A Vector Bond Graph Method of Kineto-static Analysis for Spatial Multibody Systems

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Keywords: Vector Bond Graph, Kineto-static Analysis, Spatial Multibody System, Joint Constraint, Causality

Abstract. In order to increase the reliability and efficiency of the kineto-static analysis of complex multibody systems, the corresponding vector bond graph procedure is proposed. By the kinematic constraint condition, spatial multibody systems can be modeled by vector bond graph. For the algebraic difficulties brought by differential causality in system automatic kineto-static analysis, the effective decoupling method is proposed, thus the differential causalities in system vector bond graph model can be eliminated. In the case of considering EJS, the unified formulae of driving moment and constraint forces at joints are derived based on vector bond graph, which are easily derived on a computer in a complete form and very suitable for spatial multibody systems. As a result, the automatic kineto-static analysis of spatial multibody system on a computer is realized, its validity is illustrated by the spatial multibody system with three degrees of freedom.

Introduction

System kineto-static analysis is essential for the control and dynamic design of modern mechanical systems. For complex spatial multibody systems, determining driving moment (or force) and the constraint forces at joints is a very tedious and error-prone task on account of the nonlinearities and couplings involved. Although different procedures have been proposed to increase the reliability and efficiency of this process\textsuperscript{[1,2]}, most of them can not be used to deal with systems that simultaneously include various physical domains in a unified manner.

Bond graph technique\textsuperscript{[3]} was chosen because it is a computer oriented method which can describe all type of physical systems, thus allowing a single model to represent the dynamic interactions of the spatial multibody system with electrical, hydraulic, pneumatic, and other components. Compared with scalar bond graph\textsuperscript{[3]}, vector bond graph is more suitable for modelling spatial multibody systems because of its more concise representation manner\textsuperscript{[4]}. Since complex spatial multibody systems posses highly nonlinear and coupled dynamic character, these result in differential causality and nonlinear junction structure. Current vector bond graph procedures\textsuperscript{[4]} were found to be very difficult algebraically in derivation of system driving moment (or force) and the constraint forces equations automatically on a computer. To solve above problems, a more efficient and practical kineto-static analysis procedure for complex spatial multibody system based on vector bond graph\textsuperscript{[4]} is proposed here.

The Unified Formulae of Driving Moment and Constraint Forces for Spatial Multibody Systems

To eliminate the differential causality of system vector bond graph model, the constraint force vectors at joints can be considered as unknown effort source vectors and added to the corresponding 0-junctions of the system vector bond graph model. In what follows, the unified formulae of driving moment (or force) and constraint forces at joints are derived in the case of considering EJS\textsuperscript{[5]} (Euler-junction structure), which are easily derived on a computer in a complete form.

The basic fields and junction structure of system bond graph is shown in Fig.1\textsuperscript{[3]}, where Euler-junction structure\textsuperscript{[5]} (EJS) is added. $x_j$ represents energy vector variable of independent storage energy field corresponding to independent motion, $x_i$ represents energy vector variable of...
independent storage energy field corresponding to dependent motion, \( Z_{iz} \) and \( Z_{is} \) are the corresponding coenergy vector variables. \( D_{in} \) and \( D_{out} \) represent input and output vector variables in resistive field, \( U \) and \( V \) represent input and output vector variables of source field respectively, \[ \begin{bmatrix} U_1 & U_2 & U_3 \end{bmatrix}^T, \begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix}^T. \] Where \( U_1 \) is driving moment (or force) vector, \( U_2 \) is the constraint force vector of joint, and \( U_3 \) is known source vector. \( E_{in} \) and \( E_{out} \) are the input and output vector variables in Euler-junction structure (EJS).

For independent energy storage field, we have

\[
Z_{iz} = F_{iz} X_{iz}, \quad Z_{is} = F_{is} X_{is}
\]  

(1)

where \( F_{iz} \) and \( F_{is} \) are the \( m_i \times m_i \) and \( m_j \times m_j \) matrices respectively.

For resistive field, we have

\[
D_{out} = RD_{in}
\]  

(3)

where \( R \) is \( L \times L \) matrix.

For Euler-junction structure (EJS), we have

\[
E_{out} = R_E E_{in}
\]  

(4)

where \( R_E \) is \( L_E \times L_E \) matrix[5].

The corresponding junction structure equations can be written as

\[
\begin{align}
\dot{X}_{iz} &= J_{iz} Z_{iz} + J_{iz,i} D_{out} + J_{iz,\mu} U_1 + J_{iz,\mu_2} U_2 + J_{iz,\mu_3} U_3 + J_{iz,E} E_{out} \\
\dot{X}_{is} &= J_{is} Z_{is} + J_{is,i} D_{out} + J_{is,\mu} U_1 + J_{is,\mu_2} U_2 + J_{is,\mu_3} U_3 + J_{is,E} E_{out} \\
D_{in} &= J_{ii} Z_{is} + J_{ii,i} D_{out} + J_{ii,\mu} U_1 + J_{ii,\mu_2} U_2 + J_{ii,\mu_3} U_3 + J_{ii,E} E_{out} \\
E_{in} &= J_{ii} Z_{is} + J_{ii,i} D_{out} + J_{ii,\mu} U_1 + J_{ii,\mu_2} U_2 + J_{ii,\mu_3} U_3 + J_{ii,E} E_{out}
\end{align}
\]  

(5)

(6)

(7)

(8)

From the flow summation of 0-junctions corresponding to \( m_i \) constraint force vectors in system vector bond graph model, we have

\[
0 = J_{Ci} Z_{i} + J_{Ci} Z_{i} + J_{CL} D_{out} + J_{Ci} U_3 + J_{CE} E_{out}
\]  

(9)

By the algebraic manipulation from Eq.(1)~Eq.(9), the system driving moment and constraint force equations can be written as

\[
\begin{align}
U_1 &= S_{\mu \mu} (S_{\mu \mu} X_i + S_{\mu \mu} X_i + S_{\mu \mu} X_i + T_{\mu \mu} \dot{X}_{i} + T_{\mu \mu} \dot{X}_{i} + H_{\mu \mu} J_{\mu \mu} U_3) \quad (a) \\
U_2 &= (-H_{\mu \mu}) (H_i X_i + H_i X_i + H_i X_i + J_{\mu \mu} U_3) \quad (b)
\end{align}
\]  

(10)

where

\[
\bar{A} = [I_2 - J_{EL} (I_1 - J_{LL} R)^{-1} J_{LE} R - J_{EE} R E]^{-1}, \quad A_2 = J_{EL} F_{i} + J_{EL} (I_1 - J_{LL} R)^{-1} J_{LLU} F_{i}
\]

\[
A_3 = J_{EL} F_{i} + J_{EL} (I_1 - J_{LL} R)^{-1} J_{LLU} F_{i}, \quad A_4 = J_{LLU} + J_{EL} (I_1 - J_{LL} R)^{-1} J_{LLU}
\]

\[
A_5 = J_{EL} F_{i} + J_{EL} (I_1 - J_{LL} R)^{-1} J_{LLU}, \quad A_6 = J_{ELU} + J_{EL} (I_1 - J_{LL} R)^{-1} J_{LLU}
\]

\[
B_1 = (I_1 - J_{LL} R)^{-1} (J_{LLU} F_{i} + J_{LE} R E A_1), \quad B_2 = (I_1 - J_{LL} R)^{-1} (J_{LLU} F_{i} + J_{LE} R E A_2)
\]

\[
B_3 = (I_1 - J_{LL} R)^{-1} (J_{LLU} + J_{LE} R E A_1), \quad B_4 = (I_1 - J_{LL} R)^{-1} (J_{LLU} + J_{LE} R E A_2)
\]

\[
B_5 = (I_1 - J_{LL} R)^{-1} (J_{LLU} + J_{LE} R E A_2), \quad T_{\mu \mu} = J_{\mu \mu} F_{i} + J_{\mu \mu} R B_1 + J_{\mu \mu} E A_2
\]

\[
T_{\mu \mu} = J_{\mu \mu} F_{i} + J_{\mu \mu} R B_1 + J_{\mu \mu} E A_2, \quad T_{\mu \mu} = J_{\mu \mu} R B_1 + J_{\mu \mu} E A_2
\]

\[
H_1 = J_{EL} F_{i} + J_{EL} F_{i} + J_{EL} F_{i} + J_{CL} F_{i} T_{\mu \mu}, \quad H_2 = J_{EL} F_{i} + J_{EL} F_{i} T_{\mu \mu} + J_{CL} F_{i} T_{\mu \mu}
\]

\[
H_3 = J_{EL} F_{i} T_{\mu \mu} + J_{CL} F_{i} T_{\mu \mu}, \quad H_4 = J_{CL} F_{i} T_{\mu \mu} + J_{CL} F_{i} T_{\mu \mu}
\]
\[
H = \dot{J}_{Cu} + J_{Cy} \dot{F}_i T_{i\eta_i} + J_{Cz} \dot{F}_j T_{j\eta_j} + S_{u_\eta_1} = T_{i\eta_1} \left[ T_{i\eta_1} + T_{i\eta_2} (-H_1) \right] + \frac{1}{H_2} T_{j\eta_2} \nonumber
\]
\[
S_{u_{\eta_2}} = T_{i\eta_1} (T_{i\eta_1} H_1 - T_{i\eta_2}), \quad S_{u_{\eta_2}} = T_{i\eta_2} (T_{j\eta_2} H_2 - T_{i\eta_2}), \quad S_{u_{\eta_3}} = T_{i\eta_2} (T_{i\eta_3} H_3 - T_{i\eta_3})
\]

If \( J_{CL} \neq 0 \) or \( J_{CE} \neq 0 \)

\[
\begin{align*}
U_1 &= D_{u\eta_1}^{-1} \left( D_{u\eta_1} X_i + D_{u\eta_2} X_j + D_{u\eta_3} U_1 + T_{i\eta_1} \dot{X}_i \right) \\
U_2 &= (-T_{Cu_j})^{-1} (T_{Cu_j} X_i + T_{Cu_j} X_j + T_{Cu_j} U_1 + T_{Cu_j} U_3)
\end{align*}
\]

(11)

where

\[
\begin{align*}
T_{Cu_j} &= J_{Cl} F_i + J_{Cl} R B_1 + J_{CE} R E A_i A_2 \\
T_{Cy_i} &= J_{Cl} F_i + J_{Cl} R B_2 + J_{CE} R E A_i A_3 \\
T_{Cu_i} &= J_{Cl} R B_1 + J_{CE} R E A_i A_4, \quad T_{Cu_j} = J_{Cl} R B_4 + J_{CE} R E A_i A_5 \\
T_{Cu_j} &= J_{Cl} R B_3 + J_{CE} R E A_i A_6, \quad D_{u\eta_1} = T_{i\eta_1} \left( T_{i\eta_1} T_{i\eta_2}^{-1} T_{i\eta_2} \right) \nonumber
\] \]

Giving the system independent moving state variable vector \( X_i \) and its derivative \( \dot{X}_i \), the corresponding system driving moment (or force) vector \( U_1 \) and constraint force vector \( U_2 \) can be determined from Eq.(10) and Eq.(11) directly.

### Example Systems

Fig.2 shows a spatial multibody system with three degrees of freedom. The components for this example are three rigid bodies, two revolute joints \( J_1, J_2 \) and a prismatic joint \( J_3 \). These components are parameterized with the following data: \( m_1 = m_2 = 1 \) Kg are the mass of the rigid body \( m_1 \) and \( m_2 \), \( I_x = I_y = I_z = 4.167 \times 4 \text{Kgm}^2 \) are the principal moment of inertia of \( m_1 \) and \( m_2 \). \( F_1 = F_2 = -9.8 \text{N} \) are the weight of rigid body \( m_1 \) and \( m_2 \). \( K = 500 \text{N/m} \) is spring stiffness, \( a = 0.3 \text{m} \), \( b = 0.1 \text{m} \) are the distances shown in Fig.2. \( M_1 \) and \( M_2 \) are two moments imposed to joint \( J_1 \) and \( J_2 \) respectively, \( F \) is a driving force imposed to joint \( J_3 \). The system input motion are as following, \( \theta_1 = \sin(\pi t) \), \( \theta_2 = \sin(2\pi t) \), \( c = \cos(\pi t) \).

The revolute joint element is a joint that allows turning the bodies jointed between them. Therefore, three translations and two rotation degrees of freedom are constrained, leaving only one rotation degree of freedom free. For prismatic joint, two translations and three rotation degrees of freedom are constrained, leaving only one translation degree of freedom free. From these kinematic constraint conditions, the vector bond graph model of revolute joint and prismatic joint can be obtained. By assembling the vector bond graph of a single space moving rigid body[4], the revolute joints, and the prismatic joint, the overall system vector bond graph model can be obtained and shown as Fig.3, where EJS is equivalent to R element of bond graph[5].
The constraint force vectors of joints can be considered as unknown source vectors, such as $S_{e_2}, S_{e_3}, S_{e_5}, S_{e_{10}}$ in Fig.3, and added to the corresponding 0-junctions to eliminate differential causality. As a result, all differential causalities in this system vector bond graph can be eliminated, thus the procedure presented here can be used.

In Fig.3, $J_{J_2} = [I_{Z_2}], J_{J_3}^b = \text{diag}[I_{X_2}, I_{Y_2}, I_{Z_2}], J_{J_3}^b = \text{diag}[I_{X_3}, I_{Y_3}, I_{Z_3}]. \dot{r}_{C_2}, \dot{r}_{C_3}$ are the mass center velocity vector of body 2 and body 3 in global coordinates, $\omega_{J_2}^b, \omega_{J_3}^b$ are the angular velocity vector of body 2 and body 3 in body frame respectively.

Inputting the physical parameters of the spatial multibody systems, the coefficient matrices of Eq.(1)~Eq.(9), known source vector $U_3$, system independent moving state variable vector $\dot{X}_i$, and its derivative $\ddot{X}_i$ into the program associated with the procedure presented here based on MATLAB\cite{6}, the system driving moment (or force) and constraint force equations in the form of Eq.(11) can be derived on a computer, and the corresponding driving moment (or force) and constraint forces can be determined. Some of results are shown in Fig.4~Fig.7.

For this example, the Newton-Euler method\cite{1,2} was used to determine the corresponding driving moment (or force) and constraint forces, the results are in good agreement with that obtained by the procedure in this paper.
Conclusions

Compared with standard scalar bond graph method, the vector bond graph procedure presented here is more concise. Because nonlinear junction structure and differential causality exit in the vector bond graph of spatial multibody system, current vector bond graph methods are found to be very difficult algebraically in the derivation of system driving moment and constraint force equations. Thus the constraint force vectors of joints can be considered as unknown source vectors, and added to the corresponding 0-junctions to eliminate differential causality. In the case of considering EJS, the unified formulae of system driving moment and constraint force equations are derived, which are easily derived on a computer. These lead to a more efficient and practical automated procedure for kineto-static analysis of complex spatial multibody systems over a multi-energy domains in a unified manner, its validity is illustrated by the spatial multibody system with three degrees of freedom.

Acknowledgements

This work was financially supported by National Natural Science Foundation of China (Grant No. 51175272).

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