shows that when shocking applied on the node supporting vibration beam gyrooscope reaches to 200 g, the error of the scale factor of output signal over the limit of the system requirements. When the shocking reaches to 210 g, the parameter of short-term bias stability and the output error will become worse. And when the shocking reaches to 220 g, the nonlinearity can’t be accepted. So the traditional node supporting vibration beam gyrooscope can work well when the shocking less than 200 g.

It is evident that the NPSG have better shocking resistance characteristics than the traditional node supporting vibration beam gyrooscope which based on the supporting structure of cylinder pins. The short-term bias stability of the gyrooscope is less than 0.003, error of the Scale factor is less than 2%, error of the output is less than 0.004, and the nonlinearity of the output is less than 0.4% when the shocking is less than 10000 g. The Figure 10 to Figure 13 shows that the NPSG can suffer the largest shock up to 10000 g.

![Figure 12](image3.png)  
Figure 12 the error of the output of the both kind of gyrooscope after different shock

![Figure 13](image4.png)  
Figure 13 the nonlinear of the both kind of gyrooscope after different shock
7. Conclusion and discussion

Node-Plane supporting vibration beam gyroscopes is a new kind of solid state angular rate sensor. The sensing part of the gyroscope is a vibration beam. In the traditional node supporting vibration beam gyroscope, the sensing part was supported by the cylinder pin which connects with the substrate. The presented supporting structure of the NPSG has a better capability against shaking and impacting caused by high-g or other conditions. In this paper, the working principle of the NPSG was introduced; the value of shocking resistant of the two kinds of the solid vibration beam gyroscope was derived and compared. The analyzing results shows that the traditional node supporting vibration beam gyroscope can suffer 407 g of shocking, while the Node-Plane supporting vibration beam gyroscope is 6*104 g. The experimental results of two kinds of the solid vibration beam gyroscope proved the principle analyzing. The work in this paper provides a theoretical and experimental foundation for the shocking resistance ability of the node supporting vibration beam gyroscope.

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9. References