This paper discusses the $H_\infty$ filtering problem of time-delayed singular Markovian jump systems (SMJSs) with time-varying transition rate matrix (TRM). In this paper, the underlying TRM is studied by the quantized method, whose difficulties used in system analysis and synthesis with infinite precision are overcome. Based on the proposed quantization principle, the time-varying TRM is firstly quantized into a series of finite TRMs with norm bounded uncertainties. Then, new criteria depending on time delay and quantization density simultaneously are developed such that the corresponding system is stochastic admissible with an $H_\infty$ performance. Several sufficient conditions for the existence of the desired filter are given in the form of linear matrix inequalities (LMIs). Finally, numerical examples are used to demonstrate the correctness and superiorities of the proposed methods.

1. Introduction

In the past decades, the filtering problem has been studied as a hot topic. It is well-known that the filtering technique is very useful to estimate unavailable states of a given system through noisy measurement, which plays important roles in the areas of control and signal processing. When the system and the statistics of noise disturbances are known exactly, Kalman filtering approach [1–4] is effective. But sometimes, such assumptions are difficult to be satisfied. In order to remove these limitations, another filtering technique referred to $H_\infty$ filtering has been proposed. Compared with Kalman filtering technique, $H_\infty$ filtering has powerful signal estimation and good robustness performance. Such features have motivated the study of $H_\infty$ filtering problem for variant systems, see [5–10] and the references therein.

Markovian jump system (MJS) such as [11,12] is usually modeled by a class of subsystems driven by a Markov chain taking values in a finite set. Two kinds of mechanisms are simultaneously contained in such a system, which are named as time-evolving and event-driven mechanisms respectively. The first one is continuous in time and related to the state vector; the latter one is referred to be operation mode or system mode and driven by a Markov process. Many practical systems experiencing system parameters or structures changed abruptly, such as aircraft control, manufacturing system, power systems, and solar receiver system, are very appropriately described by MJSs. Up to now, the $H_\infty$ filtering problems of various kinds of MJSs have emerged, see, e.g., [13–18]. Especially, singular Markovian jump systems (SMJSs) [19–22] are usually used to describe singular systems having abrupt changes. Due to singular systems having additional impulsive and non-dynamic modes, the analysis and synthesis of such systems are usually more complicated than normal systems. In singular systems, the regularity, impulse elimination
continuous signal into piecewise continuous signals. Recent
seen that the utility of the quantizer is to convert the
time-varying TRM appropriately becomes the first problem
in system analysis and synthesis. Thus, how to deal with the
time-varying, it is impossible to use it with infinite precision
another general case of time-varying TRM. Due to TRM
many problems of MJSs with general TRMs referred above
SMJSs were considered in [26,30]. When an TRM is partially
practical applications. Instead, the corresponding TRM may
TRM should be available exactly. This is not true in many
r e s u l t so f M J S s i n c l u d i n g S M J S s h a v e a n a s s u m t i o n t h a t

be some quantization constraints on the proposed quantizer.

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(3) stochastically admissible, if it is regular, impulse-free and stochastically stable.

**Definition 2.** Time-delayed SMJS (4) with time-varying TRM (2) is said to be stochastic admissible with an $H_{\infty}$ performance, if it is stochastic admissible, and the filtering error $e(t)$ under zero initial condition and any nonzero $\omega(t) \in L_2[0, \infty)$, satisfies

$$E \left( \int_0^\infty \xi^T(t) \xi(t) dt \right) \leq \frac{1}{\gamma^2} \int_0^\infty \omega^T(t) \omega(t) dt$$

In order to achieve the goal of this paper, the time-varying TRM (2) will be handled by a quantized method first. The corresponding quantizer is selected to be logarithmic static and time-invariant, whose quantized level set is given by

$$U = \{ \pm \mu_i ; \mu_i = \rho \mu_0, i = 0, \pm 1, \pm 2, \pm 3, \ldots \} \cup \{0\}$$

where $\mu_0 > 0$ is a scaling parameter and $\rho \in [0, 1]$. The logarithmic quantizer is defined as

$$q(\alpha) = \begin{cases} \mu_i & \text{if } \frac{1}{1+\sigma} \mu_i < \alpha < \frac{1}{1-\sigma} \mu_i, \quad \alpha > 0 \\ 0 & \text{if } \alpha = 0 \\ -q(-\alpha) & \text{if } \alpha < 0 \end{cases}$$

where

$$\sigma = \frac{1-\rho}{1+\rho}$$

Then, the density of quantizer $q(\cdot)$ is defined as

$$\rho_q = \limsup_{\varepsilon \to 0} \frac{N(\varepsilon)}{-\ln \varepsilon}$$

where the number of quantization levels in the interval $[\varepsilon, 1/\varepsilon]$ is denoted as $N(\varepsilon)$. Then, the finite quantization density grows logarithmically as the interval $[\varepsilon, 1/\varepsilon]$ increases. Moreover, it is easy to testify that $\rho_q = 2/\ln(1/\rho)$. That means that the smaller the $\rho$, the smaller the $\rho_q$. Based on this, instead of $\rho_q$, we will use $\rho$ as the quantization density in the following section. A logarithmic quantizer $q(\cdot)$ is illustrated in **Fig. 1**.

Based on (6), we use an analogous quantizer to handle time-varying TRM (2). That is

$$q(\pi_i(t)) = \begin{cases} x_i^{(k)} & \text{if } \frac{1}{1+\sigma} \pi_i^{(k)} < \pi_i(t) < \frac{1}{1-\sigma} \pi_i^{(k)}, \quad j \neq i \\ - \sum_{j \neq i} \pi_j^{(k)} & \text{if } j = i \end{cases}$$

where $\pi_i^{(k)}$ denotes the quantized value of $\pi_i(t)$. The quantization level set of $\pi_i(t)$ is

$$\Pi_q = \{x_i^{(k)}; \pi_i^{(k)} = \rho^k \pi_i^{(k-1)}, k = 0, \pm 1, \pm 2, \ldots \} \cup \{0\}$$

The quantization level set of $\Pi(t)$ is

$$\hat{\Pi} = (\Pi_1, \Pi_2, \ldots, \Pi_M)$$

where $\Pi_k = (\pi_i^{(k)})$ and $M = \{1, 2, \ldots, M\}$, and $M$ is an integer.

**Remark 1.** Applying the proposed quantizer (7), time-varying TRM (2) will be transformed into a series of constant TRMs with norm bounded uncertainties. Because of the quantized set of time-varying TRMs finite, it makes the analysis and synthesis of system (1) or (4) with time-varying TRM (2) impossible. However, due to singular derivative matrix, Markov process and quantized error of TRMs existing together, all these factors make the corresponding analysis and synthesis complicated. What method to deal with the quantized error appropriately should be developed.

### 3. Main results

**Theorem 1.** Given logarithmic quantizer (7), scalars $\tau > 0$ and $\gamma > 0$, system (1) with time-varying TRM (2) is stochastically admissible with an $H_{\infty}$ performance, if there are matrices $P_i, Q_i > 0, Z > 0, M_i = M_i^T$ and $T_{ij} > 0$ satisfying

$$E^T P_i = P_i E \geq 0, \quad \forall i \in S$$

$$E^T P_i - E^T P_i - M_i = 0, \quad \forall j \neq i \in S$$

$$\begin{bmatrix} \overline{\mathbf{F}}_{i1}^{11} & \overline{\mathbf{F}}_{i2}^{12} & P_i^T B_i & \overline{\mathbf{A}}_i^T & E^T & \Psi_{i1} \\ * & \overline{\mathbf{F}}_{i2}^{12} & 0 & \overline{\mathbf{A}}_i^T & 0 & 0 \\ * & * & -\gamma^2 I & \tau B_i & 0 & 0 \\ * & * & * & -Z^{-1} & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & \Psi_{i2} \end{bmatrix} < 0, \quad \forall i \in S, \quad \forall k \in M$$

where

$$\overline{\mathbf{F}}_{i1}^{11} = (A_i^T P_i)^* + Q - E^T Z E + \delta \sum_{j \in S} \pi_j^{(k)} E^T P_j$$

$$\overline{\mathbf{F}}_{i2}^{12} = \sum_{j \in S} \{0.25 \delta^2 (\pi_j^{(k)})^2 T_{ij} + \delta \pi_j^{(k)} M_i\}$$

$$\delta = 1 - \delta, \quad \delta = \sigma/(1-\sigma), \quad \overline{\mathbf{F}}_{i2}^{12} = P_i^T A_{ik} + E^T Z E,$$

$$\Psi_{i1} = \begin{bmatrix} M_{i1} & \cdots & M_{i(i-1)} & M_{i(i+1)} & \cdots & M_{iN} \end{bmatrix}$$

$$\Psi_{i2} = \begin{bmatrix} \tau & \cdots & \tau & M_{i1} & \cdots & M_{i(i-1)} & \tau & \cdots & \tau \end{bmatrix}$$

**Fig. 1.** The simulation of logarithmic quantizer $q(\cdot)$. 
Proof. Firstly, we prove the regularity and impulse-free of system (1). It is known that for system (1), there are always two nonsingular matrices $M$ and $N$ such that

$$MEN = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad MA_N = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad M^{-T}P_N = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}$$

where matrix $P_1$ comes from

$$(A_1^T P_1)^* + \sum_{j \in S} \pi_j(t) E^T P_j - E^T Z E < 0$$

(13)

By pre- and post-multiplying (10) with $N^T$ and $N$, we get $P_{12} = 0$. Applying the similar technique to (13), one has

$$\begin{bmatrix} \pi_{ij}^* & 0 \\ \pi_{ij}^T & \pi_{ij}^* \end{bmatrix} < 0$$

(14)

where $\pi_{ij}$ represents the terms not used here. Then, it is known that $A_4$ is nonsingular. Therefore, system (1) is regular and impulse-free. Define $x_i = x(t+s)$, $-\tau \leq s \leq 0$, it is known that the new process $\{(x_i, t), t \geq \tau\}$ is also a Markov process. Now, choose a Lyapunov functional as

$$V(x_i, t, r) = V_1(t, r_i) + V_2(t, r_i) + V_3(t, r_i)$$

(15)

where

$$V_1(x_i, t, r_i) = \chi^T(t) E^T P_i \chi(t)$$

$$V_2(x_i, t, r_i) = \int_{t-\tau}^{t} x^T(\alpha) Q_x(\alpha) d\alpha$$

$$V_3(x_i, t, r_i) = \tau \int_{t-\tau}^{t} \hat{x}^T(t) \hat{x}^T \hat{Z} \hat{x}(\alpha) d\alpha$$

Let $L$ be the weak infinitesimal generator of stochastic process $(x_i, r_i)$, for each $r_i \in \mathbb{S}$, $k \in \mathbb{M}$, we have

$$LV(x_i, t, r_i) + \zeta^T(t) \zeta(t) - \gamma^2 \omega^2 E^2 \omega(t) \leq 2 \chi^T(t) P_i^T E \chi(t)$$

$$+ \chi^T(t) \sum_{j \in S} \pi_j(t) E^T P_j \chi(t) + \chi^T(t) Q_x(\alpha) d\alpha$$

$$- \chi^T(t-\tau) Q_x(t-\tau) + \tau^2 \chi^T(t) \hat{Z} \hat{x}(\alpha) d\alpha + \hat{x}^T(t) \hat{L}_i \hat{L}_i \chi(t)$$

$$- \gamma^2 \omega^2 \hat{E}^2 \omega(t)$$

(16)

Based on the quantized method, one has

$$\sum_{j \in S} \pi_j(t) E^T P_j = \sum_{j \in S} \left[ q(\pi_j(t)) + \pi_j(t) - q(\pi_j(t)) \right] E^T P_j$$

$$= \sum_{j \in S} \left[ \pi_j^{(k)} + e_j(t) \right] E^T P_j$$

(17)

where

$$e_j(t) = \pi_j(t) - q(\pi_j(t))$$

and

$$e_j(t) = - \sum_{j \neq i} \pi_{ij}(t) + \sum_{j \neq i} q(\pi_{ij}(t))$$

(18)

Moreover, $e_j(t)$, $\forall j \neq i \in \mathbb{S}$, is further rewritten as

$$e_j(t) = \Delta_j(t) q(\pi_j(t))$$

(19)

where $\Delta_j(t) \in [-\delta, \delta]$. Based on (18), we have

$$\sum_{j \in S} \pi_j(t) E^T P_j = \sum_{j \neq i} \left[ 1 + \Delta_j(t) \right] \pi_j^{(k)} (E^T P_j - E^T P_i)$$

$$= \sum_{j \in S} \pi_j^{(k)} E^T P_j + \sum_{j \neq i} \left[ \Delta_j(t) + \delta \pi_j^{(k)} \right] E^T P_j - E^T P_i$$

which implies the following inequality for any $T_{ij} > 0$,

$$\sum_{j \in S} \pi_j(t) E^T P_j = \sum_{j \in S} \pi_j^{(k)} E^T P_j + \sum_{j \neq i} \left[ \Delta_j(t) + \delta \pi_j^{(k)} \right] M_{ij}$$

$$+ \sum_{j \neq i} \left[ \Delta_j(t) + \delta \pi_j^{(k)} \right] E^T P_j - E^T P_j$$

$$\leq \delta \sum_{j \in S} \pi_j^{(k)} E^T P_j + \sum_{j \neq i} \left[ 0.25 \delta^2 \pi_j^{(k)} \right] T_{ij}$$

$$+ \delta \pi_j^{(k)} M_{ij} + M_{ij} T_{ij}^{-1} M_{ij} + \sum_{j \neq i} \left[ \Delta_j(t) + \delta \pi_j^{(k)} \right] E^T P_j - E^T P_j - M_{ij}$$

(20)

Substituting (20) into (16) and by Jensen's inequality and condition (12), it is obtained that

$$\sum_{j \in S} \pi_j(t) E^T P_j - E^T P_j - M_{ij}$$

(21)

where

$$\pi_j(t) = \begin{bmatrix} x(t) \\ x(t-\tau) \\ \omega(t) \end{bmatrix}, \quad \Phi_i = \begin{bmatrix} \Phi_{i1} & \Phi_{i2} & \Phi_{i3} \\ \Phi_{i1} & \Phi_{i2} & \Phi_{i3} \end{bmatrix}$$

$$\Phi_{i1} = \left( A_i^T P_i \right)^* + \tau^2 A_i^T Z A_i + Q - \tau^2 E^T Z E + L_i^T L_i + \delta \sum_{j \in S} \pi_j^{(k)} E^T P_j$$

$$+ \sum_{j \neq i} \left[ 0.25 \delta^2 \pi_j^{(k)} \right] T_{ij} + \delta \pi_j^{(k)} M_{ij} + M_{ij} T_{ij}^{-1} M_{ij}$$

$$\Phi_{i2} = P_i^T A_i + \tau^2 A_i^T Z A_i - E^T Z E$$

$$\Phi_{i3} = \tau^2 A_i^T Z A_i$$

It is obvious that $\Phi_i < 0$ is equivalent to (11), which implies (21). Next, by the methods in [28–30], it is obtained that system (1) is stochastically admissible with an $H_{\infty}$ performance. Finally, we prove condition (13). Moreover, based on $\Phi_{i1} < 0$ and taking into account (20), one has

$$\Phi_{i2} = P_i^T A_i + \tau^2 A_i^T Z A_i - E^T Z E$$

$$\Phi_{i3} = \tau^2 A_i^T Z A_i$$

(22)

which implies (13). This completes the proof. □

Remark 2. As we know, a great deal of results on various kinds of MJSs have emerged, such as [1,11,13,16,17,23,24,27,28]. By investigating such existing references, it is obtained that they are effective to MJSs with constant TRM is constant. Unfortunately, they cannot be applied to problem considered here. Based on proposed quantizer (7), sufficient condition for time-delayed SMJS (1) with time-varying TRM (2) is developed successfully. Though the singular derivative matrix, Markov property and quantized error of TRMs are contained simultaneously, they are handled in Theorem 1 appropriately. Because of the quantization density included, it is said that it will play important roles in system analysis and synthesis.

Theorem 2. Given logarithmic quantizer (7), scalars $\tau > 0$ and $\gamma > 0$, there exists a filter (3) such that system (4) with
time-varying TRM (2) is stochastically admissible with an $H_\infty$ performance, if there are matrices $P_1 > 0$, $P_2$, $P_3 > 0$, $Q_1 > 0$, $Q_2$, $Q_3 > 0$, $Z_1 > 0$, $Z_2$, $Z_3 > 0$, $T_{i}^0 > 0$, $T_{i}^{j} > 0$, $T_{i}^{j+1} > 0$, $G_{i1}$, $G_{i2}$, $X_i$, $J_{i1}$, $J_{i2}$, $M_{i1}$, $M_{i2}$, $M_{i3}$, $S_{i1}$, $S_{i2}$, $S_{i3}$, $S_{i4}$, $\bar{A}_{fi}$, $\bar{B}_{fi}$ and $T_{fi}$, satisfying

$$
\begin{bmatrix}
\Omega_{i1} & \Omega_{i2} & \Omega_{i3} & \Omega_{i4} & \Omega_{i5} & \tau_i \\
* & \Omega_{i6} & \Omega_{i7} & \Omega_{i8} & \Omega_{i9} & 0 \\
* & * & \Omega_{i10} & 0 & 0 & 0 \\
* & * & * & \Omega_{i11} & 0 & 0 \\
* & * & * & * & \Omega_{i12} & 0 \\
* & * & * & * & * & \Omega_{i13}
\end{bmatrix} < 0, \quad \forall i \in \mathbb{S}, \quad \forall k \in \mathbb{M}
$$

(23)

$$
\begin{bmatrix}
E^TP_{i1}E - E^TP_{i1}E - M_{i1}^T & E^TP_{i2}E - E^TP_{i2}E - M_{i2}^T & \cdots & E^TP_{i6}E - E^TP_{i6}E - M_{i6}^T \\
\cdots & \cdots & \cdots & \cdots \\
\end{bmatrix} \leq 0, \quad \forall j \neq i \in \mathbb{S}
$$

(24)

where

$$
\begin{align*}
\Omega_{i1} &= \begin{bmatrix} \Omega_{i1}^1 & \Omega_{i1}^2 \\ \Omega_{i1}^3 & \Omega_{i1}^4 \end{bmatrix}, & \Omega_{i2} &= \begin{bmatrix} \Omega_{i2}^1 & \Omega_{i2}^2 \\ \Omega_{i2}^3 & \Omega_{i2}^4 \end{bmatrix}, & \Omega_{i3} &= \begin{bmatrix} \Omega_{i3}^1 & \Omega_{i3}^2 \\ \Omega_{i3}^3 & \Omega_{i3}^4 \end{bmatrix} \\
\Omega_{i4} &= \begin{bmatrix} \Omega_{i4}^1 \\ \Omega_{i4}^2 \end{bmatrix}, & \Omega_{i5} &= \begin{bmatrix} \Omega_{i5}^1 & \Omega_{i5}^2 \\ \Omega_{i5}^3 & \Omega_{i5}^4 \end{bmatrix}, & \Omega_{i6} &= \begin{bmatrix} \Omega_{i6}^1 & \Omega_{i6}^2 \\ \Omega_{i6}^3 & \Omega_{i6}^4 \end{bmatrix} \\
\Omega_{i7} &= \begin{bmatrix} \Omega_{i7}^1 \\ \Omega_{i7}^2 \end{bmatrix}, & \Omega_{i8} &= \begin{bmatrix} \Omega_{i8}^1 & \Omega_{i8}^2 \\ \Omega_{i8}^3 & \Omega_{i8}^4 \end{bmatrix}, & \Omega_{i9} &= \begin{bmatrix} \Omega_{i9}^1 & \Omega_{i9}^2 \\ \Omega_{i9}^3 & \Omega_{i9}^4 \end{bmatrix} \\
\Omega_{i10} &= \begin{bmatrix} \Omega_{i10}^1 & \Omega_{i10}^2 \\ 0 & 0 \end{bmatrix}, & \Omega_{i11} &= \begin{bmatrix} \Omega_{i11}^1 & \Omega_{i11}^2 \\ \Omega_{i11}^3 & \Omega_{i11}^4 \end{bmatrix} \\
\Omega_{i12} &= \begin{bmatrix} \Omega_{i12}^1 & \Omega_{i12}^2 \\ \Omega_{i12}^3 & \Omega_{i12}^4 \end{bmatrix}, & \Theta_{i1} &= \begin{bmatrix} \Theta_{i1}^1 \\ \Theta_{i1}^2 \end{bmatrix}
\end{align*}
$$

where $U \in \mathbb{R}^{(n-r) \times n}$ satisfies $UE = 0$. Then, the parameters of filter (3) are given by

$$
A_{fi} = X_i^{-1} \bar{A}_{fi}, \quad B_{fi} = X_i^{-1} \bar{B}_{fi}, \quad L_{fi} = L_{fi}
$$

(25)

**Proof.** Based on Theorem 1, it is known that the filtering error system (4) is stochastically admissible with an $H_\infty$ performance, if there are matrices $P_i$, $Q_i > 0$, $Z_i > 0$, $M_{ij} = M_{ij}^T$ and $T_{ij} > 0$, such that (10) and the following LMI's, $\forall i \in \mathbb{S}$, $\forall k \in \mathbb{M}$, hold

$$
\begin{bmatrix}
\Omega_{i1}^{11} & \Omega_{i1}^{12} & P_{i1}^T & \bar{A}_{fi}^T & \bar{B}_{fi}^T & \bar{E} & \bar{F}_i & \bar{G}_{i1}^T & \Psi_{i1}
\end{bmatrix}
\begin{bmatrix}
P_{i1} & \bar{A}_{fi}^T & \bar{B}_{fi}^T & \bar{E} & \bar{F}_i & \bar{G}_{i1}^T & \Psi_{i1}
\end{bmatrix}^T < 0,
$$

(26)

where

$$
\begin{align*}
\Omega_{i1}^{11} &= (\bar{A}_{fi})^T P_i + Q_i - \bar{E}^T Z_i E + \bar{G}_{i1}^T E^T P_i J_i \\
+ & \begin{bmatrix} \Omega_{i1}^{12} & P_{i1}^T & \bar{A}_{fi}^T & \bar{B}_{fi}^T & \bar{E} & \bar{F}_i & \bar{G}_{i1}^T & \Psi_{i1} \end{bmatrix} \begin{bmatrix} \Omega_{i1}^{12} & P_{i1}^T & \bar{A}_{fi}^T & \bar{B}_{fi}^T & \bar{E} & \bar{F}_i & \bar{G}_{i1}^T & \Psi_{i1} \end{bmatrix}^T < 0,
\end{align*}
$$

Let $P_i = P_i E + \bar{U}^T \bar{S}_i^T$ with $P_i > 0$, and define
we conclude that $\mathbf{UF} = 0$ and $\mathbf{E}^T P_1 = P_1^T \mathbf{E} \succeq 0$. Moreover, it is obtained that (26) is guaranteed by the following LMIs:

$$
\begin{bmatrix}
\mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{A}_{13} & \mathcal{A}_{14} & \mathcal{T} & \mathcal{T}_1^T & \Psi_{\Omega 1}
\end{bmatrix} < 0, \quad \forall \mathbf{i} \in \mathbb{S}, \quad \forall \mathbf{k} \in \mathbb{M}
$$

where

$$
\mathcal{A}_{11} = (\mathcal{A}_i^T G_i)^* + \mathcal{Q} - \mathcal{E}^T \mathcal{Z} + \mathcal{G}_i \sum_{j=5}^{15} \mathcal{N}_{i,j} \mathcal{P}_j^T + \mathcal{E}^T \mathcal{P}_j
$$

$$
+ \mathcal{Q} - \mathcal{E}^T \mathcal{Z} + \mathcal{G}_i \sum_{j=5}^{15} \mathcal{N}_{i,j} \mathcal{P}_j^T + \mathcal{E}^T \mathcal{P}_j
$$

by pre- and post-multiplying its both sides with

$$
\begin{bmatrix}
1 & \mathcal{A}_i^T & 0 & 0 & 0 & 0
\end{bmatrix}
$$

and its transpose, respectively. If matrices $G_i$ and $J_i$ are nonsingular, (27) can be written as

$$
\begin{bmatrix}
\mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{A}_{13} & \mathcal{A}_{14} & \mathcal{T} & \mathcal{T}_1^T & \Psi_{\Omega 1}
\end{bmatrix} < 0, \quad \forall \mathbf{i} \in \mathbb{S}, \quad \forall \mathbf{k} \in \mathbb{M}
$$

As for $-G_i^T Z^{-1} G_i$, we have

$$
-G_i^T Z^{-1} G_i \leq -(G_i)^* + Z
$$

When matrices $G_i$, $J_i$, $Q$, $Z$, $M_{ij}$ and $T_{ij}$ are defined as

$$
G_i = \begin{bmatrix} G_{i1} & G_{i2} \\ X_i & X_i \end{bmatrix}, \quad J_i = \begin{bmatrix} J_{i1} & J_{i2} \\ X_i & X_i \end{bmatrix}, \quad Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix}, \quad Z = \begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix}, \quad M_{ij} = \begin{bmatrix} M_{ij}^1 & M_{ij}^2 \\ * & M_{ij}^3 \end{bmatrix}, \quad T_{ij} = \begin{bmatrix} T_{ij}^1 & T_{ij}^2 \\ * & T_{ij}^3 \end{bmatrix}
$$

by (23), it is concluded that both $G_i$ and $J_i$ are nonsingular. Based on (25) and taking into account (29), we get (23) implying (28). On the other hand, based on the representations of matrices $E$, $P_i$ and $M_{ij}$, it is obtained that (24) is equivalent to (12). This completes the proof. \hfill \Box

**Remark 3.** By the LMI approach, sufficient condition for mode-dependent filter (3) has been developed, which could be solved easily. When system mode $R_i$ is not available online, filter (3) will be failed. However, the design method for filter (3) can also be applied to mode-independent case. That is, when matrix $X_i$ in $G_i$ and $J_i$ is selected to be a common matrix $X$, one will have a mode-independent filter. But, the exploited Lyapunov functional is still mode-dependent, which will reduce the conservatism of ones obtained by a common Lyapunov functional for all operation modes.

Particularly, when matrix $E$ is an unit matrix, system (1) will become

$$
\begin{align*}
\dot{x}(t) &= A(t)x(t) + A_d(t)x(t - \tau(t)) + B(t)u(t) \\
\dot{y}(t) &= C(t)x(t) + C_d(t)x(t - \tau(t)) + D(t)v(t) \\
z(t) &= L(t)x(t) \\
x(t) &= \phi(t), \quad \forall t \in [-\tau, 0]
\end{align*}
$$

whose TRM is also time-varying and described by (2). The corresponding filter is

$$
\begin{align*}
\dot{x}_f(t) &= A_f(t)x_f(t) + B_f(t)u(t) \\
z_f(t) &= L_f(t)x_f(t)
\end{align*}
$$

Thus, the error system is

$$
\begin{align*}
\dot{x}(t) &= \mathcal{A}_x x(t) + \mathcal{A}_{dx} x(t - \tau) + \mathcal{B}_d u(t) \\
x(t) &= \mathcal{L}_x x(t)
\end{align*}
$$

Then, we have the following corollary.

**Corollary 1.** Given logarithmic quantizer (7), scalars $\tau > 0$ and $\gamma > 0$, there exists a filter (30) such that system (32) with time-varying TRM (2) is stochastically stable with an $H_\infty$ performance, if there exist matrices $P_{11} > 0$, $P_{12} > 0$, $P_{13} > 0$, $Q_1 > 0$, $Q_2$, $Q_3 > 0$, $Z_1 > 0$, $Z_2$, $Z_3 > 0$, $T_{ij} > 0$, $T_{ij}^1 > 0$, $T_{ij}^2 > 0$, $G_{11}$, $G_{12}$, $X_i$, $J_{i1}$, $J_{i2}$, $M_{ij}$, $M_{ij}^1$, $M_{ij}^2$, $M_{ij}^3$, $A_{di}$, $B_{di}$ and $L_{ji}$, satisfying

$$
\begin{bmatrix}
\Omega_i & \tilde{\Omega}_{ij} & \tilde{\Omega}_{i3} & \Omega_{i4} & \Omega_{i5} & \Omega_{i6} & \Omega_{i7} & \Omega_{i8} & \Theta_{i1}
\end{bmatrix} < 0, \quad \forall \mathbf{i} \in \mathbb{S}, \quad \forall \mathbf{k} \in \mathbb{M}
$$

As for $-G_i^T Z^{-1} G_i$, we have

$$
-G_i^T Z^{-1} G_i \leq -(G_i)^* + Z
$$

When matrices $G_i$, $J_i$, $Q$, $Z$, $M_{ij}$ and $T_{ij}$ are defined as

$$
\begin{bmatrix}
G_{i1} & G_{i2} \\ X_i & X_i \\
J_{i1} & J_{i2} \\ X_i & X_i \\
Q_1 & Q_2 \\
* & Q_3 \\
Z_1 & Z_2 \\
* & Z_3 \\
M_{ij}^1 & M_{ij}^2 \\
* & M_{ij}^3 \\
T_{ij}^1 & T_{ij}^2 \\
* & T_{ij}^3
\end{bmatrix}
$$

where

$$
\begin{align*}
\tilde{\Omega}_{11} &= \begin{bmatrix} \tilde{\Omega}_{11}^1 & \tilde{\Omega}_{11}^2 \\ * & \tilde{\Omega}_{11}^3 \end{bmatrix} \\
\tilde{\Omega}_{12} &= \begin{bmatrix} \tilde{\Omega}_{12}^1 & \tilde{\Omega}_{12}^2 \end{bmatrix} \\
\tilde{\Omega}_{13} &= \begin{bmatrix} \tilde{\Omega}_{13}^1 & \tilde{\Omega}_{13}^2 \end{bmatrix} \\
\tilde{\Omega}_{14} &= \begin{bmatrix} \tilde{\Omega}_{14}^1 & \tilde{\Omega}_{14}^2 \end{bmatrix} \\
\tilde{\Omega}_{15} &= \begin{bmatrix} \tilde{\Omega}_{15}^1 & \tilde{\Omega}_{15}^2 \end{bmatrix} \\
\tilde{\Omega}_{16} &= \begin{bmatrix} \tilde{\Omega}_{16}^1 & \tilde{\Omega}_{16}^2 \end{bmatrix} \\
\tilde{\Omega}_{17} &= \begin{bmatrix} \tilde{\Omega}_{17}^1 & \tilde{\Omega}_{17}^2 \end{bmatrix} \\
\tilde{\Omega}_{18} &= \begin{bmatrix} \tilde{\Omega}_{18}^1 & \tilde{\Omega}_{18}^2 \end{bmatrix}
\end{align*}
$$

$$(33)$$

$$
\begin{bmatrix}
P_{11} - P_{11} - M_{ij}^1 \\
P_{12} - P_{12} - M_{ij}^2 \\
P_{13} - P_{13} - M_{ij}^3
\end{bmatrix} \succeq 0, \quad \forall \mathbf{j} \neq \mathbf{i}, \quad \forall \mathbf{k} \in \mathbb{M}
$$

(34)
\[ \hat{\Omega}_{12} = \begin{bmatrix} \hat{\Omega}^1_{12} & \hat{\Omega}^2_{12} \\ \hat{\Omega}^3_{12} & \hat{\Omega}^4_{12} \end{bmatrix}, \quad \hat{\Omega}_{19} = \begin{bmatrix} \hat{\Omega}^1_{19} & \hat{\Omega}^2_{19} \\ \hat{\Omega}^3_{19} & \hat{\Omega}^4_{19} \end{bmatrix} \]

\[ \hat{\Omega}_{11} = \begin{bmatrix} \hat{\Omega}^1_{11} & \hat{\Omega}^2_{11} \\ \hat{\Omega}^3_{11} & \hat{\Omega}^4_{11} \end{bmatrix} \]

\[ \hat{\Omega}_{11} = (A_i^T G_{i1} + C_i^T B_{i})^* + Q_i - Z_f + \bar{\sigma} \sum_{j \in \mathcal{S}} \pi_{ij}^k P_{j1} + \sum_{j \in \mathcal{S}} [0.25 \delta^2 (\pi_{ij}^k)^2 T_{ij} + \delta \pi_{ij}^k M_{ij}^1] \]

\[ \hat{\Omega}_{11}^2 = A_i^T G_{i2} + C_i^T B_{i} + \bar{\sigma}^T + Q_{i2} - 2Z_f + \bar{\sigma} \sum_{j \in \mathcal{S}} \pi_{ij}^k P_{j2} + \sum_{j \in \mathcal{S}} [0.25 \delta^2 (\pi_{ij}^k)^2 T_{ij} + \delta \pi_{ij}^k M_{ij}^2] \]

\[ \hat{\Omega}_{11}^3 = (\bar{\sigma} P_i + \bar{\sigma} P_{i3} - C_i^T) + \sum_{j \in \mathcal{S}} [0.25 \delta^2 (\pi_{ij}^k)^2 T_{ij} + \delta \pi_{ij}^k M_{ij}^3] \]

Then, the parameters of filter (31) are given by (25).

When \( \tau = 0 \), system (1) will be reduced to

\[
\begin{align*}
E(x(t)) &= A(r_i) x(t) + B(r_i) o(t) \\
y(t) &= C(r_i) x(t) + D(r_i) o(t) \\
z(t) &= L_i x(t) \\
x(t) &= x_0
\end{align*}
\]

The filter is still (3), but the filtering error system becomes

\[ \{ E(x(t)) = \bar{\Delta} x(t) + B_i o(t) x(t) = L_i x(t) \] (36)

**Corollary 2.** Given logarithmic quantizer (7), scalar \( \gamma > 0 \), there exists a filter (3) such that system (36) with time-varying TRM (2) is stochastically admissible with an \( H_\infty \) performance, if there exist matrices \( P_{i1} > 0 \), \( P_{i2} > 0 \), \( T_{ij} > 0 \), \( T_{ij}^2 > 0 \), \( G_{i1} \), \( G_{i2} \), \( X_i \), \( J_{i2} \), \( M_{i1} \), \( M_{i2} \), \( S_{i1} \), \( S_{i2} \), \( S_{33} \), \( S_{34} \), \( A_{i} \), \( B_{i} \) and \( L_i \), satisfying (24) and

\[
\begin{bmatrix}
\hat{\Omega}_{i1} & \hat{\Omega}_{i2} & \hat{\Omega}_{i4} & \hat{\Omega}_{i8} & \hat{\Omega}_{i6} \\
* & \hat{\Omega}_{i6} & \hat{\Omega}_{i8} & \hat{\Omega}_{i2} & \hat{\Omega}_{i4} \\
* & * & -\gamma^2 I & 0 & 0 \\
* & * & * & -I & 0 \\
* & * & * & * & \Theta_{i1}
\end{bmatrix} < 0, \quad \forall i \in \mathcal{S}, \quad \forall k \in \mathcal{M}
\]

where

\[
\begin{bmatrix}
\hat{\Omega}_{i1} & \hat{\Omega}_{i2} & \hat{\Omega}_{i4} & \hat{\Omega}_{i8} & \hat{\Omega}_{i6} \\
* & \hat{\Omega}_{i6} & \hat{\Omega}_{i8} & \hat{\Omega}_{i2} & \hat{\Omega}_{i4} \\
* & * & -\gamma^2 I & 0 & 0 \\
* & * & * & -I & 0 \\
* & * & * & * & \Theta_{i1}
\end{bmatrix} < 0, \quad \forall i \in \mathcal{S}, \quad \forall k \in \mathcal{M}
\]

**Corollary 3.** Given scalars \( \tau > 0 \) and \( \gamma > 0 \), system (4) with TRM (38) is stochastically admissible with an \( H_\infty \) performance, if there exist matrices \( P_{i1} > 0 \), \( P_{i2} > 0 \), \( Q_i > 0 \), \( Z_1 > 0 \), \( Z_2 > 0 \), \( G_{i1} \), \( G_{i2} \), \( X_i \), \( J_{i2} \), \( M_{i1} \), \( M_{i2} \), \( S_{i1} \), \( S_{i2} \), \( S_{33} \), \( S_{34} \), \( A_{i} \), \( B_{i} \), and \( L_i \), satisfying (25) and

\[
Pr(r_{i1} + h|r_i = i) = \begin{cases} 
\pi_{ih} + o(h), & i \neq j \\
1 + \pi_{ih} + o(h), & i = j
\end{cases}
\] (38)

In this case, we will have the following corollaries.

**Corollary 4.** Given scalars \( \tau > 0 \) and \( \gamma > 0 \), there exists a filter (3) such that system (4) with TRM (38) is stochastically admissible with an \( H_\infty \) performance, if there exist matrices \( P_{i1} > 0 \), \( P_{i2} > 0 \), \( Q_i > 0 \), \( Z_1 > 0 \), \( Z_2 > 0 \), \( G_{i1} \), \( G_{i2} \), \( X_i \), \( J_{i2} \), \( M_{i1} \), \( M_{i2} \), \( S_{i1} \), \( S_{i2} \), \( S_{33} \), \( S_{34} \), \( A_{i} \), \( B_{i} \), and \( L_i \), satisfying (25) and

\[
\phi_i^{(11)} = (A_i^T P_{i1})^* + Q - E^T Z E + \sum_{j \in \mathcal{S}} \pi_{ij}^k E^T P_{j}
\]
the following LMI:

\[
\begin{bmatrix}
\hat{\Omega}_1 & \hat{\Omega}_2 & \hat{\Omega}_3 & \hat{\Omega}_4 & \hat{\Omega}_5 \\
* & \hat{\Omega}_6 & \hat{\Omega}_7 & \hat{\Omega}_8 & \hat{\Omega}_9 \\
* & * & \hat{\Omega}_11 & \hat{\Omega}_12 & \hat{\Omega}_13 \\
* & * & * & -\gamma I & \hat{\Omega}_11 \\
* & * & * & * & \hat{\Omega}_12 \\
* & * & * & * & -I
\end{bmatrix}
< 0 \quad \forall i \in \mathbb{S}, \quad \forall k \in \mathbb{M}
\]

(40)

where

\[
\hat{\Omega}_1 = \begin{bmatrix}
\hat{\Omega}_1^1 \\
\hat{\Omega}_1^2 \\
\end{bmatrix}
\]

\[
\begin{align*}
\hat{\Omega}_1^1 &= (A_1^T C_1 + C_1^T B_1)^* + Q_1 - E^T Z_1 E + \sum_{j \in \mathbb{S}} \pi_{ij} E^T P_j E \\
\hat{\Omega}_1^2 &= A_1^T C_1 + C_1^T B_1 + A_1^T + Q_2 - E^T Z_2 E + \sum_{j \in \mathbb{S}} \pi_{ij} E^T P_j E \\
\hat{\Omega}_1^3 &= (\bar{A}_1)^* + Q_3 - E^T Z_3 E + \sum_{j \in \mathbb{S}} \pi_{ij} E^T P_j E
\end{align*}
\]

In this case, the parameters of filter (3) are solved by (25).

4. Numerical examples

In this section, we will present two numerical examples to show the applicability and superiority of the developed theories.

Example 1. Consider a time-delay SMJS of form (1) with two modes, whose parameters are given by

\[
A_1 = \begin{bmatrix}
-0.5 & -1 \\
1 & -1.5
\end{bmatrix}, \quad A_{d1} = \begin{bmatrix}
0.5 & 0 \\
-0.2 & 0.3
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
0.3 \\
-0.1
\end{bmatrix}
\]

\[
C_1 = \begin{bmatrix}
0.1 & 0.5 \\
\end{bmatrix}, \quad C_{d1} = \begin{bmatrix}
-0.5 & -1
\end{bmatrix}, \quad D_1 = 1,
\]

\[
L_1 = \begin{bmatrix}
-0.5 & 0.1
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
-0.7 & 0.6 \\
-1 & -0.9
\end{bmatrix},
\]

\[
A_{d2} = \begin{bmatrix}
0.7 & -0.6 \\
0 & 0.5
\end{bmatrix}, \quad B_2 = \begin{bmatrix}
-0.1 \\
0.2
\end{bmatrix}
\]

\[
C_2 = \begin{bmatrix}
-0.3 & -0.2
\end{bmatrix}, \quad C_{d2} = \begin{bmatrix}
-0.5 & 0.3
\end{bmatrix}, \quad D_2 = 0.3,
\]

\[
L_2 = \begin{bmatrix}
0.2 & -0.3
\end{bmatrix}
\]

Singular matrix E is

\[
E = \begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix}
\]

The time-varying TRM is described as

\[
\Pi(t) = \begin{bmatrix}
\pi_{11}(t) & \pi_{12}(t) \\
\pi_{21}(t) & \pi_{22}(t)
\end{bmatrix}
\]

whose values are demonstrated in Fig. 2.

By the proposed quantizer (7) with quantization density \( \rho = 0.4 \), one could have the quantization of \( \Pi(t) \) as follows:

\[
\Pi_1 = \begin{bmatrix}
-0.6373 & 0.6373 \\
0.3199 & -0.3199
\end{bmatrix}, \quad \Pi_2 = \begin{bmatrix}
-0.6373 & -0.3199 \\
0.3199 & 0.6373
\end{bmatrix}, \quad \Pi_3 = \begin{bmatrix}
-0.2549 & 0.2549 \\
0.3199 & -0.3199
\end{bmatrix}, \quad \Pi_4 = \begin{bmatrix}
-0.2549 & 0.2549 \\
0.3199 & -0.3199
\end{bmatrix}
\]

The quantization effects of \( \pi_{12}(t) \) and \( \pi_{21}(t) \) are given in Figs. 3 and 4 respectively.

Let \( \tau = 0.9 \) and by Theorem 2, we get the parameters of filter as

\[
A_{f1} = \begin{bmatrix}
-0.4547 & -0.2955 \\
0.6063 & -0.6468
\end{bmatrix}, \quad B_{f1} = \begin{bmatrix}
-0.2611 \\
0.2647
\end{bmatrix}
\]

\[
L_{f1} = \begin{bmatrix}
0.4414 & -0.0531
\end{bmatrix}
\]

\[
A_{f2} = \begin{bmatrix}
-1.4883 & 0.2430 \\
0.1432 & -0.5762
\end{bmatrix}, \quad B_{f2} = \begin{bmatrix}
1.9337 \\
-1.4794
\end{bmatrix}
\]

\[
L_{f2} = \begin{bmatrix}
-0.1737 & 0.3106
\end{bmatrix}
\]

where the minimum disturbance index is \( \gamma^* = 0.265 \). Let the initial condition \( \phi(t) = [1 \ -1]^T \), after applying the desired filter, one has the simulation of filtering error \( \xi(t) \) demonstrated in Fig. 5. Then, it is said that the desired filter approximates the original system very well.
Fig. 4. The quantization effect of $x_{21}(t)$.

Fig. 5. The simulation of filtering error $\zeta(t)$.

Fig. 6. The comparison simulation of $\gamma^*$ with given $\tau$.

Table 1: Minimum $\gamma^*$ for different $\tau$.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^*_1$</td>
<td>3.97</td>
<td>4.14</td>
<td>4.26</td>
<td>4.40</td>
<td>4.50</td>
<td>4.65</td>
<td>4.80</td>
<td>4.96</td>
</tr>
<tr>
<td>$\gamma^*_2$</td>
<td>2.65</td>
<td>2.67</td>
<td>2.71</td>
<td>2.74</td>
<td>2.76</td>
<td>2.78</td>
<td>2.78</td>
<td>2.78</td>
</tr>
</tbody>
</table>

When the TRM is constant, without loss of generality, it is given by

$$II = \begin{bmatrix} -1.6 & 1.6 \\ 0.2 & -0.2 \end{bmatrix}$$

Under this condition, Table 1 gives the comparison results of this paper and [28], where $\gamma^*_1$ and $\gamma^*_2$ are obtained by [28] and Corollary 3 respectively. From Table 1, it is seen that for this example, our results are less conservative.

Fig. 6 further demonstrates the comparison of $\gamma^*$ with given different $\tau$, and also shows the superiority of Corollary 3. Moreover, when $\tau = 1.3$, by the filtering method of [28], it is obtained that $\gamma^* = 0.502$. On the other hand, under the same condition, by Corollary 4, we could have $\gamma^* = 0.109$, and the filtering parameters are as follows:

$$A_1 = \begin{bmatrix} -0.6386 & -0.0233 \\ -0.6132 & -1.1264 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.4312 \\ 0.1337 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.4319 & -0.2440 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1.5296 & 0.3673 \\ -0.4241 & -0.3634 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.7415 \\ -0.9389 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} -0.1768 & 0.1677 \end{bmatrix}$$

This comparison on $H_\infty$ filter design also shows that our results are less conservative.

**Example 2.** Consider the following system controlled by a DC motor, which is illustrated in Fig. 7. It is described by

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \gamma \sin x_1(t) + \frac{NK_m}{m} x_3(t) \\ L_0 \dot{x}_3(t) = K_b N x_2(t) - R(r) x_3(t) + u(t) \end{cases}$$

where $x_1(t) = \theta_p(t)$, $x_2(t) = \dot{\theta}_p(t)$, $x_3(t) = I_d(t)$, $u(t)$ is the control input, $K_m$ is the motor torque constant, $K_b$ is the back emf constant, $N$ is the gear ratio, $R(r)$ is defined as

$$R(r) = \begin{cases} R_a & \text{if } r_t = 1 \\ R_b & \text{otherwise } r_t = 2 \end{cases}$$

where $\{r_t, t \geq 0\}$ is a Markov process taking values in a finite set $S = \{1, 2\}$. Let $L_0 = c H$, $g = 9.8 \text{ m/s}^2$, $I = 1 \text{ m}$, $m = 1 \text{ kg}$, $N = 10$, $I = 1 \text{ m}$, $K_m = 0.1 \text{ Nm/A}$, $K_b = 0.1 \text{ V s/rad}$, $R_a = 1 \Omega$, $R_b = 0.5 \Omega$ and $u(t) = -20x_1(t) - 2x_2(t)$, system (41) becomes

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = x_3(t) + 9.8 \sin x_1(t) \\ c \dot{x}_3(t) = -20x_1(t) - 3x_2(t) - R(r) x_3(t) \end{cases}$$

(42)
Letting $e = 0$, one has
\[
\begin{cases}
\dot{x}_1(t) = x_2(t) \\
\dot{x}_2(t) = x_3(t) + 9.8 \sin x_1(t) \\
0 = -20x_1(t) - 3x_2(t) - R(t)x_3(t)
\end{cases}
\]
and its linearized model is
\[
E \dot{x}(t) = A(r(t))x(t)
\]
where
\[
x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]
\[
A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ -20 & -3 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 9.8 & 0 & 1 \\ -20 & -3 & -0.5 \end{bmatrix}
\]
\[
A_{d1} = \begin{bmatrix} 0 & 0.3 & 0 \\ 0.7 & 0 & 0.6 \\ -0.1 & -1 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}
\]
\[
A_{d2} = \begin{bmatrix} 0.1 & 0 & 0.6 \\ 0 & 2 & 0 \\ -0.1 & -1 & -0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0.4 \end{bmatrix}
\]
\[
C_1 = \begin{bmatrix} 0.2 & 0 & 0 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} 0.3 & 0 & 0 \end{bmatrix}
\]
\[
C_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad C_{d2} = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}
\]
\[
L_1 = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}, \quad D_1 = 1
\]
\[
L_2 = \begin{bmatrix} 0.2 & 0 & 0 \end{bmatrix}, \quad D_2 = 0.3
\]

For this example, the TRM is also time-varying, which is described as $\pi_{12}(t) \in [0.5, 1.1]$ and $\pi_{21}(t) \in [0.2, 0.6]$. Based on the proposed quantized method, under $\rho = 0.6$, one has the quantized TRMs as
\[
P_{1} = \begin{bmatrix} -0.6127 & 0.6127 \\ 0.3347 & -0.3347 \end{bmatrix}, \quad P_{2} = \begin{bmatrix} -0.6127 & 0.6127 \\ 0.1874 & -0.1874 \end{bmatrix}
\]
\[
P_{3} = \begin{bmatrix} -0.3431 & 0.3431 \\ 0.3347 & -0.3347 \end{bmatrix}, \quad P_{4} = \begin{bmatrix} -0.3431 & 0.3431 \\ 0.1874 & -0.1874 \end{bmatrix}
\]

Then, by Theorem 2 and letting $\gamma = 0.3$, we have the filtering parameters computed as
\[
A_{f1} = \begin{bmatrix} 1.0002 & 1.0525 & 0.2252 \\ 12.7495 & -2.3904 & 1.4620 \\ -28.6679 & -1.6181 & -2.5351 \end{bmatrix}
\]
\[
A_{f2} = \begin{bmatrix} -0.4351 & 1.2435 & 0.2073 \\ 7.1931 & -5.0925 & 0.7171 \\ -22.5650 & -4.4411 & -2.4705 \end{bmatrix}
\]
\[
B_{f1} = \begin{bmatrix} -0.0180 \\ -0.1288 \\ -0.9444 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} -0.0789 \\ -0.2049 \\ -1.0022 \end{bmatrix}
\]
\[
L_{f1} = \begin{bmatrix} 0.0583 & 0.0164 & -0.0030 \end{bmatrix}, \quad L_{f2} = \begin{bmatrix} -0.0906 & -0.0259 & 0.0033 \end{bmatrix}
\]

Letting the initial condition $x_0 = [1 - 1 0.5]^T$, the response of filtering error $\tau(t)$ is demonstrated in Fig. 8. Based on such simulations, it is seen that the desired filtering method proposed in this paper is effective.

5. Conclusions

In this paper, we have investigated the $H_{\infty}$ filtering for a class of time-delayed SMJSs with time-varying TRMs. A quantized approach has been used to deal with such a time-varying TRM, and the time-varying TRM can be quantized into multiple finite TRMs with norm bounded uncertainties. By exploiting the proposed quantizer, new conditions involving both time delay and quantization density are firstly developed. Then, an $H_{\infty}$ filter is designed such that the filtering error system is stochastic admissible and has an $H_{\infty}$ performance. All the conditions on the design of filter are formulated as LMI forms. Finally, we give two numerical examples to demonstrate the correctness of the proposed theories.
Acknowledgments

The authors would like to thank the associate editor and the reviewers for their very helpful comments and suggestions. This work was supported by the National Natural Science Foundation of China under Grants 61104066, 61203001, 61374043 and 61473140, the China Postdoctoral Science Foundation funded project under Grant 2012M521086, the Program for Liaoing Excellent Talents in University under Grant LJQ2013040, the Natural Science Foundation of Liaoning Province under Grant 2014020106.

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