Revisiting immunological algorithms for numerical optimization

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Abstract

In this paper, we aim to give a tight survey on the topic of the immunological algorithms for numerical optimization. As the rapid interests on bio-inspired algorithms grow in the computational intelligence community, many scientists have developed various models and algorithms for solving different complex engineering problems. Immunological algorithms are such a class of immune inspired ones and have been applied on a number of areas. Specifically, immunological algorithms for numerical optimization have been investigated in many years. Diverse models and theories have been proposed. However, according to the best of our knowledge, there is no such study summarizing the general characteristics of these models. Thus, immunological algorithms which are applied on numerical optimization problems with single-objective and multi-objective are reviewed. The powerful features of the algorithms are extracted and discussed. In addition, some general insights on the research directions are also highlighted.

Keywords: Artificial Immune Algorithm, Computational Intelligence, Optimization, Complex Function

1. Introduction

In recent years, bio-inspired computational intelligence (CI) which imitates the principles of information processing of senior mammals has received a rapid growing interests \cite{1}. The emerged techniques are often referred to as bio-inspired algorithms. The motivation of these algorithms is primarily to extract useful metaphors from natural biological systems, aiming to create effective and robust computational solutions for practical problems in a wide range of domain areas. The more notable developments have been the evolutionary algorithms (EA) \cite{2} motivated by neo-Darwinian theory of evolution, the swarm intelligence (SI) \cite{3} inspired by social behavior of gregarious insects and other animals, the artificial immune system (AIS) \cite{4} imitating biological immune principles, and so on. It has been demonstrated in many applications fields that these bio-inspired algorithms are complementary to many existing theories and technologies, and further showing promising computing features in many aspects \cite{5}.

Among these bio-inspired algorithms, artificial immune algorithms are recently developed research branch and exhibit many characteristics over specific applications such as network security, fault diagnosis, etc. In the literature, there are a number of review articles on artificial immune algorithms \cite{4, 6-20} from theoretical analysis to practical applications perspectives. Hunt et al \cite{6} discussed the learning capacity of AIS for the first time, claiming that AIS is an inherently generalized, self-organizing and explicitly represented system, thus capable of solving pattern recognition tasks. As initial works done on AIS often encountered a problem that the computing architecture of AIS is similar with that of artificial neural networks and evolutionary algorithms, therefore, many studies such as in \cite{7} had been done to distinguish these similarities, in other words, to extract the distinctive properties of AIS. Castro et al. gave a comprehensive summary of the biological immune mechanisms and theories used in AIS, and further proposed a seminal model called clonal selection algorithm (CSA) for learning, optimization and pattern recognition problems \cite{8, 9}. Thereafter, AIS has attracted many researchers around world to devote themselves to develop AIS.

In 2002, the first international conference on AIS was held, and more and more surveys have proposed. For instances, Nasaroui et al. summarized the fuzzy AIS \cite{10}. Aickelin et al. gave some reviews and remarks on AIS when using it to intrusion detection \cite{11}. Garrett discussed the evaluation criterions to judge the effectiveness and robustness of AIS \cite{12}. Stepney et al. proposed a common
meta-framework to unify several domains of AIS and further discuss mathematical techniques for analyzing its state dynamics [13]. Timmis stated in [14] that The limited capacities of AIS, such as the lack of theoretical advances, the adaption of a limited immune inspiration and the limited application of AIS to hard problems, had been a big challenge in AIS community, thus motivating us to use novel and accurate metaphors from biological immune system, to integrate with other computational technologies, to develop theoretical advances, and to solve many other complex practical problems. In [15], Ji et al. gave a short survey on negative selection algorithm and its applications. In [16], Freschi et al. gave some insights into the application of AIS on multi-objective optimization. In [17], Haktanirlar et al. reported an up-to-date review on clonal selection algorithm and its application. Four recent and comprehensive review papers regarding the developing history, immune modeling, theoretical AISs and applied AISs can be found in [4, 18-20].

Although there are many amounts of review articles on AIS in the literature, however, to the best of our knowledge, rare works have been done to summarize the AIS on numerical optimizations. Therefore, in this paper we aim to give a tight survey on the topic of the immunological algorithms for numerical optimization with single-objective and multiple objectives. It is worth emphasizing that our purposed is not to present a comprehensive reference collection, but to extract some common and promising characteristics of immunological mechanisms or strategies. By doing so, it can be expected that there are some guidelines in AIS to follow for afterwards researches.

2. Immunological algorithms

In the last decade, the biological immune mechanisms and theories has drawn significant attentions as an inspiration for designing immunological models and algorithms. Compared with brain nerve together with neuron mechanism, or the neo-Darwinian theory of evolution, the biological immune system exhibits many instinct properties, such as its highly distributed, self-adaptive, self-organizing nature, as well as its diversity, learning, memory, feature extraction functions. These features have enabled AIS to offer rich metaphors for its artificial counterpart.

Generally, there are five typical models of AIS [4], i.e. artificial immune network, clonal selection algorithm, negative selection algorithm, danger theory inspired algorithm, and dendritic cell algorithm. (1) Artificial immune network [21] mainly imitates the idiotypic network theory where the B cells not only stimulate each other but also suppress connected cells to regulate the residue stimulation of B cells in order to maintain a stable immunity. (2) Clonal selection algorithm [22] uses the clonal selection theory proposed by Brunet which interprets the response of lymphocytes in the face of an antigenic stimulus. Once B cells encountered an antigen, they will be stimulated to proliferate and eventually mature into antibody secreting cells. Proliferation of B cells is a mitotic process whereby the cells divide themselves, creating a set of identical clones and the proliferation rate is directly proportional to the affinity level. (3) Negative selection algorithm mimics the selection rule taken place in thymus gland where T cells mainly take effects. The selection rule is actually the maturation progress of T cells, i.e. T cells will be exposed to self-proteins in a binding process. If this binding activates the T cell, then the corresponding T cell will be eliminated, otherwise it is allowed to undergo the lymphatic system to refine the binding process. (4) Danger theory based algorithm is a recently developed model which utilizes the self/non-self discrimination capacities of immune cells. In detail, the immune system will pay more attention on the antigen presenting cells which will be activated by danger signals from injured cells or antigens. (5) Dendritic cell algorithm is motivated by the innate immune response where the dendritic cells perform stimulation function on T cells.

The above immunological algorithms are constructed by using different biological immune mechanisms, including the clonal selection principle, positive/negative selection, diversity, memory acquirement, immunity tolerance, self/non-self discrimination, functions of immune cells, etc. The combination of these mechanisms might form into different flowcharts of the algorithm under the unique framework, and gathering all the effects of each component. Thus, the resultant algorithm can be expected to perform better when the different immune mechanisms are suitably modeled and manipulated. Till now, immunological algorithms have been applied in a wide spectrum of fields, including data mining, networking and computer security, optimization, automation and design, anomaly detection, bioinformatics, text processing, pattern recognition, clustering and classification, etc [4]. In this paper, we mainly give a brief discussion on immunological algorithms when applied on
numerical optimization problems. It should be noticed that the negative selection algorithm, danger theory based algorithm, together with dendritic cell algorithm are mainly capable of solving learning or anomaly detection tasks. Thus, only the variants of immune network or clonal selection based algorithms are discussed in the next section.

3. Applications on numerical optimization

In general, a global numerical optimization problem can be defined as to minimize the objective function $f(X)$, where $X=(x_1, x_2, ..., x_D)$ belongs to the D-dimensional search space. The problem is to find a solution $X^*$ in the search space such that $f(X^*)$ is the global minimum.

3.1. Single-objective optimization

A single-objective optimization problem involves only one objective function which is needed to be minimized or maximized. Many practical industrial or engineering problems are related to single-objective optimization, thus it is significant important to solve the single-objective optimization problems effectively and precisely. As benchmark test functions, a number of unimodal functions and multimodal functions with fixed dimensions have been collected in CEC conference, and various immunological algorithms have been proposed to solve single-objective optimization functions.

Castro et al. proposed an adaption model of immune network which is called opt-aiNet to solve multimodal function optimization [23]. The main features of opt-aiNet are automatic determination of the population size, combination of local with global search performed by Gaussian mutation, and capability of locating and maintaining stable local optima solutions. The learning and optimization capacities of CLONALG were verified in [24] where it mainly imitates the clonal selection principle and the diversity of repertoire. Different from previous immune models, CLONAL did not use any evolution-like mutation operators. Based on the binary representation of immune cells, CLONAL utilized the sharing and tournament selection, together with the random hypermutation to track the global optimal solution. Inspired with functions of B cells in mammalian immune system, Kelsey et al. proposed a B-cell algorithm (BCA) to solve numerical optimization with dimensionality from one to twenty [25]. The novelty of BCA was that it incorporated a multiple-point and contiguous mutation into the algorithm to facilitate the search. Thus, BCA required fewer evaluations to converge on a promising solution. Garrett has presented an attempt to remove all parameters from the CLONGAL to self-evolve various parameters during a single run, and the resultant algorithm was named ACS [26].

To further improve the performance of CLONGAL, the algorithm MIOA proposed by Song et al. introduced the mechanism of concentration of antibodies [27]. Thus, all antibodies were clustered into many groups within which the elitist ones are always retained as memory set. Cutello et al. proposed an effective binary immune algorithm (opt-IA), not only to solve dimension-limited optimization instances, but also to solve large-scale instances [28]. This was realized by incorporating three efficient immune operators, i.e. cloning, hypermutation and aging. The real-valued encoding version of opt-IA was presented in [29], where the new algorithm (opt-IMMALG) also combined an inversely proportional hypermutation operator. To locate the solutions onto the local optimal peaks as more as possible for the multimodal function, DPIA [30] combined the clonal selection with immune network mechanisms. Another attempt was shown in [31] where the proposed algorithm ICCSA introduced a cluster architecture and hybrid hypermutation.

Recently, Khilwani [32] proposed a fast clonal selection algorithm (FCA) where chaotic initialization, parallel mutation operator, and the modified elitist mechanism were incorporated into the algorithm to improve its performance in terms of effectiveness and speediness. By imitating the whole process of immune response, a general computational framework and an optimizer called PAIS were proposed [33] by Gong et al. Besides, the authors also proposed a secondary response clonal programming algorithm (SRCPA) [34] to further strength the search ability. In addition, they also proposed a hybrid immune algorithm (BCSA) by combing the Baldwin effect to expand the search area of mutation operator [35]. Based on the concepts and principles of quantum computing, a quantum-inspired algorithm (QICA) divided all antibodies into a set of subpopulation groups, and utilized the inherent functions of quantum, thus improving the performance of the algorithm in terms of the quality of solution and computational complexity [36]. Kumlachew et al. [37] proposed a vaccine enhanced immunological algorithm to solve multimodal function. The whole search space was firstly divided into many grids, and the center point in each grid was regarded as vaccine to help the algorithm locate...
as many local optimal solutions as possible onto the peaks. To speed up the convergence of each individual and reduce the computational effort of the algorithm, Leonardo et al. [38] proposed a cluster and gradient based immune algorithm (CGbAIS). The gradient information was used to guide the search to promising areas, while memory cells and a maturation control strategy aimed to eliminate possible redundancies, thus accelerating the convergence speed.

3.2. Multiple-objective optimization

A multi-objective optimization problem involves more than one objective function which is needed to be minimized or maximized. In practice real world problems, the simultaneous optimization of various and conflicting objectives takes place. Since 1999, there have been a number of immunological algorithms which attempted to solve multiple-objective optimization problems [39]. In [16], Freschi et al. summarized the works of immunological algorithms until the year of 2008. According to their classification, those works can be divided into six groups: the general reviews, algorithmic papers, theoretical papers, hybrid algorithm descriptions, application papers, and analogy papers. The superiority of the algorithms proposed for multi-objective optimization is mainly demonstrated by applying them on real world problems, including electromagnetism [40], computer science [41], bioinformatics [42], control theory [43], scheduling research [44] and networking [45]. Thus, in this paper, we attempted to give some insights into the algorithm with application on numerical optimization using up-to-date references.

As far as multi-objective numerical optimization concerned, MISA [46, 47] can be considered the real first proposal in the literature. MISA encoded the decision variables of the problem to be solved by binary strings, cloned the Pareto-optimal and feasible solutions, and applied two types of mutation to the clones and other individuals, respectively. MOIA [48] considered the distinction of heavy chains and light chains of immune cells, to separate the levels of importance of bits in the encoding scheme. In the light chain, only nondominated antibodies were chosen for hypermutation and then stored as memory cells. Besides, a cumulative scalar index called avidity was also proposed to construct the germ-line DNA libraries. Campelo et al. [49] proposed a real coded multiobjective variant (MOCSA) of CLONALG, where nondominated sorting, niching, as well as suppression were applied. Freschi et al. [50] presented a vector artificial immune system (VAIS) for multiobjective optimization based on opt-aiNet. VAIS adopted the flowchart of opt-aiNet. In [51], Tan et al. proposed an evolutionary artificial immune system (EMOIA) for multi-objective optimization. To maintain the balance between exploration and exploitation of the search and the diversity repertoire of antibodies, a new cloning selection strategy and an information-theoretic based density preservation mechanism were introduced. Compared with other evolutionary based algorithms, EMOIA showed highly competitive solutions in terms of convergence, diversity and distribution. In [52], Gong et al. proposed a nondominated neighbor immune algorithm (NNIA) where an immune inspired operator, two heuristic search operators, and elitism were incorporated. The novelty of NNIA was mainly reflected by the unique selection technique which only selected minority isolated nondominated individuals in the population.

Most recently, Chen et al. proposed a hybrid immune multiobjective optimization algorithm (HIMO) based on the clonal selection principle [53]. A novel hybrid mutation operator which combined the Gaussian and polynomial mutation operators utilized an adaptive switching parameter to control the mutation procedure, enabling the algorithm to be easily implemented but to effectively find the near-global optimal solutions. Abd El-Wahed et al. [54] proposed a hybrid framework of optimizer (MISA-NN) to solve multiobjective optimization problems by combing immunological algorithm with neural network. To reduce the sensitivity to the initial values of initial population of antibodies, the neural network was used to initialize the boundary of the antibodies to guarantee that all the initial population of antibodies was feasible. Gao et al. proposed [55] a novel weight-based multiobjective artificial immune system (WBMOMAIS) based on opt-aiNET for multi-modal optimization. The distinct characteristic of WBMOMAIS was that it utilized a randomly weighted sum of multiple objectives to compute the fitness function, rather than the traditional Pareto ranking scheme. In addition, a new truncation algorithm was also introduced to eliminate similar individuals in memory and obtain a well-distributed spread of non-dominated solutions. Yang et al. [56] proposed an improved version of NNIA, called NNIA2. In the presented algorithm, an adaptive selection scheme and an adaptive ranks clone scheme by the online discovered solutions in different ranks were proposed. Besides, a k-nearest neighbor list was established and maintained to eliminate the solutions in the archive population. In
[56], an improved version of WBMOAIS was proposed by using the quantum encoding scheme and introduced a chaos-based rotation gate to perform mutation operator.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Encoding strategy</th>
<th>Main immune metaphor</th>
<th>Diversity</th>
<th>Selection</th>
<th>Populati on</th>
</tr>
</thead>
<tbody>
<tr>
<td>opt-aiNet</td>
<td>Decimal</td>
<td>Immune network</td>
<td>Gaussian random mutation</td>
<td>Elitist replacement, Network suppression</td>
<td>Flexible</td>
</tr>
<tr>
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<td>Binary</td>
<td>Clonal selection theory</td>
<td>Cloning, affinity proportional mutation</td>
<td>Elitist selection</td>
<td>Fixed</td>
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<tr>
<td>BCA</td>
<td>Binary</td>
<td>Clonal selection theory</td>
<td>Somatic contiguous mutation</td>
<td>Meta-dynamics</td>
<td>Fixed</td>
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<td>Clonal selection theory</td>
<td>Gaussian mutation</td>
<td>Elitist selection, death</td>
<td>Scalable</td>
</tr>
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<td>Gaussian mutation, antibody concentration</td>
<td>Antibody suppression, clustering (Mu+lamda)-selection, aging</td>
<td>Fixed</td>
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<tr>
<td>opt-IA</td>
<td>Binary</td>
<td>Clonal selection theory</td>
<td>Cloning, hypermutation</td>
<td>(Mu+lamda)-selection, aging</td>
<td>Fixed</td>
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<tr>
<td>opt-IMMALG</td>
<td>Decimal</td>
<td>Clonal selection theory</td>
<td>Cloning, inversely proportional hypermutation</td>
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<td>Multi-point mutation, receptor editing</td>
<td>Suppression, elitist</td>
<td>Fixed</td>
</tr>
<tr>
<td>ICCSA</td>
<td>Decimal</td>
<td>Clonal selection theory</td>
<td>Gaussian mutation, Cauchy mutation, cloning</td>
<td>Antibody supplement and suppression, clustering</td>
<td>Fixed</td>
</tr>
<tr>
<td>FCA</td>
<td>Binary/Decimal/Permutation</td>
<td>Clonal selection theory</td>
<td>Parallel implementation of Gaussian and Cauchy mutation</td>
<td>Chaotic, elitist selection</td>
<td>Fixed</td>
</tr>
<tr>
<td>PAISA</td>
<td>Decimal</td>
<td>Whole immune response process</td>
<td>Random hypermutation, receptor editing, immune memory</td>
<td>Self-learning and feedback rules</td>
<td>Fixed</td>
</tr>
<tr>
<td>SRCPA</td>
<td>Decimal</td>
<td>Clonal selection, secondary response</td>
<td>Cloning, secondary response operator</td>
<td>Elitist</td>
<td>Fixed</td>
</tr>
<tr>
<td>BCSA</td>
<td>Decimal</td>
<td>Clonal selection theory</td>
<td>Clonal proliferation, Baldwinian learning, hypermutation</td>
<td>Clonal selection</td>
<td>Fixed</td>
</tr>
<tr>
<td>QICA</td>
<td>Quantum</td>
<td>Clonal selection theory</td>
<td>Quantum rotation gate, dynamic angle adjusting</td>
<td>Clonal selection</td>
<td>Fixed</td>
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<td>Clonal selection theory, vaccine</td>
<td>Vaccines extraction and injection, Gaussian hypermutation, suppression of memory</td>
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<td>Fixed</td>
</tr>
<tr>
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<td>Clonal selection theory</td>
<td>Gradient information utilization, hypermutation</td>
<td>Stochastic ranking, clustering</td>
<td>Fixed</td>
</tr>
</tbody>
</table>

4. General promising features of immunological algorithms

In the previous section, we have given a relatively comprehensive literature review for the immunological algorithms used to solve numerical optimization problems. It is easily to find out that, for both single-objective and multi-objective optimization problems, the immune network together with clonal selection theory based algorithms exhibit useful applicability. The instinct features of negative selection, danger theory and dendritic cells’ functions make the resultant algorithms not suitable for solving optimization problems. Nevertheless, the reasons why immune network or clonal selection theory are appropriate to perform searching in complex hyper-dimensional spaces are still unclear. Thus, by analyzing the existent immunological algorithms, we tried to extract some general promising features of the immune system, thus making a deep insight into its theoretical foundation.
To realize the above purpose, we summarized the main features of different immunological algorithms for single-objective and multi-objective optimization problems in Table 1 and 2, respectively. Then we can conclude some general remarks as:

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Encoding strategy</th>
<th>Main immune metaphor</th>
<th>Diversity/Memory</th>
<th>Pareto optimality</th>
<th>Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>MISA</td>
<td>Binary</td>
<td>Diversity, Clonal selection theory</td>
<td>Uniform and non-uniform mutation, secondary population</td>
<td>Non-dominated solutions</td>
<td>Elitist</td>
</tr>
<tr>
<td>MOIA</td>
<td>Binary</td>
<td>Light chains of immune cells</td>
<td>Suppression and generation of new population</td>
<td>Non-dominated sorted as memory</td>
<td>Recombination/diversification</td>
</tr>
<tr>
<td>MOCSA</td>
<td>Decimal</td>
<td>Clonal selection theory</td>
<td>Ranking, hypermutation</td>
<td>Dominated and non-dominated solutions</td>
<td>Niching</td>
</tr>
<tr>
<td>VAIS</td>
<td>Decimal</td>
<td>Immune network</td>
<td>Pareto ranking</td>
<td>Entropy-based density assessment</td>
<td>Elitist</td>
</tr>
<tr>
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<td>Decimal</td>
<td>Clonal selection theory</td>
<td>Naïve uniform crossover, uniform mutation</td>
<td>Minority isolated nondominated individuals</td>
<td>Tournament</td>
</tr>
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<td>Clonal selection theory</td>
<td>Heuristic search operator</td>
<td>Cloning, archive update</td>
<td>Elitist</td>
</tr>
<tr>
<td>HIMO</td>
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<td>Clonal selection theory</td>
<td>Gaussian and polynomial mutation, recombination</td>
<td>Elitist</td>
<td></td>
</tr>
<tr>
<td>MISA-NN</td>
<td>Mixed</td>
<td>Clonal selection theory, neural network</td>
<td>Uniform and non-uniform mutation, secondary population</td>
<td>Dominance principle</td>
<td>Elitist</td>
</tr>
<tr>
<td>WBMOAIS</td>
<td>Decimal</td>
<td>Immune network</td>
<td>Clonal suppression and supplement</td>
<td>Truncation of similar non-dominated solutions</td>
<td>Elitist</td>
</tr>
<tr>
<td>NNIA2</td>
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<td>Clonal selection theory</td>
<td>Adaptive ranks clone scheme</td>
<td>k-nearest neighbor list</td>
<td>Elitist</td>
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<tr>
<td>HQIA</td>
<td>Quantum</td>
<td>Clonal selection theory</td>
<td>Chaos-based rotation gate, Q-gates</td>
<td>Truncation of similar non-dominated solutions</td>
<td>Elitist</td>
</tr>
</tbody>
</table>

(1) **Encoding strategy**: for numerical optimization problems, the diversity metaphor of natural immune system is widely imitated. The diversity of repertoire is presented by using a population of candidate immune cells. Each immune cell depicts a feasible solution of the problem on hand. As a result, the data structure (or encoding scheme) directly makes significant influences on the searching performance of the algorithm. By comparing the experimental results in the literature, we can draw a conclusion empirically that, for optimization instances with small dimension size, the binary encoding scheme seems to be a good choice, due to its simple implementation; for complex functions, both real-valued and quantum encoding strategy perform well. Especially, when the algorithm is sophisticated designed and implemented, the quantum encoding can facilitate the search to become more powerful.

(2) **Mutation/Memory**: As all mentioned immunological algorithms for numerical optimization problems are population based iteration algorithm. The global optimal solution is found iteratively along the running time goes. Thus, the use of mutation operator is the key issue to drive the search. Effective mutation operators can find promising solutions with satisfied distribution within reasonable iteration numbers, and vice verse. By analyzing all the mutation operators, we can find that the design of mutation is sophisticated and the capacity of the operator becomes larger if some of the problem-oriented knowledge is incorporated into the process of design. In addition, for multiobjective optimization, the purpose of mutation is to locate the solutions of the Pareto optimal set normally. Therefore, many adaptive ranking strategies are used to manipulate the immune cells. Besides, most of the immunological algorithm adopted memory cells to record elitist solutions during iteration, and the memory pool is also updated with some mechanism. It is expected that the thereafter searching around memory cells is within promising areas.
(3) **Selection:** Although immunological algorithms have shown competitive performance with respect to other computational intelligence, such as evolutionary algorithms, the selection operator and the population control strategy are almost directly taken from non-immune inspired algorithms. For instance, neither (Mu+lambda)-selection, aging, clustering or non-dominated solution sorting is motivated by immune systems.

(4) **Population:** It is the inherent ability of immune system that it can adaptively control the population size of B cells once the system is exposed to antigen. Thus, in immune networks, the strategy of antibody suppression and supplement are also utilized. On the other hand, in clonal selection principle based algorithms, the elitist scheme is usually used, and it is often related with the population information indices (such as entropy). It is worth emphasizing that this feature is significant for real world applications to reduce the computational cost for immunological algorithms.

5. **Conclusions**

In this survey, we have reviewed most of the up-to-date immunological algorithms for solving numerical optimization problems. The single-objective and multi-objective optimization problems are discussed separately. By extracting some general promising features of each algorithm, we have drawn some instructive guidelines. In the future, we plan to present the current work by extending to the numerical optimization problems with constrained conditions, and dynamic environments.

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7. **References**


