Adaptive output feedback fault-tolerant control design for hypersonic flight vehicles

Jingjing He, Ruiyun Qi*, Bin Jiang, Jiasong Qian

College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, 29 Yuedao Street, Nanjing 210016, China

Received 2 September 2014; received in revised form 4 December 2014; accepted 15 January 2015
Available online 31 January 2015

Abstract

In this paper, an adaptive output feedback fault-tolerant controller is developed for the longitudinal dynamics of a generic hypersonic flight vehicle in the presence of parameter uncertainties, actuator faults and external disturbances. Firstly, the derivatives of the output are calculated repeatedly so that the relative degree of the system is obtained. Then feedback linearization is used to design the nominal controller. Considering the occurrence of actuator faults, a fault-tolerant controller is developed based on the nominal feedback linearization controller to accommodate the effect of actuator fault, ensure system stability and recover desirable tracking performance. Since some of the states are difficult to measure during actual hypersonic flight, the high-gain observer technique is adopted to achieve output feedback fault-tolerant control. Adaptive laws are designed for updating the controller parameters when both the plant parameters and actuator fault parameters are unknown. Closed-loop stability and output tracking performance are analyzed rigorously. Simulation results verify the effectiveness of the proposed adaptive fault-tolerant control scheme.

© 2015 The Franklin Institute. Published by Elsevier Ltd. All rights reserved.

1. Introduction

The various interests of hypersonic vehicles have been identified for a long time due to their promise for high speed transportation and affordable space access. Hypersonic vehicles are sensitive to changes in flight conditions as well as peculiar structure due to their design and flight conditions of high altitudes and Mach numbers. But it is difficult to measure the atmospheric
properties and aerodynamic characteristics at the hypersonic flight altitude. In addition, the strong parameter coupling and modeling uncertainty bring much more challenges to control technologies.

There have been several approaches for control of hypersonic vehicles in recent years [3–13]. An adaptive sliding controller is designed and analyzed for the longitudinal dynamics of a generic hypersonic air vehicle in [3,4]. The paper [5] proposes a reference command tracking controller for the linearized dynamics of a hypersonic vehicle using LQR design with integral augmentation. An integrated guidance and control architecture, including ascent and entry trajectory-planning methods suitable for in-flight updates and adaptive trajectory-following flight control, has been developed in [6]. The paper [7,8] address issues related to output feedback control for a model of an air-breathing hypersonic vehicle. The paper [9] proposes an adaptive fuzzy control strategy for hypersonic aircraft via backstepping method. In [10], the design of a nonlinear robust controller for a non-minimum phase model of an air-breathing hypersonic vehicle is presented, using a combination of small-gain arguments and adaptive control techniques for the design of a state-feedback controller that achieves asymptotic tracking of a family of velocity and flight-path angle reference trajectories. The paper [11] designs a controller for reference command tracking control for the longitudinal model of an air-breathing hypersonic vehicle subject to high nonlinearity, uncertain parameters and input constraints. Based on the functional decomposition, the adaptive discrete-time nonlinear controllers are developed using feedback linearization and neural approximation for the two subsystems in [12], to make the altitude and velocity to follow a given desired trajectory in the presence of aerodynamic uncertainties. The paper [13] investigates the problem of robust output feedback control and simulation for the longitudinal model of a flexible air-breathing hypersonic vehicle.

In practical application, actuator faults, sensor faults and system faults may cause serious safety problems. To improve system reliability and performance, a controller has to be able to accommodate those faults. Fault diagnosis and identification (FDI) and fault-tolerant control (FTC) have been considered as one of the most promising control technologies for ensuring system stability and maintaining acceptable system performance in the presence of abrupt faults [14]. Both model-based FDI/FTC and data-driven FDI/FTC have attracted many research interests in recent year [31,32]. A few fault-tolerant control methods have been proposed for hypersonic vehicles during the past decades [15–27]. A reconfigurable control law for the full X-33 flight envelope has been designed to accommodate a failed control surface and redistribute the control effort among the remaining working surfaces to retain satisfactory stability and performance in [16]. An integrated adaptive guidance and control program is presented in [18], where a reconfigurable control, adaptive-guidance, and trajectory-command-reshaping system are developed. Most control techniques in this program have been flight-tested. However, mainly linear control methods were used, which may not be suitable for highly nonlinear hypersonic vehicles. Recent contributions have begun to address the design of nonlinear fault-tolerant controllers for nonlinear vehicle models. In [20], an adaptive backstepping control scheme is developed for the spacecraft attitude stabilization system, in which the external disturbances and partial loss of actuator effectiveness fault are considered. Based on fuzzy control and sliding mode observer technique, a fault accommodation strategy is proposed for hypersonic vehicles with actuator fault in [21]. In [24], an adaptive fault-tolerant tracking control scheme for hypersonic vehicles using Takagi–Sugeno fuzzy models is proposed. In [27], an adaptive fault-tolerant controller for hypersonic vehicle is developed based on the combination of backstepping control scheme and dynamic surface control technique. Most proposed fault-tolerant control schemes use state feedback and assume all the states could be measured accurately in flight.
However, not all the states are easy to measure, i.e., the angle of attack and the flight path angle are difficult to measure in actual hypersonic flight because they are very small.

In this paper, we propose an output feedback adaptive fault-tolerant controller for hypersonic flight vehicles based on feedback linearization and high gain observer. Both parameter uncertainties and actuator fault uncertainties are considered in our design. A fault-tolerant control scheme is proposed and the necessary matching conditions for accommodating elevator fault are given. Considering the facts that some states are difficult to measure, the high-gain observer technique is used to achieve an output feedback fault-tolerant controller. Parameter adaptive laws are designed to estimate both the unknown flight dynamics parameters and the actuator fault. Parameter projection algorithm is also employed to keep the parameters in admissible ranges. The stability of the overall control scheme is proven and simulation results under different cases are presented to demonstrate the effectiveness and efficiency of the proposed scheme.

This paper is organized as follows: Section 2 briefly presents the longitudinal dynamics of the hypersonic vehicle and control problem. In Section 3, a state feedback adaptive fault-tolerant controller is proposed to accommodate the elevator fault. Section 4 presents an output feedback adaptive fault-tolerant control scheme based on high-gain observers to solve the problem that some system states are difficult to measure in hypersonic flight. A velocity control scheme is presented for the velocity subsystem in Section 5. Simulation studies are conducted to demonstrate the effectiveness of the propose scheme in Section 6, followed by conclusions in Section 7.

2. Vehicle model and problem simulation

2.1. Longitudinal dynamics of hypersonic vehicle

The study in this paper considers a model for the longitudinal dynamics of a generic hypersonic vehicle developed at NASA Langley Research Center [1,2]. The equations of motion include an inverse-square-law gravitational model and the centripetal acceleration for the nonrotating Earth. The model of the longitudinal vehicle dynamics is given by as follows:

\[
\dot{V} = \frac{T \cos \alpha - D}{m} - \frac{\mu \sin \gamma}{r^2}
\]

\[
\dot{\gamma} = \frac{L + T \sin \alpha}{mV} - \frac{(\mu - V^2 r) \cos \gamma}{V r^2}
\]

\[
\dot{h} = V \sin \gamma
\]

\[
\dot{\alpha} = q - \dot{\gamma}
\]

\[
\dot{q} = \frac{M_{yy}}{I_{yy}}
\]

where

\[ T = \frac{1}{2} \rho V^2 S C_T, \quad L = \frac{1}{2} \rho V^2 S C_L, \quad D = \frac{1}{2} \rho V^2 S C_D \]

\[ M_{yy} = \frac{1}{2} \rho V^2 S \left[ C_M(\alpha) + C_M(\delta) + C_M(q) \right], \quad r = h + R_e. \]

The nomenclature is given in Table 1. For the control design in this study, the aerodynamic coefficients are simplified around the nominal cruise flight with the trimmed cruise condition [3]:

```plaintext
J. He et al. / Journal of the Franklin Institute 352 (2015) 1811–1835
```

1813
Table 1
Variables and parameters of interest.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Velocity, ft/s</td>
</tr>
<tr>
<td>h</td>
<td>Altitude, ft</td>
</tr>
<tr>
<td>γ</td>
<td>Flight-path angle, rad</td>
</tr>
<tr>
<td>α</td>
<td>Angle of attack, rad</td>
</tr>
<tr>
<td>q</td>
<td>Pitch rate, rad/s</td>
</tr>
<tr>
<td>m</td>
<td>Mass, slug</td>
</tr>
<tr>
<td>ρ</td>
<td>Density of air, slug/ft³</td>
</tr>
<tr>
<td>τ</td>
<td>Mean aerodynamic chord, ft</td>
</tr>
<tr>
<td>S</td>
<td>Reference area, ft²</td>
</tr>
<tr>
<td>Re</td>
<td>Radius of Earth, ft</td>
</tr>
<tr>
<td>T</td>
<td>Thrust, lbf</td>
</tr>
<tr>
<td>L</td>
<td>Lift, lbf</td>
</tr>
<tr>
<td>D</td>
<td>Drag, lbf</td>
</tr>
<tr>
<td>Myy</td>
<td>Pitching moment, lbf ft</td>
</tr>
<tr>
<td>Iyy</td>
<td>Moment of inertia, slug ft²</td>
</tr>
<tr>
<td>Cₖ, Cₗ, C₉</td>
<td>Aerodynamic coefficients</td>
</tr>
<tr>
<td>r</td>
<td>Radial distance, ft</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
<tr>
<td>δₑ</td>
<td>Elevator deflection, rad</td>
</tr>
<tr>
<td>β</td>
<td>Throttle setting</td>
</tr>
</tbody>
</table>

$M = 5$, $V = 15,060$ ft/s, $h = 110,000$ ft, $γ = 0$ rad, $q = 0$ rad/s. The aerodynamic coefficients are

$$C_T = \begin{cases} 0.02576\beta & \text{if } \beta < 1 \\ 0.0224 + 0.00336\beta & \text{otherwise} \end{cases}$$

$$C_L = 0.6203\alpha$$

$$C_D = 0.6450\alpha^2 + 0.0043378\alpha + 0.003772$$

$$C_M(\alpha) = -0.035\alpha^2 + 0.036617\alpha + 5.3261 \times 10^{-6}$$

$$C_M(q) = (c/2V)q(-6.796\alpha^2 + 0.3015\alpha - 0.2289)$$

$$C_M(\delta_e) = c_e(\delta_e - \alpha), \ \delta_e = d_1\delta_{e1} + d_2\delta_{e2},$$

where $\delta_e$ is the elevator deflection generated by a two-piece elevator to provide some control redundancy, with $\delta_{e1}$ and $\delta_{e2}$ being the deflection angles of the two pieces, $d_1$ and $d_2$ being the gains of the two deflection angles. The engine dynamics are modeled by a second-order system:

$$\ddot{\beta} = -2\zeta\omega_n\dot{\beta} - \omega_n^2\beta + \omega_p^2\beta_c.$$  \hspace{1cm} (2)

In this study, we consider the elevator deflection $\delta_e$ and the throttle setting $\beta$ as the inputs, the altitude $h$ and the velocity $V$ as the outputs.

2.2. Control objectives

The control objective is to ensure that all the closed-loop signals are bounded and the altitude $h$ and velocity $V$ asymptotically track given reference signals $h_d$ and $V_d$ when there exist nonlinear dynamics uncertainties and actuator faults.

The following assumption is made on the reference trajectories:
Assumption 1. The reference trajectories $V_d(t)$ and $h_d(t)$ are bounded and with bounded derivatives of any order.

In cruise flight, the angel-of-attack $\alpha$ is small and the thrust item $T \sin \alpha$ is far less than the lift $L$ in Eq. (1b). It can be seen from Eqs. (1a) and (1e) that the altitude $h$ mainly depends on the elevator deflection $\delta_e$ while the velocity $V$ is controlled mainly by the throttle setting $\beta$. Therefore, for the control design in this study, the longitudinal dynamics are divided into two subsystems: altitude control subsystem and velocity control subsystem.

In the following sections, we first present the fault-tolerant control design for the altitude subsystem, and then present the velocity control design.

3. State feedback fault-tolerant altitude control design

Feedback linearization is an effective method for designing a controller for a nonlinear system. The nonlinear system can be transformed into a linear system by means of nonlinear state feedback when satisfying the relative degree condition. Then, we can design the controller in the way of linear system theory. Therefore, we first need to examine whether its model can be linearized completely.

3.1. Input/output linearization

Define the state vector

$$x = [x_1, x_2, x_3, x_4]^T = [h, \gamma, \theta_p, q]^T$$

where $\theta_p = \alpha + \gamma$. Then the altitude subsystem (1) can be transformed into the following form:

$$\begin{align*}
\dot{x}_1 &= V \sin x_2 \\
\dot{x}_2 &= f_1(x_2, V) + g_1(V)x_3 \\
\dot{x}_3 &= f_2 + g_2x_4 \\
\dot{x}_4 &= f_3(x_2, x_3, x_4, V) + g_3(V)u \\
y &= x_1,
\end{align*}$$

where

$$\begin{align*}
f_1(x_2, V) &= -(\mu - V^2 r) \cos \gamma / V r^2 - 0.6203qS/(mV) \times \gamma, \\
g_1(V) &= 0.6203qS/(mV), \\
f_2 &= 0, \quad g_2 = 1, \\
f_3(x_2, x_3, x_4, V) &= \frac{qSc(C_M(\alpha) + C_M(q) - c_e a)}{I_{yy}}, \\
g_3(V) &= \frac{qSc_c}{I_{yy}} u = \delta_e.
\end{align*}$$

The altitude subsystem output $y = x_1$ is indirectly linked to the input $u$. In order to obtain the relationship between $y = x_1$ and $u$, it is necessary to differentiate the output $y$ repeatedly until the
input $u$ appears in the resulting equation. Differentiating $y$ as follows:

\[
\dot{y} = V \sin x_2 \tag{5a}
\]

\[
\ddot{y} = V \cos x_2 \cdot \dot{x}_2 \tag{5b}
\]

\[
y^{(3)} = V (- \sin x_2 \cdot (\dot{x}_2)^2 + \cos x_2 \cdot \ddot{x}_2) \tag{5c}
\]

\[
y^{(4)} = V (- \cos x_2 \cdot (\dot{x}_2)^3 - 3 \sin x_2 \cdot \dot{x}_2 \cdot \ddot{x}_2 + \cos x_2 \cdot x_2^{(3)}) \tag{5d}
\]

In Eq. (5d),

\[
\dot{x}_2 = - (\mu - V^2 r) \cos \frac{x_2}{r^2} - g_1 (x_2 - x_3) \tag{6}
\]

\[
\ddot{x}_2 = \left( \frac{\mu}{V r} - V \right) \cdot \frac{\sin x_2 \cdot \dot{x}_2}{r} + \left( \frac{\mu}{V r} - \frac{V}{2} \right) \cdot \frac{V \sin 2x_2}{r^2} - g_1 (\dot{x}_2 - x_4) \tag{7}
\]

\[
x_2^{(3)} = - \frac{\mu}{r^3} \cdot \sin x_2 \left( \sin x_2 \cdot \dot{x}_2 + \frac{V}{r} \sin 2x_2 \right)
+ \left( \frac{\mu}{V r} - V \right) \cdot \frac{(\cos x_2 \cdot (\dot{x}_2)^2 + \sin x_2 \cdot \ddot{x}_2) r - V (\sin x_2)^2 \cdot \dot{x}_2}{r^2}
+ \left( \frac{\mu}{V r} - \frac{V}{2} \right) \cdot \frac{2V \cos 2x_2 \cdot \dot{x}_2 \cdot r - 2V^2 \sin 2x_2 \cdot \sin x_2}{r^3}
- g_1 (\ddot{x}_2 - f_3) + g_1 g_3 u \tag{8}
\]

Substituting Eqs. (6)–(8) into Eq. (5d), we obtain

\[
y^{(4)} = A_0 + A_1 g_1 + A_2 g_1^2 + A_3 g_1^3 + V \cos x_2 g_1 f_3 + V \cos x_2 g_1 g_3 u \tag{9}
\]

where $A_0$, $A_1$, $A_2$ and $A_3$ are nonlinear functions of $x$ whose detailed expressions are given in Appendix A.

In reality, there exists disturbance. To include the effect of disturbance, the model (9) becomes

\[
y^{(4)} = A_0 + A_1 g_1 + A_2 g_1^2 + A_3 g_1^3 + V \cos x_2 g_1 f_3 + V \cos x_2 g_1 g_3 u + o(t), \tag{10}
\]

where $o(t)$ represents the external disturbance.

**Assumption 2.** The external disturbance $o(t)$ is bounded, that is, there exists a positive constant $\overline{\omega}$ such that $|o(t)| \leq \overline{\omega}$, where $\overline{\omega}$ is unknown.

The control input $u$ appears in Eq. (10) such that the relative degree of the altitude subsystem is four, which equals to the order of the system. Thus, the nonlinear model (3) can be linearized completely, and the closed-loop system has no zero dynamics.

### 3.2. Nominal altitude controller

For the given reference $h_d$, the tracking error and its derivatives can be defined as $e_1 = y - h_d$, $e_2 = \dot{y} - \dot{h}_d$, $e_3 = \ddot{y} - \ddot{h}_d$, $e_4 = y^{(3)} - h_d^{(3)}$. We obtain

\[
\dot{e} = Ae + B(A_0 + A_1 g_1 + A_2 g_1^2 + A_3 g_1^3 + V \cos x_2 g_1 f_3 + V \cos x_2 g_1 g_3 u + o(t) - h_d^{(4)}),
\]
where
\[ e = [e_1 \ e_2 \ e_3 \ e_4]^T, \]
\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \]

Suppose that parameters \( m, S, \rho, I_{yy}, \mu, c, \) and \( \bar{\omega} \) are known. The state feedback altitude control law can be designed as
\[
\begin{align*}
    u &= (g_1^{-1}g_3^{-1}(-Ke - A_0 + h_d^{(4)}) - g_3^{-1}A_1 - g_1g_3^{-1}A_2 - g_1^2g_3^{-1}A_3 \\
    &\quad - g_3^{-1}f_3V \cos x_2 - g_3^{-1}\bar{\omega})/(V \cos x_2),
\end{align*}
\]
which \( K = [k_1, k_2, k_3, k_4] \) is designed to assign the eigenvalues of \( A - BK \) at desired locations in the open left-half complex plane.

**Remark 2.** It can be seen that \( \cos x_2 \) in Eq. (11) does not equal zero except on a vertical flight path, where \( x_2 = \gamma \). This paper considers the altitude and velocity control in cruising flight, during which the flight-path angle \( \gamma \) would not reach \( \pm 90^\circ \).

### 3.3. Adaptive altitude control

Now we develop an adaptive control scheme for the system (3) with unknown parameters \( m, S, \rho, I_{yy}, \mu, c, \) and \( \bar{\omega} \). To design the parameter adaptive laws, we need to parameterize some functions which contain the unknown parameters. Let \( g_1^{-1}g_3^{-1} = \theta_1 \varphi_1, \quad g_3^{-1} = \theta_2 \varphi_2, \quad g_1g_3^{-1} = \theta_3 \varphi_3, \quad g_2^2g_3^{-1} = \theta_4 \varphi_4, \quad g_3^{-1}f_3 = \theta_5^T \varphi_5, \quad g_1^{-1}g_3^{-1}\bar{\omega} = \theta_6 \varphi_6, \) where
\[
\theta_1 = \frac{4mI_{yy}}{0.6203\rho^2S^2c_ee}, \quad \varphi_1 = \frac{1}{V^3}
\]
\[
\theta_2 = \frac{2I_{yy}}{\rho S c_ee}, \quad \varphi_2 = \frac{1}{V^2}
\]
\[
\theta_3 = \frac{0.6203I_{yy}}{m c_ee}, \quad \varphi_3 = \frac{1}{V}
\]
\[
\theta_4 = \frac{0.6203^2\rho S I_{yy}}{2m^2c_ee}, \quad \varphi_4 = 1
\]
\[
\theta_5 = \begin{bmatrix} -0.035 \ c_ee \\ 0.036617 - c_ee \\ 5.3261 \times 10^{-6} \ c_ee \\ -6.796c_ee \\ 0.3015c_ee \\ -0.2289c_ee \end{bmatrix}^T
\]
\[
\varphi_5 = \begin{bmatrix} \alpha^2 & \alpha & 1 & \frac{q\alpha^2}{V} & \frac{q\alpha}{V} & \frac{q}{V} \end{bmatrix}^T
\]
\[
\theta_6 = \frac{4mI_{yy}\bar{\omega}}{0.6203\rho^2S^2c_ee}, \quad \varphi_6 = \frac{1}{V^3}
\]

Then the adaptive control law is
\[
 u = (\hat{\theta}_1 \varphi_1 (-K_x - A_0 + h_0^{(4)}) - \hat{\theta}_2 \varphi_2 A_1 - \hat{\theta}_3 \varphi_3 A_2 - \hat{\theta}_4 \varphi_4 A_3 \\
 - \hat{\theta}_5 \varphi_5 V \cos x_2 - \hat{\theta}_6 \varphi_6)/(V \cos x_2),
\]
(12)

where \( \hat{\theta}_i, i = 1, 2, \ldots, 6 \), are the estimates of \( \theta_i \).

The adaptive laws for updating \( \hat{\theta}_i, i = 1, 2, \ldots, 6 \), are chosen as
\[
\begin{align*}
\dot{\hat{\theta}}_1 &= -\Gamma_1 e^T PB(-K_x - A_0 + h_0^{(4)})\varphi_1 \\
\dot{\hat{\theta}}_2 &= \Gamma_2 e^T PBA_1 \varphi_2 \\
\dot{\hat{\theta}}_3 &= \Gamma_3 e^T PBA_2 \varphi_3 \\
\dot{\hat{\theta}}_4 &= \Gamma_4 e^T PBA_3 \varphi_4 \\
\dot{\hat{\theta}}_5 &= \Gamma_5 e^T P \varphi_5 V \cos x_2 \\
\dot{\hat{\theta}}_6 &= \Gamma_6 e^T P \varphi_6,
\end{align*}
\]
(13a–f)

where \( \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4 \) and \( \Gamma_6 \) are positive constants, \( \Gamma_5 \in \mathbb{R}^{6 \times 6} \) and \( P \in \mathbb{R}^{4 \times 4} \) are positive definite matrices.

3.4. Adaptive fault-tolerant controller

In this section, we consider the actuator stuck fault. When a stuck fault occurs in one elevator, the output \( u_j(t) \) of the faulty elevator \( j \) becomes as
\[
u_j(t) = \bar{u}_j, \quad t \geq t_j, \quad j = j_f
\]
(14)

where \( \bar{u}_j \) is a constant value, \( j_f \) is the number of the faulty actuator (\( j_f = 1 \) or 2) and \( t_j \) is the occurrence time of the fault. To take into account both the normal and fault cases, the following model is employed:
\[
u_j(t) = v_j(t) + \sigma_j(\bar{u}_j - v_j(t)), \quad j = 1, 2
\]
(15)

where \( v_j(t) \) is a designed control input, and \( \sigma_j \) is the fault pattern index, and
\[
\sigma_j = \begin{cases} 
1 & \text{if the jth actuator fails,} \\
0 & \text{otherwise.}
\end{cases}
\]

To make the control law capable of accommodating faults, the reconfigurable control law \( v_j(t) \) is designed as
\[
v_j(t) = k_{1j}v_0 + k_{2j}, \quad j = 1, 2
\]
(16)

where \( v_0 \) is the nominal control law defined in Eq. (12), and \( k_{1j}, k_{2j} \) are two constants which are designed to satisfy the matching condition as follows:
\[
[k_{11} k_{12}](I - \sigma)d^T = 1 \\
[k_{21} k_{22}](I - \sigma)d^T = -d\sigma\bar{u},
\]
(17)

with \( \sigma = \text{diag}(\sigma_1, \sigma_2) \), \( d = [d_1, d_2] \) and \( \bar{u} = [\bar{u}_1, \bar{u}_2]^T \).
For the fault pattern \( \sigma \) and the fault value \( \bar{u} \) unknown, an adaptive version of controller (16) is proposed:

\[
v_j(t) = \hat{k}_{1j}(t)v_0 + \hat{k}_{2j}(t), \quad j = 1, 2
\]

where \( \hat{k}_{1j}(t) \) and \( \hat{k}_{2j}(t) \) are estimates of \( k_{1j} \) and \( k_{2j} \). Since \( k_{1j} \) and \( k_{2j} \) exist to meet the matching condition (17), the standard adaptive control design method can be applied to solve this problem.

To derive the adaptive laws for \( \hat{k}_{1j}(t) \) and \( \hat{k}_{2j}(t) \), we define parameter errors as follows:

\[
\hat{\theta}_1 = \frac{\hat{\theta}_1}{C_0} \frac{\bar{u}}{u_0}, \quad \hat{\theta}_2 = \frac{\hat{\theta}_2}{C_0} \frac{\bar{u}}{u_0}, \quad j = 1, 2
\]

\[
\hat{\theta}_j = \hat{\theta}_j - \hat{\theta}_j_0, \quad j = 1, 2
\]

Then, we choose the adaptive laws for \( \hat{k}_{1j}(t) \) and \( \hat{k}_{2j}(t) \) as

\[
\hat{k}_{1j} = -\frac{\hat{\theta}_1}{C_0} \frac{\bar{u}}{u_0}, \quad \hat{k}_{2j} = -\frac{\hat{\theta}_2}{C_0} \frac{\bar{u}}{u_0}
\]

4. Output feedback fault-tolerant altitude control design

To implement the control law (12), it is required that all the states were measurable. It is practical to measure the states \( V \) (velocity), \( h \) (altitude) and \( q \) (pitch rate) in flight. However, it is difficult to measure the angle of attack \( \alpha \) and the flight-path angle \( \gamma \) in an actual hypersonic flight since they are generally very small. Even the measurements are possible, it is usually costly. Hence, an output feedback controller is designed in this section based on the high-gain observer technique [28].

4.1. High-gain observer design

To estimate the derivatives of the tracking error \( (e_1 = y - y_d) \), the following high-gain observer is constructed:

\[
\dot{\hat{e}}_1 = \hat{e}_2 + \frac{\alpha_1}{\varepsilon} (e_1 - \hat{e}_1)
\]

\[
\dot{\hat{e}}_2 = \hat{e}_3 + \frac{\alpha_2}{\varepsilon^2} (e_1 - \hat{e}_1)
\]

\[
\dot{\hat{e}}_3 = \hat{e}_4 + \frac{\alpha_3}{\varepsilon^3} (e_1 - \hat{e}_1)
\]

\[
\dot{\hat{e}}_4 = \frac{\alpha_4}{\varepsilon^4} (e_1 - \hat{e}_1),
\]

where \( \hat{e}_i \) are the estimates of \( e_i \), \( \varepsilon \) is a small positive parameter to be specified. The constants \( \alpha_i > 0 \) are chosen such that the roots of

\[
s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1} s + \alpha_n = 0
\]
From Eqs. (5)–(8) and with the definition of $e$, we can express the state variables as

$$x_1 = \Phi_1(e_1 + \hat{h}_d)$$  

(23a)

$$x_2 = \Phi_2(e_2 + \hat{h}_d)$$  

(23b)

$$x_3 = g_1^{-1}\Phi_3(e_2 + \hat{h}_d, e_3 + \tilde{h}_d) + x_2$$  

(23c)

$$x_4 = g_1^{-1}\Phi_4(e_2 + \hat{h}_d, e_3 + \tilde{h}_d, e_4 + \hat{h}_d^{(3)}) + \frac{e_3 + \tilde{h}_d}{V \cos x_2},$$  

(23d)

where $\Phi_1$, $\Phi_2$, $\Phi_3$ and $\Phi_4$ are given in Appendix B.

Substituting Eq. (23) into Eq. (11), we can obtain the controller as

$$u = (g_1^{-1}g_3^{-1}(-Ke + \hat{h}_d^{(4)} + H_1) - g_3^{-1}H_2)/V \cos (\Phi_2(e_2 + \hat{h}_d)) - g_3^{-1}f_3,$$  

(24)

where $H_1$ and $H_2$ are nonlinear functions of $e$ whose expressions are given in Appendix B.

To derive an adaptive output feedback controller, we need to parameterize some functions. Let $g_1^{-1}g_3^{-1} = \theta_{o1}q_{o1}$, $g_3^{-1} = \theta_{o2}q_{o2}$, $g_3^{-1}f_3 = \theta_{o3}q_{o3}$, where

$$\theta_{o1} = \frac{4mI_{yy}}{0.6203\rho^2S^2\tau c_e}, \quad \rho_{o1} = \frac{1}{V^3}$$

$$\theta_{o2} = \frac{2I_{yy}}{\rho S\tau c_e}, \quad \rho_{o2} = \frac{1}{V^2}$$

$$\theta_{o3} = \begin{bmatrix}
-0.0354m^2 \\
\frac{0.6203\rho S c_e}{\rho c_e}
\end{bmatrix}, \quad \rho_{o3} = \begin{bmatrix}
\frac{\Phi_2(e)}{V} \\
\frac{\Phi_3(e)}{V} \\
1 \\
\frac{\Phi_4(e)}{V}
\end{bmatrix}$$

Then, the adaptive output feedback fault-tolerant controller is

$$v_0 = (\hat{\theta}_{o1}q_{o1}(-K\hat{e} + \hat{h}_d^{(4)} + H_1(\hat{e}_1 + h_d, \hat{e}_2 + \hat{h}_d, \hat{e}_3 + \tilde{h}_d, \hat{e}_4 + \hat{h}_d^{(3)}))$$

$$-\hat{\theta}_{o2}q_{o2}H_2(\hat{e}_1 + h_d, \hat{e}_2 + \hat{h}_d, \hat{e}_3 + \tilde{h}_d, \hat{e}_4 + \hat{h}_d^{(3)})/V \cos (\Phi_2(\hat{e}_2 + \hat{h}_d))$$

$$-\hat{\theta}_{o3}q_{o3}(\hat{e}_1 + h_d, \hat{e}_2 + \hat{h}_d, \hat{e}_3 + \tilde{h}_d, \hat{e}_4 + \hat{h}_d^{(3)})$$

$$v_j = \hat{k}_{1j}v_0 + \hat{k}_{2j}, \quad j = 1, 2.$$  

(25)
We choose the adaptive laws as

\[
\dot{\theta}_{o1} = -\Gamma_{o1} \hat{e}^T PB \varphi_{o1} (-K \hat{e} + h_d^{(4)} + H_1 (\hat{e}_1 + h_d, \hat{e}_2 + \hat{h}_d, \hat{e}_3 + \hat{h}_d, \hat{e}_4 + h_d^{(3)})) \\
\dot{\theta}_{o2} = \Gamma_{o2} \hat{e}^T PB \varphi_{o2} H_2 (\hat{e}_1 + h_d, \hat{e}_2 + \hat{h}_d, \hat{e}_3 + \hat{h}_d, \hat{e}_4 + h_d^{(3)}) \\
\dot{\theta}_{o3} = \Gamma_{o3} \hat{e}^T PB \varphi_{o3} \hat{e} + h_d^{(3)} V \cos (\Phi_2 (\hat{e}_2 + \hat{h}_d))
\]

\[
\dot{k}_{ij} = -\text{sgn} \{d_j \} \Gamma_{ij} \hat{e}^T PB V \cos (\Phi_2 (\hat{e}_2 + \hat{h}_d)), \quad j = 1, 2,
\]

where \( \Gamma_{o1}, \Gamma_{o2} \) are positive constants, \( \Gamma_{o3} \in \mathbb{R}^{9 \times 9} \) is a positive definite matrix.

### 4.2. Projection operator and control saturation

In order to guarantee the boundedness of estimated parameters and avoid parameter drift, we use the idea of parameter projection [30]. Suppose \( \Omega_\theta, \Omega_\delta \) are two convex sets: \( \Omega_\theta = \{ \theta| a_i \leq \theta_{oi} \leq b_i, 1 \leq i \leq 3 \} \), \( \Omega_\delta = \{ k| a_{ij} \leq \delta_{ij} \leq \delta_{ij}, 1 \leq i \leq 3 \} \). Let \( \Omega_{\delta_0} = \{ \theta_{oi} - \delta_{ij} \leq \theta_{oi} \leq b_i + \delta_{ij}, 1 \leq i \leq 3 \} \), \( \Omega_{\delta_0} = \{ k| a_{ij} - \delta_{ij} \leq k_{ij} \leq b_{ij} + \delta_{ij}, i, j = 1, 2 \} \), the projection operator \( \text{Proj}(\theta_{oi}, \phi_i) \) is taken as

\[
\text{Proj}(\theta_{oi}, \phi_i) = \left\{ \begin{array}{ll}
\Gamma_{oi} \phi_i & \text{if } a_i \leq \theta_{oi} \leq b_i \\
n \theta_{oi} > b_i \text{ and } \phi_i \leq 0 & \text{if } \theta_{oi} \leq a_i \text{ and } \phi_i \geq 0,
\end{array} \right.
\]

\[
\text{Proj}(\theta_{oi}, \phi_i) = \left\{ \begin{array}{ll}
\Gamma_{oi} \phi_i & \text{if } \theta_{oi} > b_i \text{ and } \phi_i > 0 \\
n \theta_{oi} < a_i \text{ and } \phi_i < 0 & \text{if } \theta_{oi} < a_i \text{ and } \phi_i < 0.
\end{array} \right.
\]

where

\[
\bar{\phi}_i = \left[ 1 + \frac{b_i - \hat{\theta}_{oi}}{\delta_{ij}} \right] \phi_i, \quad \bar{\phi}_i = \left[ 1 + \frac{\hat{\theta}_{oi} - a_i}{\delta_{ij}} \right] \phi_i, \quad i = 1, \ldots, 3.
\]

\[
\phi_1 = \hat{e}^T PB \varphi_{o1} (-K \hat{e} + h_d^{(4)} + H_1 (\hat{e}_1 + h_d, \hat{e}_2 + \hat{h}_d, \hat{e}_3 + \hat{h}_d, \hat{e}_4 + h_d^{(3)}))
\]

\[
\phi_2 = \hat{e}^T PB \varphi_{o2} H_2 (\hat{e}_1 + h_d, \hat{e}_2 + \hat{h}_d, \hat{e}_3 + \hat{h}_d, \hat{e}_4 + h_d^{(3)}))
\]

\[
\phi_3 = \hat{e}^T PB \varphi_{o3} \hat{e} + h_d^{(3)} V \cos (\Phi_2 (\hat{e}_2 + \hat{h}_d))
\]

and the projection operator \( \text{Proj}(\hat{k}_{ij}, \phi_{ij}) \) is taken as

\[
\text{Proj}(\hat{k}_{ij}, \phi_{ij}) = \left\{ \begin{array}{ll}
\Gamma_{ij} \phi_{ij} & \text{if } a_{ij} \leq \hat{k}_{ij} \leq b_{ij} \text{ or } \\
n \hat{k}_{ij} > b_{ij} \text{ and } \phi_{ij} \leq 0 & \text{if } \hat{k}_{ij} \leq a_{ij} \text{ and } \phi_{ij} \geq 0, \\
n \hat{k}_{ij} \phi_{ij} & \text{if } \hat{k}_{ij} \leq a_{ij} \text{ and } \phi_{ij} > 0 \\
n \hat{k}_{ij} \phi_{ij} & \text{if } \hat{k}_{ij} < a_{ij} \text{ and } \phi_{ij} < 0 \\
\end{array} \right.
\]
where
\[
\Phi_{ij} = \left[1 + \frac{b_{ij} - \hat{k}_{ij}}{\delta_k}\right] \phi_{ij}, \quad \omega \phi_{ij} = \left[1 + \frac{\hat{k}_{ij} - a_{ij}}{\delta_k}\right] \phi_{ij},
\]
\[
\phi_{ij} = -\text{sgn}(d)[\hat{e}^T P B V \cos(\Phi_2(\hat{e}_2 + \hat{h}_d))]v_0
\]
\[
\phi_{2j} = -\text{sgn}(d)[\hat{e}^T P B V \cos(\Phi_2(\hat{e}_2 + \hat{h}_d))], \quad i, j = 1, 2.
\]

The parameter adaptive laws with projection are
\[
\dot{\theta}_{oi} = \text{Proj}(\hat{\theta}_{oi}, \phi_i), \quad i = 1, \ldots, 3 \quad (28a)
\]
\[
\dot{k}_{ij} = \text{Proj}(\hat{k}_{ij}, \phi_{ij}), \quad i, j = 1, 2. \quad (28b)
\]

It can be easily verified that \( \dot{\theta}_{oi}(t) \in \Omega_{\theta}, \hat{k}_{ij}(t) \in \Omega_k, \forall t \geq 0 \). The values of \( a_i, b_i, a_{ij} \) and \( b_{ij} \) are given in simulation study.

We assume that all initial conditions are bounded, in particular, \( \dot{\theta}_{oi}(0) \in \Omega_{\theta}, \hat{k}_{ij}(0) \in \Omega_k, e(0) \in E_0 \), where \( E_0 \) is a compact set. Let
\[
c_1 = \max_{e \in E_0} \frac{1}{2g_1 g_3} e^T P e,
\]
\[
c_2 = \max_{\theta \in \Omega_\theta, \hat{k} \in \Omega_k} \left( \frac{1}{2} \Gamma_{1 \theta}^{-1} \hat{\theta}_{1 \theta}^2 + \frac{1}{2} \Gamma_{2 \theta}^{-1} \hat{\theta}_{2 \theta}^2 + \frac{1}{2} \hat{\theta}_{3 \theta}^2 \right),
\]
\[
c_3 = \max_{k \in \Omega_k, \hat{k} \in \Omega_{k \neq j}, k} \left( \frac{1}{2} \Gamma_{1 \hat{k}}^{-1} \hat{k}_{1 \hat{k}}^2 + \frac{1}{2} \Gamma_{2 \hat{k}}^{-1} \hat{k}_{2 \hat{k}}^2 \right),
\]
\[
c_4 > c_1 + c_2 + c_3.
\]

Considering the admissible ranges of elevator deflection, we need to saturate our control signal \( v_0 \). Define the saturated function as
\[
v_0 = \kappa \text{ sat}(v_0/\kappa), \quad (29)
\]
where \( \kappa \) represents the maximum admissible deflection angle, and \( \text{sat}(\cdot) \) is the saturation function defined by
\[
\text{sat}(y) = \begin{cases} 
-1 & \text{for } y < -1, \\
y & \text{for } |y| \leq 1, \\
1 & \text{for } y > 1.
\end{cases}
\]

In summary, with parameter projection and control saturation function, the adaptive output feedback fault-tolerant altitude controller is given by
\[
v_0 = (\dot{\theta}_{oi} \phi_{oi}(-K \hat{e} + h_d^{(4)} + H_1(e_1 + h_d, \hat{e}_2 + \hat{h}_d, \hat{e}_3 + \hat{h}_d, \hat{e}_4 + h_d^{(3)}))
\]
\[\quad - \dot{\theta}_{o2} \phi_{o2} H_2(e_1 + h_d, \hat{e}_2 + \hat{h}_d, \hat{e}_3 + \hat{h}_d, \hat{e}_4 + h_d^{(3)})/V \cos(\Phi_2(\hat{e}_2 + \hat{h}_d))
\]
\[\quad - \dot{\theta}_{o3} \phi_{o3} (e_1 + h_d, \hat{e}_2 + \hat{h}_d, \hat{e}_3 + \hat{h}_d, \hat{e}_4 + h_d^{(3)})
\]
\[
v_j = \hat{k}_{1j} v_0 + \hat{k}_{2j}, \quad j = 1, 2
\]
\[
\dot{\theta}_{oi} = \text{Proj}(\hat{\theta}_{oi}, \phi_i(\hat{e})), \quad i = 1, \ldots, 3
\]
\[
\dot{k}_{ij} = \text{Proj}(\hat{k}_{ij}, \phi_{ij}(\hat{e}, v_0^i)), \quad i, j = 1, 2.
\] 

(31)

4.3. Stability analysis

Define the observer error as
\[
\xi_i = \frac{e_i - \hat{e}_i}{\epsilon^{\mu - i}}, \quad 1 \leq i \leq n, \quad n = 4.
\]

Let \( \xi = [\xi_1, \ldots, \xi_n]^T \), we represent the closed-loop system in the standard singularly perturbed form
\[
\begin{align*}
\dot{e} &= A_m e + B(K e - H_1 + g_1 H_2 + g_1 f_3 V \cos x_2 \\
&\quad + g_1 g_3 V \cos x_2 u^\epsilon - h_d^{(4)}) \quad \text{(32)} \\
\epsilon \dot{\xi} &= (A - HC)\xi + \epsilon B(-H_1 + g_1 H_2 + g_1 f_3 V \cos x_2 \\
&\quad + g_1 g_3 V \cos x_2 u^\epsilon - h_d^{(4)}), \quad \text{(33)} \\
u^\epsilon &= \sum_{j \neq j_p} d_j (\hat{k}_j v_0^j + \hat{k}_j) + d_{j_p} \bar{u}_{j_p}, \quad \text{(34)}
\end{align*}
\]

where \( j_p \) is the index number of the failed actuator, \( H = [\alpha_1, \alpha_2, \alpha_3, \alpha_4]^T \), \( C = [1, 0, 0, 0] \), and the characteristic equation of \( A - HC \) is Eq. (22), hence it is Hurwitz.

We have the following stability results.

**Theorem 1.** For the system (3), suppose that Assumptions 1–2 are satisfied. The fault-tolerant controller (30) with the adaptive laws (31) ensures the boundedness of all the closed-loop signals, and the mean-square tracking error satisfies the following inequality:
\[
\lim_{T \to \infty} \frac{1}{T} \int_0^T e^T(t) Q e(t) \, dt \leq 2g_1 g_3 k_4 \epsilon,
\]

where \( T \) and \( k_4 \) are positive constants.

The proof of Theorem 1 is presented in Appendix C.

5. Velocity control

In this section, the adaptive feedback linearization control scheme is also applied to regulate the vehicle velocity. Eq. (1a) can be written into
\[
\dot{V} = f_v(x_2, x_3, x_4, V) + g_v(x_2, x_3, V)\beta, \quad \text{(35)}
\]

where
\[
f_v(x_2, x_3, x_4, V) = \begin{cases} 
- \left( \frac{D}{m} + \frac{\mu \sin \gamma}{r^2} \right), & \beta < 1, \\
0.02247 S \cos \alpha \frac{D}{m} - \left( \frac{D}{m} + \frac{\mu \sin \gamma}{r^2} \right), & \beta > 1,
\end{cases}
\]
The adaptive laws are

\[
\dot{\theta}_{v1} = \frac{2m}{0.0257\rho S^2}, \quad \dot{\varphi}_{v1} = \frac{1}{V^2\cos\alpha},
\]

\[
\dot{\theta}_{v2} = \begin{bmatrix}
-0.6450m & -0.0043378m & -0.003772m & 2mu \\
0.0257 & 0.0257 & 0.0257 & 0.0257\rho S
\end{bmatrix}^T,
\]

\[
\dot{\varphi}_{v2} = \begin{bmatrix}
\alpha^2 & \alpha & 1 & \sin\gamma \\
\cos\alpha & \cos\alpha & \cos\alpha & V^2\cos\alpha \alpha^2
\end{bmatrix}^T.
\]

Then the adaptive velocity controller is

\[
\beta = \dot{\theta}_{v1}\varphi_{v1}(-k_ve_v + \dot{V}_d) - \dot{\theta}_{v2}\varphi_{v2}.
\] (37)

The adaptive laws are

\[
\dot{\theta}_{v1} = -\Gamma_{v1}\varphi_{v1}(-k_ve_v + \dot{V}_d)
\]

\[
\dot{\theta}_{v2} = \Gamma_{v2}\varphi_{v2},
\] (38)

where \(k_v\) and \(\Gamma_{v1}\) are positive constant, \(\Gamma_{v2} \in \mathbb{R}^{4 \times 4}\) is a positive definite matrix. It can be easily proven that the velocity tracking error asymptotically converges to zero.

**Remark 1.** By following the similar procedure in deriving the adaptive output feedback fault-tolerant controller for the altitude control subsystem, an adaptive output feedback control for the velocity control subsystem can also be developed.

### 6. Simulation results

In this section, we use simulation results to verify the effectiveness of the proposed adaptive control schemes. It is assumed that the vehicle is at its cruise mode with the initial conditions: \(h = 110,000\) ft, \(V = 15,060\) ft/h, \([\gamma, \theta, \phi] = [0\, \text{rad}, 0.01\, \text{rad}, 0\, \text{rad/s}]\).

The initial values of parameters in altitude controller are chosen as \(\dot{\theta}_{a1}(0) = 10^{13}, \dot{\theta}_{a2}(0) = 5 \times 10^7, \dot{\theta}_{a3}(0) = [-10^{11}, 7 \times 10^4, 1.5 \times 10^{-4}, -3 \times 10^{20}, 2 \times 10^{14}, -8 \times 10^7, -9 \times 10^{14}, 1 \times 10^8, -250]^T, \dot{k}_{a1}(0) = 0.8, \dot{k}_{a2}(0) = -0.08 (j = 1,2)\).

The parameters in altitude controller are chosen as \(\Gamma_{a1} = 1, \Gamma_{a2} = 2 \times 10^8, \Gamma_{a3} = \text{diag}(1, 100, 10^{-8}, 1, 1, 1, 1, 1, 100), \Gamma_{ij} = 0.00000008, \Gamma_{2j} = 0.00000008 (j = 1,2), k_v = 0.01, K = [153.6, 275.2, 118.4, 18.8], Q = \text{diag}[5, 5, 5, 5]\).

The maximum and minimum values of parameters in Eqs. (26) and (27) are chosen as \(a_1 = 10^4, b_1 = 10^{15}, a_2 = 10^9, b_2 = 10^9, a_3 = [-3 \times 10^{11}, 10^4, 10^{-5}, -10^{21}, 10^{14}, -10^8, -10^{15}, 1 \times 10^7, -500], b_3 = [-10^{10}, 10^5, 3 \times 10^{-4}, -2 \times 10^{20}, 4 \times 10^{14}, -10^7, -5 \times 10^{14}, 2 \times 10^8, -50], a_{11} = 0.1, b_{11} = 1, a_{12} = 0.5, b_{12} = 1.5, a_{21} = -0.1, b_{21} = -0.01, a_{22} = -0.2, b_{22} = -0.05\).
Considering the maximum deflection angle that can be provided by the elevator, we choose \( \kappa = 0.3 \) rad.

The initial values of parameters in velocity controller are chosen as \( \hat{\theta}_v(0) = 0, \hat{\theta}_p(0) = [0, 0, 0, 0]^T \).

The parameters in velocity controller are chosen as \( \Gamma_v = 0.001, \Gamma_p = \text{diag}[0.0008, 0.0008, 0.0008, 0.0008] \).

The parameters in high-gain observer are set as \( \varepsilon = 0.02, \alpha_1 = 10, \alpha_2 = 35, \alpha_3 = 50, \alpha_4 = 24 \).

The simulations are done for three different cases.

Case 1: All actuators are normal. The desired altitude command is generated from the following first-order filter:

\[
\frac{h_d}{h_c} = \frac{1}{5s + 1}, \quad h_c = 2000 \text{ ft}
\]  

The tracking response to a 2000-ft step-altitude command for the nominal model is shown in Fig. 1. It is observed that the altitude converges to the desired value in a short time while the velocity remains almost unchanged. The responses of flight-path angle \( \gamma \), pitch angle \( \theta_p \), pitch rate \( q \) and angle-of-attack \( \alpha \) are shown in Fig. 2. It can be seen that all the state variables are bounded. The controller parameter adaption is shown in Fig. 3. The estimated tracking error tracks the actual one after a short time in Fig. 4. In Fig. 5, it is shown that the thrust component \( T \sin \alpha \) is far less than the lift \( L \), so that Assumption 1 is reasonable.

Case 2: In this case, we consider the second elevator is stuck at 100 s:

\[
u_1(t) = v_1(t) \\
u_2(t) = \begin{cases} v_2(t), & 0 < t < 100 \text{ s}, \\ 0.1 \text{ rad}, & t \geq 100 \text{ s}. \end{cases}
\]  

Fig. 1. Adaptive altitude tracking response in Case 1.
The simulation results for 2000-ft step change in altitude are shown in Figs. 6–9. It can be seen from Fig. 6 that the altitude $h$ can track the desired value with parameter uncertainties and actuator faults. It also indicates that when one of the two elevators is stuck at some fixed position at 100 s, the other one can accommodate the effect caused by the faulty elevator and make the output achieve the desired performance after some transient response. All the state variables remain bounded in spite of the effects of actuator faults in Fig. 7. The fault-tolerant controller parameter adaptation is shown in Fig. 8.
Case 3: We consider velocity tracking under the elevator fault that is the same as in Case 2. The desired velocity $V_d$ is obtained from the following first-order filter:

$$\frac{V_d}{V_c} = \frac{1}{2s + 1}, \quad V_c = 100 \text{ ft/s}. \quad (41)$$

Fig. 10 presents the trajectories of the velocity and altitude, and elevator angle and the throttle, respectively. The velocity $V$ can track the desired velocity despite the elevator fault.
Fig. 11 shows the responses of flight-path angle $\gamma$, pitch angle $\theta_p$, pitch rate $q$ and angle-of-attack $\alpha$.

Remark 2. In the simulation study, we only show the dynamic performances when one of the elevators is stuck at 0.1 rad. In fact, our designed adaptive fault-tolerant controller can accommodate the effect caused by the faulty elevator which is stuck between $-0.3$ rad and 0.3 rad, which is the admissible range for elevator deflection.
This paper develops a novel adaptive output feedback fault-tolerant controller for hypersonic air vehicles in the presence of unknown parameters, external disturbances and uncertain actuator failures. The nominal controller is designed using feedback linearization technique. When all the failure information, including which elevator fails, the failure time instant and the magnitude of the fault, are required, both the necessary matching conditions for the existence of such a fault-tolerant controller and the fault-tolerant controller structure are derived. When the actuator failure parameters are unknown, an adaptive fault-tolerant version is proposed. Parameter adaptive laws are designed to deal with uncertainties both in the dynamic model and the actuator failures. A projection operator is also introduced to guarantee the boundedness of parameters in the process.
of adaptation. Considering the fact that some states are difficult to measure, the high-gain observer technique is used to derive the output feedback controller. The closed-loop longitudinal dynamics under the proposed adaptive fault-tolerant controller is proven to be bounded and simulation studies are presented to show the effectiveness of the proposed scheme.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (61374116, 61273171), the Fundamental Research Funds for the Central Universities (No. NE2014202), and the Foundation of Graduate Innovation Center in NUAA (kjj201421).
Appendix A. Nonlinear functions in Eq. (9)

The nonlinear functions in Eq. (34) are given as follows:
\[
A_0 = -V \cos x_2 f_{10}^2 - 3V \sin x_2 f_{10} f_{20} + V \cos x_2 (f_{30} + f_{10} f_{31} + f_{10} f_{32} + f_{10} f_{33} + f_{21} f_{33}) \\
A_1 = (3V \cos x_2 f_{10}^2 + 6V \sin x_2 f_{10} f_{20} - V \cos x_2 (f_{20} f_{33} + f_{31} + 2f_{10} f_{32})) (x_2 - x_3) \\
+ 3V \sin x_2 f_{10}^2 + V \cos x_2 (-f_{10} f_{33} + f_{33} x_4 - f_{10} f_{20} - f_{21}) \\
A_2 = (-3V \cos x_2 f_{10} - 3V \sin x_2 f_{20} + V \cos x_2 f_{32}) (x_2 - x_3)^2 \\
+ (-6V \sin x_2 f_{10} + V \cos x_2 (f_{33} + f_{20})) (x_2 - x_3) + V \cos x_2 (f_{10} - x_4) \\
A_3 = V \cos x_2 (x_2 - x_3)^3 + 3V \sin x_2 (x_2 - x_3)^2 - V \cos x_2 (x_2 - x_3). \\
f_{10} = \frac{-(\mu - V^2 r)}{V r^2} \cos x_2 \\
f_{20} = \frac{\mu}{V r} \sin x_2 \frac{r}{r}, ~ f_{21} = \left(\frac{\mu}{r} - \frac{V^2}{2}\right) \sin 2x_2 \frac{r}{r^2} \\
f_{30} = \frac{V \sin 2x_2 \sin x_2}{r^3} \left(\frac{-3\mu}{r} + \frac{V^2}{2}\right) \sin^2 x_2 \frac{r}{r^2} \\
f_{31} = -\frac{\mu}{r^3} \sin^2 x_2 + \left(\frac{\mu}{V r} - \frac{V}{2}\right) \frac{2V \cos 2x_2}{r^2} \\
f_{32} = \left(\frac{\mu}{V r^2} - \frac{V}{r}\right) \cos x_2, ~ f_{33} = \left(\frac{\mu}{V r^2} - \frac{V}{r}\right) \sin x_2.
\]

Appendix B. Nonlinear functions in Eqs. (23) and (24)

The nonlinear functions in Eqs. (23) and (24) are given as follows:
\[
\Phi_1 = e_1 + h_d \\
\Phi_2 = \arcsin \left(\frac{e_2 + h_d}{V}\right) \\
\Phi_3 = \frac{e_3 + h_d}{V \cos x_2} + \frac{\mu - V^2 r}{V r^2} \cos x_2 \\
\Phi_4 = \frac{e_4 + h_d^{(3)}}{V \cos x_2}
\]
Our analysis is to establish that there is a short transient period during which the fast variables are bounded subset of the region of attraction.

Now let us turn to the fast equation and study its solution over the interval $[0, T_2)$. We assume that all initial conditions are bounded, in particular, $\vartheta(0), \hat{\vartheta}(0) \in \Omega, e(0) = 0$, $e(t) \in E = \{e^T P e \leq c_4\}$. Let $R = E \times \Omega \times \Omega$, since $V(e(0), \vartheta(0), \hat{\vartheta}(0)) \leq c_1 + c_2 + c_3 < c_4, u^s$ is bounded, there exists a finite time $T_2$, such that $V(e(t), \vartheta(t), \hat{\vartheta}(t)) \leq c_4$ for all $t \in [0, T_2)$.

Now let us turn to the fast equation and study its solution over the interval $[0, T_2)$, taking the Lyapunov candidate $W = \xi^T P \xi$, where $P = P^T > 0$ is the solution of the Lyapunov equation $\dot{P}(A - H C) + (A - H C)^T P = -I$. Let $| - H_1 + g_1 H_2 + g_1 f_3 V \cos \theta_2 + g_1 g_3 V \cos \theta_2 u^s - \dot{\hat{\vartheta}}_2 | \leq k_2, \forall (e(0), \vartheta(0), \hat{\vartheta}(0)) \in R$.

When $W > \epsilon^2 \beta_2, \beta_2 = 16 \lambda_{\max}(P) P \| B \|^2 k_2^2 / \lambda_{\min}(P)$, we have

\[ W = \xi^T P \xi + \xi^T P \xi \]

\[ = - \frac{1}{\epsilon} \xi^T P \xi + 2 \xi^T P B \left( -H_1 + g_1 H_2 + g_1 f_3 V \cos \theta_2 + g_1 g_3 V \cos \theta_2 u^s - \dot{\hat{\vartheta}}_2 \right) \]

\[ \leq - \frac{1}{\epsilon} \xi^T P \xi + 2 k_2 \| B \| \| \xi \| \]

\[ \leq - \frac{1}{\epsilon} \frac{W}{\lambda_{\max}(P)} + 2 k_2 \| B \| \sqrt{\frac{W}{\lambda_{\min}(P)}} \]

\[ \leq - \frac{1}{2 \epsilon} \frac{W}{\lambda_{\max}(P)}. \quad (C.1) \]

We have $W(t) \leq W(0) e^{-\gamma_2} \leq (k_3 / \epsilon^{2n-2}) e^{-\gamma_2} = 1/(2 \lambda_{\max}(P))$, for $k_3 > 0$. Let $\epsilon_1$ be small enough such that

\[ T_1(\epsilon) = \frac{\epsilon}{\gamma_2} \ln \left( \frac{k_3}{\beta_2 \epsilon^{2n-2}} \right) \leq \frac{1}{2} T_2, \quad \forall \epsilon \in (0, \epsilon_1]. \quad (C.2) \]
This is possible since the left-hand side of the above inequality tends to zero as $\varepsilon \to 0$. Hence, for all $\forall \varepsilon \in (0, e_1)$, there exist $T_1 \leq \frac{1}{2} T_2$ such that $\forall t \in [T_1, T_2)$, $W(t) \leq \varepsilon^2 \beta_2$, and this implies that $\| \xi \|$ is of order $O(\varepsilon)$.

2. In the second part of the analysis, we study (31), (32) over the time interval $[T_1, T_3)$, where $T_3 \geq T_2$ is the first time $(e, \hat{\theta}, \hat{k})$ exits from the set $R_s$. Using the fact that $\| \xi \|$ is of order $O(\varepsilon)$. Taking Lyapunov candidate as

$$V = \frac{1}{2g_1g_3} e^T Pe + \frac{1}{2} \Gamma_{o1}^{-1} \tilde{\theta}_{o1}^2 + \frac{1}{2} \Gamma_{o2}^{-1} \tilde{\theta}_{o2}^2 + \frac{1}{2} \Gamma_{o3}^{-1} \tilde{\theta}_{o3}^2 + \sum_{j \neq p} \frac{|d_j|}{2} \Gamma_{kj}^{-1} \tilde{\kappa}_{kj}^2$$

$$+ \sum_{j \neq p} \frac{|d_j|}{2} \Gamma_{kj}^{-1} \tilde{\kappa}_{kj}^2.$$  

(C.3)

The time derivative of $V$ is

$$\dot{V} = \frac{1}{2g_1g_3} (\dot{e}^T Pe + e^T \dot{P} \dot{e}) + \Gamma_{o1}^{-1} \tilde{\theta}_{o1} \dot{\hat{\theta}}_{o1} + \Gamma_{o2}^{-1} \tilde{\theta}_{o2} \dot{\hat{\theta}}_{o2} + \Gamma_{o3}^{-1} \tilde{\theta}_{o3} \dot{\hat{\theta}}_{o3}$$

$$+ \sum_{j \neq p} \frac{|d_j|}{2} \Gamma_{kj}^{-1} \tilde{\kappa}_{kj} \dot{\hat{\kappa}}_{kj} + \sum_{j \neq p} \frac{|d_j|}{2} \Gamma_{kj}^{-1} \tilde{\kappa}_{kj} \dot{\hat{\kappa}}_{kj}$$

$$= -\frac{1}{2g_1g_3} e^T Qe + e^T PB \left( \dot{\theta}_{o1}\varphi_{o1}(-Ke + H_1 + h_d^{(4)}) - \dot{\theta}_{o2}\varphi_{o2}H_2 \right.$$

$$+ \dot{\theta}_{o3}\varphi_{o3} \cos x_2 + V \cos x_2 \sum_{j \neq p} d_j (\tilde{k}_{j1}v_0^j + \tilde{k}_{j2}) \bigg) + \Gamma_{o1}^{-1} \tilde{\theta}_{o1} \dot{\hat{\theta}}_{o1} + \Gamma_{o2}^{-1} \tilde{\theta}_{o2} \dot{\hat{\theta}}_{o2}$$

$$+ \Gamma_{o3}^{-1} \tilde{\theta}_{o3} + \sum_{j \neq p} \frac{|d_j|}{2} \Gamma_{kj}^{-1} \tilde{\kappa}_{kj} \dot{\hat{\kappa}}_{kj} + \sum_{j \neq p} \frac{|d_j|}{2} \Gamma_{kj}^{-1} \tilde{\kappa}_{kj} \dot{\hat{\kappa}}_{kj}$$

$$\leq -\frac{1}{2g_1g_3} e^T Qe + \|D(e)\xi\| \cdot \|PB\| \cdot \left| \dot{\theta}_{o1}\varphi_{o1}(-Ke + H_1 + h_d^{(4)}) - \dot{\theta}_{o2}\varphi_{o2}H_2 \right.$$

$$+ \dot{\theta}_{o3}\varphi_{o3} \cos x_2 + V \cos x_2 \sum_{j \neq p} d_j (\tilde{k}_{j1}v_0^j + \tilde{k}_{j2}) \bigg)$$

As $(e, \hat{\theta}, \hat{k}) \in R_s$ during $[T_1, T_3)$, $\| \xi \|$ is of order $O(\varepsilon)$ is bounded, $|\dot{\theta}_{o1}\varphi_{o1}(-Ke + H_1 + h_d^{(4)}) - \dot{\theta}_{o2}\varphi_{o2}H_2 + \dot{\theta}_{o3}\varphi_{o3} \cos x_2 + V \cos x_2 \sum_{j \neq p} d_j (\tilde{k}_{j1}v_0^j + \tilde{k}_{j2})|$ is bounded, we obtain

$$\dot{V} \leq -\frac{1}{2g_1g_3} e^T Qe + k_4 \varepsilon$$

$$\leq -\frac{1}{2g_1g_3} c_0 e^T Pe + k_4 \varepsilon$$

$$= -c_0 \left( V - \left( \frac{1}{2} \Gamma_{o1}^{-1} \tilde{\theta}_{o1}^2 + \frac{1}{2} \Gamma_{o2}^{-1} \tilde{\theta}_{o2}^2 + \frac{1}{2} \Gamma_{o3}^{-1} \tilde{\theta}_{o3}^2 + \sum_{j \neq p} \frac{|d_j|}{2} \Gamma_{kj}^{-1} \tilde{\kappa}_{kj}^2 \right) \right).$$
\[ + \sum_{j \neq j_p} \left| d_j \right| \frac{1}{2} \Gamma^{-1}_{ij} k_{2j}^2 \right) + k_4 \epsilon \]

\[ = -c_0 V + \frac{1}{2} c_0 \left( \Gamma_{o1}^{-1} \theta_{o1}^2 + \Gamma_{o2}^{-1} \theta_{o2}^2 + \theta_{o3}^2 \Gamma_{o3}^{-1} \theta_{o3} + \sum_{j \neq j_p} \left| d_j \right| \Gamma^{-1}_{ij} k_{1j}^2 \right) + k_4 \epsilon \]

\[ \leq -c_0 V + c_0 (c_2 + c_3 + k_4 \epsilon) \quad \text{(C.4)} \]

where \( c_0 = \lambda_{\min}(Q)/\lambda_{\max}(P) \), \( D(\epsilon) \) is a diagonal matrix with \( \epsilon^{n-i} \) as the \( i \)th diagonal element. It shows that when \( V > c_2 + c_3 + k_4 \epsilon/c_0 \), then \( \dot{V} < 0 \). So for sufficiently small \( \epsilon \), the set \( \{ V \leq c_4 \} \cap \{ \dot{\theta} \in \Omega_0 \} \cap \{ \dot{k} \in \Omega_0 \} \) is a positively invariant set. Thus the trajectory is trapped inside \( R_s = E \times \Omega_0 \times \Omega_0 \). Therefore, \( T_3 = \infty \), and we conclude that all the state variables are bounded for all \( t > 0 \) and \( \xi = O(\epsilon) \) for all \( t > T_1 \). From the above proof, We obtain \( \dot{V} \leq -(1/2g_1g_3) \epsilon^2 Qe + k_4 \epsilon \). Integrating it from \( t = 0 \) to \( t = T \) yields

\[ V(T) - V(0) \leq -\frac{1}{2g_1g_3} \int_0^T \epsilon^2 Qe(t) \, dt + k_4 \epsilon T \]

since \( V(T) \geq 0 \),

\[ \frac{1}{T} \int_0^T \epsilon^2 Qe(t) \, dt \leq 2g_1g_3 \left( \frac{1}{T} V(0) + k_4 \epsilon \right) \]

\[ \lim_{T \to \infty} \frac{1}{T} \int_0^T \epsilon^2 Qe(t) \, dt \leq 2g_1g_3 k_4 \epsilon. \quad \square \]

References