An experimental study on out-of-plane inelastic buckling strength of fixed steel arches

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Abstract

This paper presents an experimental study on the out-of-plane inelastic buckling strength of fixed circular steel arches under symmetric and non-symmetric loading. A test loading arrangement that allows for lateral deflections to develop freely under vertical loading is described. A finite element (FE) model consisting of the tested steel arch and the loading system is established for carrying out supplementary numerical investigation on the inelastic out-of-plane buckling strength of the fixed steel arches. The FE numerical model is validated by the experimental results. From the experimental results and supplementary FE investigation, it is found that the out-of-plane inelastic buckling strength of fixed steel arches is influenced significantly by the magnitude and distribution of initial out-of-plane geometric imperfections, as well as the out-of-plane elastic buckling modes and the in-plane loading patterns. It is also found that the out-of-plane buckling strength of a fixed steel arch under non-symmetric loading is lower than that under symmetric loading. Based on the experimental and FE results, a lower bound interaction equation is developed for predicting the out-of-plane inelastic buckling strength in the design of fixed circular steel arches against their out-of-plane failure.

1. Introduction

Fixed steel arches have been widely used in buildings and bridges. Under in-plane loading, a fixed steel arch may fail in an out-of-plane inelastic buckling if it does not have adequate lateral bracings. Studies on elastic and inelastic out-of-plane buckling strength of steel arches have focused mainly on laterally pin-ended arches [1–13]. Because the distributions of the internal actions and out-of-plane elastic buckling loads of fixed steel arches are different from those of pin-ended steel arches [14–16], it is expected that their out-of-plane inelastic buckling strengths differ from those of pin-ended steel arches [17–19].

Although a number of analytical and numerical studies of the out-of-plane elastic and inelastic buckling of pin-ended arches have been published in the literature [1–13], there are very limited investigations on the out-of-plane elastic and inelastic buckling strength of fixed steel arches. Pi and Bradford [14], Bradford and Pi [15] investigated the out-of-plane elastic buckling of fixed circular arches. Pi and Bradford [16] also studied the effects of pre-buckling deformations on the out-of-plane elastic buckling of fixed arches. Dou et al. [17] showed the significant influences of the buckling mode shapes on the accurate derivation of the out-of-plane elastic buckling load of fixed arches, and Kang and Bert [18] applied the differential quadrature method to compute the eigenvalue for elastic flexural–torsional buckling of arches with different boundary conditions. Besides, Pi and Bradford [19] studied the out-of-plane strength of fixed steel I-section arches numerically. These analytical and numerical results need to be validated against the experimental results of steel arches. As a consequence, the present paper focuses on the experimental investigations of the out-of-plane elastic and inelastic buckling of fixed steel arches.

While several experimental studies of the out-of-plane elastic buckling of arches have been conducted [20–23], only three experimental investigations of the out-of-plane inelastic strength of fixed steel arches have been reported [24–27]. Sakimoto et al. [24] conducted tests on three freestanding and nine braced circular or parabolic steel arches having a box section and a constant rise-to-span ratio of $f/L = 0.2$ under combined vertical tilting and horizontal lateral loading. Sakata and Sakimoto [25] investigated experimentally the out-of-plane inelastic buckling of steel I-section arches under eight concentrated gravity loads or tilting loads. Recently, La Poutre et al. [26] carried out an experimental study of the out-of-plane inelastic buckling of circular steel I-section arches. Fifteen arches having a constant length and included angles varying from $90^\circ$ to $180^\circ$ were tested under a centrally concentrated...
load. Dou et al. [27] carried out tests on the inelastic buckling and strength of steel I-section circular arches. All of the arches used in these experimental studies [24–27] were in-plane pin-ended and out-of-plane fixed. In the Ref. [26,27], to prevent the torsion restraining effect of the loading device on the arch rib during out-of-plane deformations, a hydrostatic load was applied at the loading point on the arch crown, resulting in a tilting load through a tension rod, while in the Refs. [24,25] both gravity and tilting loads were applied.

It is known that the out-of-plane inelastic buckling strength of a steel arch is significantly influenced by the distribution and magnitude of its initial out-of-plane geometric imperfections [19]. Unfortunately, these were not thoroughly investigated [24–26]. In addition, the in-plane boundary conditions and load cases have significant influences on the out-of-plane inelastic buckling strength of fixed arches. Hence, further experimental investigations of the out-of-plane inelastic buckling strength of fully fixed steel arches and the effects of initial out-of-plane geometric imperfections on the strength provide significant contributions to the pertinent literature.

This paper therefore presents an experimental study on the out-of-plane inelastic buckling strength of fixed circular steel arches. Symmetrical three-point loading and non-symmetrical two-point loading schemes are arranged in the experimental study. The influences of the initial out-of-plane geometric imperfections and the elastic buckling mode shapes on the out-of-plane inelastic buckling strength are investigated experimentally. Based on the test results, a FE numerical model for the out-of-plane inelastic strength of fixed steel arches is established for the supplementary numerical investigation. The experimental and numerical results are to be used to develop the design equation for the out-of-plane inelastic buckling strength of fixed circular steel arches.

2. Out-of-plane elastic buckling analysis

Because it is difficult to apply gravity point loads to a freestanding arch while it experiences out-of-plane deformations during testing (Fig. 1a) [26], the loads will be applied to the arch through vertical ties as shown in Fig. 1b. The ties are no longer vertical, but inclined during the out-of-plane deformations and so the direction of the applied load changes accordingly. Because of the influence of inclined ties, the out-of-plane elastic buckling behavior of the arch shown in Fig. 1b may be different to that of the freestanding arch shown in Fig. 1a. To facilitate the experimental investigation, it is important to determine whether the out-of-plane elastic buckling behavior of fixed arches under loads applied through vertical ties (Fig. 1b) is different from that of freestanding arches under gravity loads (Fig. 1a). For this, the commercial FE software ANSYS [28] was used to determine the respective out-of-plane elastic buckling loads and mode shapes. The arches were modeled by the Timoshenko beam element BEAM188, while the ties were assumed to be rigid and modeled by the two-node link element Link8 of ANSYS. The arches were assumed to have a doubly symmetric steel I-section with the dimensions listed in Table 1, in which \( h \) is the overall height of the I-section, \( b \) is the width of the flange, \( t_f \) is the flange thickness, and \( t_w \) is the web thickness. The rise-to-span ratios \( f/L \) of the arches were assumed to vary from 0.1 to 0.4 with a constant span of \( L = 6 \) m. An ideal bi-linear stress–strain curve was adopted for the steel, with Young’s modulus \( E = 206 \) GPa and shear modulus \( G = 81 \) GPa.

The relationships between the out-of-plane elastic buckling loads \( F \) and the rise-to-span ratios \( f/L \) for freestanding arches and for arches with vertical ties are shown in Fig. 2a and b for symmetric three-point loading (\( o_1 \) and \( o_3 \) at the quarter points from both ends and \( o_2 \) at the crown) and non-symmetric two-point loading (\( o_2 \) at one quarter point and \( o_3 \) at the crown) respectively. It can be seen that the out-of-plane elastic buckling loads of arches with vertical ties are significantly higher than those of the counterpart freestanding arches. It can also be seen that under the symmetrical three-point loading, arches with vertical ties buckle in an out-of-plane anti-symmetric two-half-wave mode while the freestanding arches buckle in an out-of-plane symmetric one-half-wave mode. Under the non-symmetric two-point loading, the out-of-plane buckling shapes are non-symmetric both for arches with vertical ties and for freestanding arches. In addition, it can be seen from the Fig. 2a and b that the out-of-plane elastic buckling load \( F \) increases when the rise-to-span ratio of arch \( f/L \) increases from 0.1 to 0.2, but decreases when the rise-to-span ratio of arch \( f/L \) increases from 0.2 to 0.4, and that the buckling load has a peak value when \( f/L \) equal to 0.25 for symmetric three-point loading or when \( f/L \) equal to 0.2 for non-symmetric two-point loading. With knowledge of the out-of-plane elastic buckling behavior of steel arches under loading through the vertical ties, an experimental investigation of the out-of-plane inelastic buckling strength was carried out.

3. Experimental investigation of out-of-plane inelastic buckling

3.1. Specimens

Steel plate designated Q235B [29] was used for fabrication of the steel I-section arch specimens. The flanges were rolled to the curved shape from steel plates by a hot-rolling machine while
the circular web was cut to size from a flat steel plate. The circular flanges and web were then welded together to form a steel I-section arch. The dimensions of the I-section are also listed in Table 1. Because no bending process for the whole I-section was involved in the fabrication of the steel arches, there were no residual rolling stresses in the specimens and so the residual stresses induced in the I-sections by welding the flanges and web were only taken as being the cross-section residual stresses. Four steel arches with welded I-sections were tested, and their dimensions are listed in Table 2. Material properties were tested on four coupons: two from the flanges and the other from the web. The test results for the material properties are listed in Table 3 and are also shown in Fig. 3.

3.2. Test setup, boundary conditions, and loading

The overall test setup is shown in Fig. 4. In the experimental investigations of Refs. [24–27], all of the arches were pin-ended in their plane and fixed out of their plane. However, in many instances of arches in engineering practice, the ends of the arches are fully fixed. Hence, the boundary conditions of the arches investigated in the present study were fully fixed both in-plane and out-of-plane by connecting the ends of arch to the floor beam through high-strength bolts, as shown in Fig. 5.

Because loading device plays an important role in applying the loads to the arch specimens properly, the loading scheme shown in Fig. 6a–d was used in the test, where loading frames were placed directly on the top flange of the arch and acted as vertical ties as shown in Fig. 6a and c. The loading frames were connected to the bottom rigid levers by U-shaped clips. Point loads were applied by hydraulic jacks acting at the mid-span of the bottom rigid levers which were simply supported at one end (Fig. 6b), and connected to the loading frame at other end by the U-shaped clips, thereby having a mechanical advantage of 1/2. The force \( Q \) generated by the hydraulic jack (Fig. 6b) was transferred through the U-shaped clips to the loading frames and then further to the loading hemispheres to apply a load \( F \) at the top flange of the arch (Fig. 6c). Because of the mechanical advantage of 1/2, \( F = Q/2 \). The loading hemispheres and U-shape clips (Fig. 6c and d) were used to ensure that the lateral displacement and twist rotation of the cross-section were not restrained by the loading frames.

3.3. Measurements

In the previous experimental investigations to the out-of-plane inelastic strength of steel arches [24–26], the central initial out-of-plane geometric imperfections were not considered. However in practice, the initial out-of-plane geometric imperfections of steel arches are inevitable in the fabrication and installation of the arches. Hence, prior to the testing, the initial out-of-plane
geometric imperfections of the specimens were measured at nine stations with equal intervals along the axis of arch specimens and the measured results are shown in Fig. 7, in which $\Theta$ represents the included angle for the arch and $\phi$ represents the angular coordinate of the stations. The maximum magnitudes of the out-of-plane geometric imperfections for all the specimens are listed in Table 2. Compared with the initial lateral imperfections, the initial twist imperfections are very small and so can be ignored.

Symmetric three-point loading or non-symmetric two-point loading were applied monotonically to the test specimens until they approached their inelastic out-of-plane strength. At the load-carrying limit stages of the test specimens, their lateral displacements began to increase rapidly, and the loading was terminated soon afterward.

The lateral displacements of the test specimens at the quarter points from both ends and at the crown were measured by the displacement transducers, with displacement transducers being placed respectively at the top and bottom flanges as shown in Fig. 8. The average value of the two measured lateral displacements was used for standing for the lateral displacement of the centroid of the cross-section at the measuring station, while the rotation of the corresponding cross-section was obtained through dividing the difference of the two displacements by the height $h$ of the cross-section.

4. Experimental results

4.1. Test Specimens No. 1 and No. 2

The specimens No. 1 and No. 2 were loaded symmetrically at the span quarter points denoted by $o_1$ and $o_3$ and at the crown denoted by $o_2$, as shown in Fig. 9. The three loads were applied by a hydraulic system simultaneously to ensure that the magnitudes of all concentrated loads were equal to each other. The profiles of the measured initial out-of-plane geometric imperfections for specimens No. 1 and No. 2 are shown in Fig. 7a and b with the maximum magnitude $S/448$ for the specimen No. 1 and $S/176$ for the specimen No. 2.

During the loading, the variations of the load $F$ with the lateral displacement $u$ and the twist angle $\theta$ of the cross-section along the arch length are shown in Fig. 9a and b for the specimen No. 1, and in Fig. 9c and d for the specimen No. 2. It is noted that the load $F$ acted directly on the top flange of arch becomes inclined due to the lateral-torsional buckling deformations of the test steel arch specimens, and this may affect the exact magnitude of the load $F$. The effects were investigated and the results show that the change of the magnitude of the load $F$ is less than 1% when the out-of-plane inelastic buckling occurs. Hence, the very small errors can be ignored if the magnitude $F = Q/2$ is used.

It can be seen from Fig. 9a–d that the out-of-plane inelastic buckling load of the specimen No. 2, which has larger initial lateral imperfections, is only 70% of that of the specimen No. 1. The magnitudes of the lateral displacements (Fig. 9c) and twist angles (Fig. 9d) of the specimen No. 2 (with larger initial out-of-plane geometric imperfections) are much larger than those of the specimen No. 1 (Fig. 9a and b) under the same load. These indicate that the initial out-of-plane geometric imperfections affect the out-of-plane inelastic buckling strength of fixed steel arches significantly. In addition, the topology of the initial out-of-plane geometric imperfections also influences significantly the out-of-plane inelastic buckling deformation mode. This conclusion was confirmed by the test specimen No. 1 in Fig. 10, from which it can be seen that...
the out-of-plane inelastic buckling of specimens No. 1 has the one
half-wave deformation mode, although its elastic counterpart has
the two half-wave deformation mode (Fig. 2). This demonstrates
that the out-of-plane inelastic buckling failure modes of fixed
arches under symmetric loading are related not only to their elastic
buckling mode, but also to the magnitude and distributions of their
initial out-of-plane geometric imperfections.

4.2. Test specimens No. 3 and No. 4

Specimens No. 3 and No. 4 were loaded in a non-symmetric
fashion, with two equal loads \( F \) being applied at the quarter points
from two ends of arch respectively denoted by \( o_3 \) and at the crown
of the arch denoted by \( o_2 \) from the hydraulic jack system simulta-
neously. The profiles of the measured initial lateral imperfections
are shown in Fig. 7c and d with the maximum magnitude of \( S/2625 \) for the specimen No. 3 and \( S/305 \) for the specimen No. 4.

The variations of the lateral displacement \( u \) and twist angle \( \theta \) of the cross-section along the arch axis with the load \( F \) are shown in Fig. 11a and b for the specimen No. 3 and in Fig. 11c and d for the specimen No. 4. Again, it can be seen that the out-of-plane inelastic buckling load for the specimen No. 4, which has larger initial lateral imperfections, is 91% of that for the specimen No. 3. To investigate the factors influencing the out-of-plane inelastic buckling deformation modes of steel arches under non-symmetric loads, a typical out-of-plane inelastic buckling failure mode is shown in Fig. 12 for the specimen No. 3. It can be seen from Fig. 12 that the out-of-plane inelastic buckling mode of the specimen No. 3 has one half-wave deformation, which is the same as its elastic buckling counterpart, although the initial out-of-plane geometric imperfections of the specimen No. 3 has two half-wave profile (Fig. 7). This indicates that while the initial out-of-plane geometric imperfections influence the out-of-plane inelastic buckling strength of steel arches under non-symmetric loading, they do not appear to affect their corresponding deformation modes.

By comparing the results of the specimens No. 3 and No. 4 with those of the specimens No. 1 and No. 2, it can be found that the out-of-plane inelastic buckling strengths of steel arches under non-symmetric loading are smaller than those of steel arches under symmetric loading. For example, the out-of-plane strength of the specimen No. 4 subjected to non-symmetric loading is only 67% of that of the specimen No. 1 subjected to symmetric loading.
It is known under load control when the ultimate load is reached, the unload phase cannot be traced. In order to trace the post-buckling equilibrium path, displacement control method should be used in the test. Because the test reported in this paper aims to the ultimate strength of fixed steel arches not to the post-buckling behavior and because of the limitation of the servo loading machine, the load control method was used in the test and so the post-buckling path was not traced.

5. Finite element simulation model

Because the experimental data are limited, it is difficult to use them exclusively for developing analytical formulae for the out-of-plane inelastic buckling strength design of fixed steel arches. To generate more useful data, FE methods can be used. For this, the commercial software package ANSYS [28] was adopted herein to establish a FE simulation model of the test process. The FE model of the arch, as shown in Fig. 13, is modeled by forty-two-node-beam elements named BEAM188, with a requirement that the central angle corresponding to one beam element is less than the five degree according to [27]. Because the point loads applied on the top flanges of the arch are transferred from the loading frames as shown in Fig. 6, the loading frame is simplified mainly as one vertical tie as shown in Fig. 13, and is modeled by two-node-link element LINK8 in analytic models. The bottom rigid lever acted by a concentrated load at the mid-span of the lever is modeled by BEAM188. It is obvious that the bottom rigid lever is simply supported on the ground at one end and connects to the loading frame.
modeled by LINK8 at the other end. All structural components of the loading frame are assumed to be infinitely rigid to avoid their deformation. When the out-of-plane deformation of arch occurs, the loading frame begins to lean, and accordingly the direction of the load applied on the top flange of the arch also leans with it, therefore the change of loading direction is automatically reflected in the FE analytic model.

Papangelis and Trahair [23] reported that rolling residual stresses have little effect on the out-of-plane buckling of arches. In addition, as discussed previously, fabrication of the arches involved no curving of the whole cross-section. Hence, no rolling residual stresses are included in the FE representation. However, the commonly-used residual stresses for welded cross-sections shown in Fig. 14 are adopted in the FE model. Because the initial out-of-plane geometric imperfections have a significant influence on the out-of-plane inelastic buckling strength of fixed steel arches, the initial out-of-plane geometric imperfections measured are included in the FE model.

For verification, the FE models are used to perform the non-linear inelastic analysis for the tested arch specimens. The FE results obtained for the load-lateral displacement curves and the load-twist angle curves are compared with the test results in Figs. 9 and 11. It can be seen that the curves of FE results are very close to the test results. It can also be seen that FE results confirmed that the out-of-plane inelastic buckling mode of fixed arches under symmetrical loading is dependent both on the out-of-plane elastic buckling mode and on the initial out-of-plane geometric imperfections, while the out-of-plane inelastic buckling shapes of fixed arches under non-symmetric loading are similar to their out-of-plane elastic buckling shapes. The out-of-plane inelastic buckling strengths obtained from the FE model are also compared with the test results in Table 4. It can be seen from Table 4 that the FE results agree very well with the test results, with relative errors of less than 8%. All of these confirm that the FE model predicts the out-of-plane buckling behavior and strength of arch specimens fairly well. Therefore, the FE model can be used to simulate the tests for the out-of-plane inelastic buckling strength of fixed steel arches for the development of formulae for the strength design of fixed steel arches against their out-of-plane inelastic buckling failure. Because the elastic-plastic nonlinear analysis in ANSYS is displacement-controlled, it can also be seen from the FE results when the ultimate out-of-plane elastic-plastic buckling strength is reached, due to effects of plasticity, the arches exhibit softening behavior, i.e., as the displacements continue to increase, the load decreases. This can explain why in the load control test, when the ultimate strength is reached, the test terminates.

It is noted that when the FE model is used to generate supplementary data for the development of design formulas against the out-of-plane inelastic buckling failure of fixed steel arches, the initial out-of-plane geometric imperfections that are consistent with the design codes for steel structures should be used. Hence, the initial out-of-plane geometric imperfections implied by the Chinese code for the design of steel structures GB 50017-2003 [29] and by the counterpart Australian standard AS4100 [30] were also modeled in the FE model as options. Both codes imply the same initial imperfections, which were assumed in the FE model as a symmetric wave given by [11,12].

$$u_0 = \frac{u_{0c}}{2} \left(1 - \cos \frac{2\pi \theta}{C_1}\right),$$

where the magnitude of the initial central lateral imperfection $u_{0c}$ is taken as
which is implied by [29,30], and \( S \) is the developed length of the arch. The initial out-of-plane geometric imperfections can also be assigned using the first out-of-plane elastic buckling mode shape with the central lateral imperfection being given by Eq. (2). The out-of-plane elastic buckling mode shape can be obtained from ANSYS [28] by the linear eigenvalue buckling analysis.

### 5.1. Proposed design equation

The transverse loads produce combined axial compressive and bending actions in fixed steel arches. The different combination of axial compressive and bending actions will lead to the different out-of-plane inelastic buckling strength. In addition, a number of other factors such as the slenderness of the arch, initial out-of-plane geometric imperfections, and residual stresses also influence the out-of-plane inelastic buckling strength of fixed steel arches. It is difficult to establish an accurate design equation for the strength design of fixed steel arches against their out-of-plane inelastic buckling failure. Hence, Pi and Bradford [19] proposed a lower bound interaction equation for the design. The test results and the FE simulating results obtained from the FE model for the out-of-plane inelastic buckling strength of fixed steel arches with different spans and slenderness are compared with the design equation proposed by Pi and Bradford [19] to investigate whether the equation provides a good prediction for lower bound for the out-of-plane inelastic buckling strength of fixed steel arches or it needs to be modified. The proposed interaction equation is expressed as

\[
\frac{N_m}{\alpha_{mN_{ac}}} + \frac{\delta_{bp} M_m}{\alpha_{mN_{ab}}} \leq \phi
\]

in which \( N_{ac} \) and \( M_{ab} \) are the design strength of a fixed steel arch in uniform compression and in uniform bending respectively, and are given in Appendices A and B, \( N_m \) and \( M_m \) are the nominal maximum axial compression and maximum moment calculated by a first-order in-plane elastic analysis for the arch, the moment amplification factor \( \delta_{bp} = 1/(1 - N_m/N_{sef}) \geq 1.0 \), where \( N_{sef} \) is the out-of-plane elastic buckling loads of fixed arches under nominal uniform axial compression [14,19], and when \( \delta_{bp} > 1.4 \), then a second-order in-plane elastic analysis should be carried out to obtain \( M_{sef} \) and \( \alpha_{mN_{ac}} \) and \( \alpha_{mN_{ab}} \) are the modified factors which account for the variations of axial compression and moment along an arch, and \( \phi \) is a safety factor and \( \phi = 0.9 \) is recommended for the design. The test and FE results show that the values of the modified factors \( \alpha_{mN_{ac}} \) and \( \alpha_{mN_{ab}} \) can empirically be derived \( \alpha_{mN_{ac}} = 2.2 \), \( \alpha_{mN_{ab}} = 1.2 \) for symmetric loads and \( \alpha_{mN_{ac}} = 1.6 \) for non-symmetric loads.

It is noted that the design strength of a fixed steel arch in uniform compression and in uniform bending \( N_{sef} \) and \( M_{sef} \) are related to the squash load and full plastic moment of the cross section, the slenderness of the arches, and the out-of-plane elastic buckling load of fixed arches under uniform compression and the out-of-plane elastic buckling moment of fixed arches under uniform bending. The details are shown in the Appendices A and B. The out-of-plane elastic buckling load and moment can be calculated according to Pi and Bradford [14], Bradford and Pi [15], Dou et al. [17].

The test and FE results are compared with the proposed interaction equation given by Eq. (3) in Fig. 15, where \( \phi = 1.0 \) is used. It can be seen that the proposed interaction equation provides satisfactory lower bound predictions for the out-of-plane inelastic buckling strength of fixed steel arches that are subjected to combined bending and axial compressive actions.

### 6. Conclusions

An experimental study on the out-of-plane inelastic buckling behavior of fixed circular steel I-section arches under symmetric three-point loading and non-symmetric two-point loading was presented in this paper. A FE model that can simulate the testing was developed and validated by the test results. It was found that the initial out-of-plane geometric imperfections have significant effects on the out-of-plane inelastic buckling strength of fixed steel arches. The out-of-plane inelastic buckling failure mode of a fixed steel arch under symmetric loading is dependent both on its out-of-plane elastic buckling mode and on the magnitude and distribution of its initial out-of-plane geometric imperfections. It was found that the fixed arches under non-symmetrical two-point loading have lower out-of-plane buckling strengths than those of the fixed arches under symmetric three-point loading. The FE model were used to perform numerical simulation to generate supplementary data for the out-of-plane inelastic buckling loads and these data together with the test results were used to develop a lower bound equation for the out-of-plane inelastic buckling strength design of fixed steel arches.

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Appendix A. Out-of-plane strength for uniform compressions

The out-of-plane strength $N_{ac}$ of a fixed steel arch that is subjected to uniform compression is given by [19]

$$N_{ac} = \varphi_{ac} N_Y$$  \hspace{1cm} (a1)

In which $N_Y$ is the squash load of the cross section, and can be calculated by $N_Y = Af_y$, where $A$ is the cross-sectional area of arch, $f_y$ is the yield stress of steel; and $\varphi_{ac}$ is the out-of-plane slenderness reduction factor of an arch in uniform compression given by

$$\varphi_{ac} = \frac{\bar{\varphi}_{ac}}{1 - \sqrt{1 - \left(\frac{90}{\bar{\varphi}_{ac} \bar{\lambda}_a} \right)^2}}$$  \hspace{1cm} (a2)

with

$$\bar{\varphi}_{ac} = \frac{(\lambda_a/90)^2 + \eta_a + 1}{2(\lambda_a/90)}$$  \hspace{1cm} (a3)

and

$$\eta_a = 0.00326(\lambda_a - 13.5) \geq 0$$  \hspace{1cm} (a4)

in which the modified slenderness ratio for out-of-plane buckling $\lambda_a$ is defined as

$$\lambda_a = \frac{L_c}{f_y \sqrt{250}}$$  \hspace{1cm} (a5)

with

$$L_c = \frac{0.55}{\sqrt{N_{def}/N_{cr}}}$$  \hspace{1cm} (a6)

where $f_y$ is the yield stress of the steel in unit of MPa, $r_y$ is the radius of gyration of the cross-section about its minor axis, $N_{cr}$ is the flexural buckling load of a fixed column with the same length of the arch under axial compression; $N_{def}$ is the out-of-plane elastic buckling of fixed arches under uniform axial compression, and can be calculated by the FE methods or the by the formulas in [14].

Appendix B. Out-of-plane strength for uniform bending

The out-of-plane strength $M_{def}$ of a fixed steel arch under uniform bending is also an important reference strength for the strength design equation of steel arches under combined compression and bending and proposed as [19]

$$M_{def} = \varphi_{db} M_p \leq M_p$$  \hspace{1cm} (a7)

where $M_p$ is the full plastic moment of the cross section; and $\varphi_{db}$ is the slenderness reduction factor of an arch in uniform bending and given by

$$\varphi_{db} = 0.6 \left( \sqrt[3]{3 + \lambda_{am}^4} - \lambda_{am}^2 \right)$$  \hspace{1cm} (a8)

where $\lambda_{am}$ is the modified slenderness of the arch in uniform bending and given by

$$\lambda_{am} = \sqrt{\frac{M_p}{M_{def}}}$$  \hspace{1cm} (a9)

with $M_{def}$ is the out-of-plane elastic buckling moment of fixed arches under uniform bending, and can be calculated by the FE methods or by the formulas in [15].

References