An optical continuous phase FSK modulation scheme with an arbitrary modulation index over long-haul transmission fiber link

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A R T I C L E   I N F O

Article history:
Received 29 September 2011
Accepted 30 January 2012
Available online 12 February 2012

Keywords:
Long haul optical fiber communication
Continuous phase frequency shift keying (CPFSK)
Modulation formats
Spectral efficiency

A B S T R A C T

This paper presents an optical continuous phase frequency shift keying (CPFSK) modulation scheme with an arbitrary modulation index. The detailed principle on the optical CPFSK generation is derived and analyzed, which includes the special case of the minimum-shift keying (MSK) with a modulation index $h=1/2$. The differential detection and the coherent detection of CPFSK are also depicted. The performances of the four kinds of the optical CPFSK modulated system with a 40 Gb/s modulation rate whose modulation index are $h=1/2$, $h=2/3$, $h=3/4$ and $h=1$ are simulated via the spectral efficiency and the receiver sensitivity over fiber link respectively. In addition, comparison with the differential phase shift keying (DPSK) is taken. Through the calculation of the spectral efficiency of each modulation formats, CPFSK has higher spectral efficiency than DPSK with the same optical devices. The transmission performances of our CPFSK over the fiber link change better as the modulation index increases under the condition of the first order dispersion of the fiber link is completely compensated. Through simulations, a 1200 km transmission distance can be achieved with a modulation index $h=1$.

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1. Introduction

A series of phase shift keying (PSK) modulation formats are demonstrated as advanced modulation formats in optical fiber transmission systems. Constant envelop modulation formats such as differential phase shift keying (DPSK) and differential quadrature phase shift keying (DQPSK) are two common modulation formats [1–5]. However, the phase of these modulation formats is not continuous which can result in some phase shifts and chirp during the transmission procedure, that may cause bit errors.

Minimum-shift keying (MSK) modulation exhibits the properties of constant amplitude and continuous phase in the bit slot and between the consecutive bits. In order to generate a high speed MSK signal at 10 Gb/s and above, a sufficient amount of schemes have been reported [6–11]. But MSK modulation format is only one kind of continuous phase frequency shift keying (CPFSK) modulation formats. The modulation index of MSK is $h=1/2$. Except for the optical MSK modulation format, a few researchers have introduced the optical CPFSK with other modulation index [12–14]. A theoretical analysis on the performance of CPFSK has been presented [12]. The external FSK modulator can generate the CPFSK whose modulation index is 1 [13]. The quaternary CPFSK system of $h=1/6$ has been used to compare the performance on nonlinear effect [14]. However, an optical CPFSK modulation system with an arbitrary modulation index has not been proposed.

Therefore, we propose an optical CPFSK modulation scheme with an arbitrary modulation index which includes the special case of MSK modulation scheme. We can choose different modulation indexes to meet different levels of spectral efficiency.

In this paper, the optical CPFSK transmitter with an arbitrary modulation index is proposed in Section 2. In Section 3, two configurations of CPFSK detectors (differential direct detection and coherent detection) are depicted. Section 4 simulates the performance of CPFSK system on frequency sensitivity and receiver sensitivity over fiber link. The conclusions are given in Section 5.

2. Principle of CPFSK with an arbitrary modulation index

Fig. 1(a) shows the configuration of the proposed optical CPFSK transmitter with an arbitrary modulation index.

In Fig. 1(a), there is a Mach–Zehnder modulator (MZM1) in the upper arm, while another Mach–Zehnder modulator (MZM2) in the lower arm. There is also a phase shifter in the lower arm to make two arms have 90° phase difference. The input waveform of MZM in CPFSK transmitter is shown in Fig. 1(b). The electrical triangular wave is used to drive the MZM to generate the ideal optical cosine signal. The output waveform of MZM in CPFSK transmitter is shown in Fig. 1(c). The period of triangular wave changes by the modulation index to adjust the frequency shift. The input waveform of MZM2 needs to perform a translation 7/2h to the right to make the generation cosine signal of the lower arm have 90° phase difference. The original data stream is put in MZM2 to achieve the frequency shift.
of modulation signal. The input waveform of MZM1 and MZM2 need to perform the same translation \( \tau(h) \) to the left to make the phase of modulation signal continuous.

In Fig. 1, the output of MZM1 can be expressed as

\[
E_{\text{out}}(t) = \frac{1}{\sqrt{2}} E_{\text{in}}(t) \cos nh/t = |E_0| e^{j2\pi f_0t} \cos nh/t
\]

(1)

where \( f_0 \) is the optical carrier frequency, \( E_{\text{in}} \) is the amplitude of optical field, \( T \) is the bit duration and \( h \) is the modulation index. The output of MZM2 can be expressed as

\[
E_{\text{out}}(t) = \frac{1}{\sqrt{2}} E_{\text{in}}(t) \cos nh(t - T/2h)/T e^{j2\pi f_0t} = |E_0| e^{j2\pi f_0(t-T)/T} \sin nh/t
\]

(2)

where \( a(t) \) is the bit stream with ‘1’ or ‘0’ within its bit duration, corresponding to the original data in Fig. 1 (a). So the output of the CPFSK modulator can be written as

\[
E_{\text{out}}(t) = E_{\text{CPFSK}}(t) = \frac{1}{2} |E_0| e^{j2\pi f_0(t+b(t)) nh/t}
\]

(3)

where \( b(t) \) is the bipolar bitstream of the original data with 1 or \(-1\) within its bit duration.

From the input of MZM1 and MZM2 in Fig. 1(a), we can see the input waveform of them has the same left translate amount \( \tau(h) \). In Fig. 1(b), we know the period of \( \varphi(t) \) is \( 2T/h \). The left translate amount \( \tau(h) \) is expressed as follows

\[
\tau(h) = nh/T = nh/2T
\]

(4)

where \( n \) is the number of prior 0’s (or 1’s) when the current bit is 1 (or 0). The expression of \( \tau(h) \) can make the phase of modulation signal continuous.

When the modulation index is a rational number, it can be written as \( h = p/m \) where \( p/m \) is an irreducible fraction. Fig. 2 shows the configuration of achieving the left translate amount \( \tau(h) \) when the denominator of modulation index is known.

In Fig. 2, data \( B \) is the remainder resulting from the division of accumulated data \( A \) by the modulus \( m \). Then data \( B \) divides into \( m \) branches. Comparing the data of adjacent branch, \( n \) in Eq. (4) can be easily achieved.

Especially, data \( B \) is the differential coding format of data \( A \) when the modulation index \( h = 1/2 \). Two branches are the odd and even bit streams which are the input of MZM1 and MZM2 in Fig. 1. The left translate amount \( \tau(h) \) has only two possible values: 0 or \( 2T \) (which is half the period of \( \varphi(t) \) in Fig. 1(b)). The modulation system equals to MSK.

3. Receiver architectures of our optical CPFSK

3.1. The differential detection

Considering the complexity of the receiver and the phase characteristic of CPFSK signal, differential detection is the first widespread receiver technology. The differential detection architecture is illustrated in Fig. 3.

The receiver consists of a \( \tau = T/2h \) delay interferometer (DI) with \( \pi/2 \) phase shift between two arms and a balanced receiver. We recall that the phase difference between two time instants \( mT \) and \((m + 1)T \), \( m \in \{0, \pm 1, \pm 2, \ldots\} \) is either \( -\pi h \) for bit 0 or \( \pi h \) for bit 1 in the time range \( mT \leq t < (m + 1)T \). Thus, the CPFSK signal can be demodulated differentially using the receiver in Fig. 3. After the demodulation, the output current of the receiver becomes

\[
I_0(t) = \frac{E_0^2}{4} \sin \Delta \varphi(t)
\]

(5)

where

\[
\Delta \varphi(t) = \varphi(t) - \varphi(t - \tau)
\]

(6)

The determination of \( \tau \) in Fig. 3 is according to the modulation index. When the modulation index has different values, \( \tau \) would be different. Fig. 4 shows the relationship between \( \tau \) and the modulation index on the reference circle.

The maximum of \( \tau \) is \( T \), otherwise the receiver can not detect the original bit. When the modulation index is given, the maximum absolute value of Eq. (6) is known. It can be expressed below as

\[
\max |\Delta \varphi(t)| = \pi h n
\]

(7)

Red arcs in Fig. 4 are the phase traces from \(-\pi h T \) to \( \pi h T \). When \( 0 \leq \tau < T, \Delta \varphi(t) = \pm \pi h n T \). The positive value represents the bit 1 and negative one represents the bit 0. The two symmetry points on the reference circle remarks the bit 1 and bit 0. The distance of the two symmetry points is \( 2 \sin[\Delta \varphi(t)] = 2 \sin[\pi h n T] \) on the reference circle. When \( \Delta \varphi(t) = \pm \pi n/2 \), the distance can achieve the maximum.

When \( h < 0.5 \), \( \max |\Delta \varphi(t)| < \pi/2 \), the distance of the two symmetry points cannot achieve the maximum thus the frequency shift is less than the minimum frequency shift. When \( h = 0.5 \), \( \max |\Delta \varphi(t)| = \pi/2 \), the distance of the two symmetry points can just achieve the maximum when \( \tau \) reaches the maximum value \( T \). When \( 0.5 < h \leq 1 \), the distance of the two symmetry points can achieve the maximum when \( \tau = T/2h \).
3.2. Coherent detection

Coherent detection can similarly recover the full information in the optical complex amplitude. A block diagram of the coherent receiver is shown in Fig. 5.

In the receiver side, the CPFSK signal is received with an offline digital coherent receiver designed for the CPFSK format. The I and Q components of the received signal project to a local oscillator (LO) light is detected with a 90-degree hybrid coupler followed by two pairs of detectors. The photocurrents from the photodiodes are sampled at the timing of middle point of each bit via amplitude-digital convectors (ADCs). Several symbols with equal spacing on the circle would appear on the IQ map similar to a constellation of phase shift keying (PSK) which is shown in Fig. 6.

This baseband signal is then submitted to the finite impulse response (FIR) filter to estimate the carrier phase. Operation of difference between estimated carrier phase can recover the original bit stream.

4. Simulations and discussions

The schematic for simulation system is shown in Fig. 7.

Generally, the transmission bit-rate is 40 Gb/s, the laser frequency is 193.1 THz, laser line-width is 100 kHz, the fiber link is several spans of 40 km standard signal mode fiber (SSMF) and 8 km dispersion compensate fiber (DCF), the first order dispersion at the optical frequency $f_c = 193.1$ THz can be compensated completely. The dispersion slope of the SMF is 0.08 ps/nm² and the nonlinear index coefficient is $\gamma = 2.6 \times 10^{-20} m^2/W(A_{eff} = 80 \mu m^2)$. The optical filter used in the receiver module has a bandwidth that is equal to 100 GHz for all the five different modulation formats including CPFSK of four different modulation indexes as well as DPSK.

First we present the feature of spectrum in different modulation formats, as shown in Fig. 8.

The main lobe of the spectrum of CPFSK with $h = 1/2$ is $3/4$ of the spectrum of DPSK and the main lobe of the spectrum of CPFSK with $h = 1$ is $3/2$ of the spectrum of DPSK. The reduction of the side lobes of the spectrum of CPFSK becomes more slowly as the modulation index becomes larger. The width of the main lobe and side lobe of the spectrum of CPFSK with given modulation index is shown in Fig. 9.

The width of the main lobe is $1 - h/2$, the width of odd side lobes is $h$ and the width of even side lobes is $1 - h$. Table 1 shows the spectral efficiency of each modulation formats.

The spectral efficiency is defined as the ratio of unit one to the width of the 99% energy of the signal on the normalized frequency axis. The main lobe of the spectrum of DPSK has only 96.5% energy of the signal. The reduction of the side lobes is more slowly than other four CPFSK modulation formats in Table 1.

![Fig. 2. Configuration of achieving the left translate amount $\tau(h)$.](image)

![Fig. 3. Proposed optical CPFSK receiver configuration.](image)

![Fig. 4. (a). The relationship between $\tau$ and the modulation index when $h = 0.5$, (b). The relationship between $\tau$ and the modulation index when $h = 0.5$, (c). The relationship between $\tau$ and the modulation index when $0.5 < h \leq 1$.](image)

![Fig. 5. Proposed optical CPFSK receiver configuration.](image)
efficiency of DPSK is the lowest one in Table 1. The width of the main lobe of the spectrum of CPFSK becomes narrower but the main lobe has less energy as the modulation index becomes larger. So the spectral efficiency of CPFSK becomes lower as the modulation index becomes larger [15].

We measure the receiver sensitivity of the five modulation formats of 40 Gb/s signal over the fiber link transmission. For fair comparison, we use the coherent detection without FIR filter. We use the back-to-back (BTB) receiver sensitivity of each format as the length of fiber link is 0 km. The usage of Monte Carlo (MC) simulation allows to calculate the power penalty at the bit rate error (BER) of $10^{-3}$. According to Fig. 10, we can find that the performance of receiver sensitivity increase with the modulation index becomes larger. Here, the largest distance of fiber link within which the signal can be demodulated at the BER lower than $10^{-3}$ is defined as the limit transmission distance.

Fig. 5. Coherent optical receiver of CPFSK.

Fig. 6. Constellations of sampled CPFSK signal.

Fig. 7. Schematic for CPFSK simulation system with different modulation index.

Fig. 8. Width of the optical spectrum of different modulation formats.

Fig. 9. Optical spectrum of CPFSK with modulation index is specified.
In Fig. 10, we know that the receiver sensitivity of CPFSK with bigger modulation index becomes 1–2 dB better than DPSK as the length of transmission fiber link increases. The limit transmission distance of CPFSK with modulation index \( h = 1/2 \) is 1200 km while that of the other three formats is 1000 km. The transmission performance over the fiber link becomes better as the modulation index increases under the condition of the first order dispersion is compensated completely.

### Table 1

<table>
<thead>
<tr>
<th>Modulation formats</th>
<th>Spectral efficiency (bit/s/Hz)</th>
<th>Width of the main lobe</th>
<th>Width of odd side lobes</th>
<th>Width of even side lobes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPFSK ( (h = 1/2) )</td>
<td>1.67</td>
<td>3/4</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>CPFSK ( (h = 2/3) )</td>
<td>1.18</td>
<td>2/3</td>
<td>2/3</td>
<td>1/3</td>
</tr>
<tr>
<td>CPFSK ( (h = 3/4) )</td>
<td>1.06</td>
<td>5/8</td>
<td>3/4</td>
<td>1/4</td>
</tr>
<tr>
<td>CPFSK ( (h = 1) )</td>
<td>0.84</td>
<td>3/2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>DPSK</td>
<td>0.43</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**5. Conclusion**

In this paper, we proposed an optical CPFSK modulation scheme with an arbitrary modulation index. Through simulations on the 40 Gb/s CPFSK with four modulation index \( h = 1/2, \ h = 2/3, \ \ h = 3/4 \) and \( h = 1 \), we know the CPFSK modulation format has higher spectral efficiency from 0.84 bit/s/Hz to 1.67 bit/s/Hz which is much better than DPSK and a 1200 km transmission distance can be achieved with a modulation index \( h = 1 \). However, we do not evaluate the performance of nonlinear effects on the CPFSK system. That is our next research step.

**Acknowledgment**

This work is supported by the China National Science Foundation Project (under granted: 6177063), the Basic Funds of Research for Central College (No. 2011QN023) and the China National Basic Research Program of China (973 Program: 2010CB328300).

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