ADAPTIVE CONTROL OF A SERVO SYSTEM BASED ON MULTIPLE MODELS
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ABSTRACT

This paper presents an alternative technology for adaptive control of a DC motor servo system based on multiple models. A dynamic mechanical model of the controlled plant is built, where the unmeasurable variables can be estimated by a filter observer. According to the mechanical model, an adaptive controller is designed. Specific attention is given to the jumping parameters in the control process, which motivate the proposition of multiple models, including fixed models, identified model, and adaptive model, to approximate the global dynamic characteristics of the plant model. A model switching rule is proposed to select the optimal model matching the plant, and the identified and adaptive models are reset when switching occurs, minimizing the effect caused by jumping parameters. Simulation results demonstrate that the introduced scheme is superior to the conventional adaptive control in that it yields a significant improvement of transient stability and response speed as well as steady accuracy, guaranteeing better low-speed performance.

Key Words: Parameter estimation, adaptive control, servo systems, filtering, observers.

I. INTRODUCTION

In the control of servo systems, performance such as response speed, accuracy, and stability, usually is influenced considerably by frequently jumping parameters of the system models due to nonlinear dynamic friction, load changes, and other external disturbances. When the parameters are not known exactly during the control process, it is challenging to maintain ideal control performance.

The traditional adaptive control can be adopted to estimate real-time parameters through adaptive laws to achieve fast transient performance and asymptotic tracking [1]. Nevertheless, if abrupt jumps occur, which indicate changing environments, it would take a long time to adjust the parameters, leading to bad transient performance.

In this paper, multiple-model adaptive control (MMAC) is introduced to cope with rapidly varying parameters and improve the adaption process in control systems [2,3]. This approach consists of multiple models and switching rules aimed at selecting the optimal models to represent the dynamics of the controlled plant from the fixed models, identified models and adaptive models, which are proposed to obtain improvement in the transient performance under large parameter uncertainties [4,5]. Meanwhile, an estimator resetting algorithm, based on a model reference adaptive control (MRAC) framework, is proposed to achieve better parameter convergence, where the identified and adaptive models are monitored online to detect parameter vectors that give a negative jump to the Lyapunov function when replacing the estimation provided by the standard adaptation law [6,7]. Moreover, an adaptive controller is presented, where adaptive single-input-single-output fuzzy systems or radial basis function neural networks approximate the discontinuous part of the control signal [8]. The advantage of this scheme is that prior knowledge of the system uncertainties is not required for guaranteeing stability. In the control scheme, a linear controller can assure the boundedness of the input and output signals, while a neutral network nonlinear controller can improve the performance of the system [9,10].

II. PLANT MODELING

The dynamic mathematical model of the DC motor servo system discussed here can be described as:

\[ b\ddot{q} = a\dot{q} + u - T_f - T_{dis} - T_l \]

where \( q, \dot{q} \) and \( \ddot{q} \) are the angular position, angular velocity and angular acceleration, respectively; \( T_f, T_{dis}, \) and \( T_l \) are the friction torque, disturbance torque, and load torque, respectively; \( u \) is the control torque; and \( a \) and \( b \) are the parameters about inertia and electrical features.

According to the LuGre friction model, the dynamic friction torque at low speed could be described as follows:
$$T_f = \sigma_0 z + \sigma_1 \dot{z} + B \dot{q}$$  \hspace{1cm} (2)

$$\dot{z} = \dot{q} - \frac{|\dot{q}|}{g(\dot{q})} z$$  \hspace{1cm} (3)

where \(z\) is the average bristle deflection, an unmeasurable intermediate friction state, and where \(\sigma_0, \sigma_1,\) and \(B\) are the parameters of bristle stiffness, viscous damping, and viscous friction, respectively. The nonlinear function \(g(\dot{q})\) is introduced to describe the Stribeck effect:

$$g(\dot{q}) = T_c + (T_s - T_c) e^{-(q/\dot{q})^2}$$  \hspace{1cm} (4)

where \(T_c\) and \(T_s\) are the Coulomb friction torque and maximum friction torque, respectively, and where \(\dot{q}_c\) is the Stribeck switching velocity, the value of which is determined according to experience. Generally speaking, \(T_s > T_c > 0\); thus, from (4), we know \(T_s > g(\dot{q}) > T_c\).

Substituting (2)–(4) into (1), the following equation is obtained:

$$b \dot{q} = a \dot{q} + u - \sigma_0 z + \sigma_1 \frac{|\dot{q}|}{g(\dot{q})} z - \beta \dot{q} - T_{\text{dis}} - T_l$$  \hspace{1cm} (5)

where \(\beta = \sigma_1 + B > 0\).

Then the dynamic model of the servo system can be simplified as:

$$\dot{\theta} = \frac{1}{b} \left[ u + (\alpha - \beta) \dot{q} + \sigma_1 \frac{|\dot{q}|}{g(\dot{q})} z - \sigma_0 z - T_{\text{dis}} - T_l \right]$$  \hspace{1cm} (6)

Define the angular position and angular velocity as the state variables and the angular position as the system output, i.e., \(x_1 = q, x_2 = \dot{q}, y = x_1\). According to (6), the state space form of the model can be described as:

$$\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = \theta_1 u + \theta_2 x_2 + \theta_3 \frac{|x_2|}{g(x_2)} z - \theta_4 z - d \\
y = x_1
\end{cases}$$  \hspace{1cm} (7)

where \(\theta_1 = \frac{1}{b}, \theta_2 = \frac{a - \beta}{b}, \theta_3 = \frac{\sigma_1}{b}, \theta_4 = \frac{\sigma_0}{b},\) and \(d = \frac{T_{\text{dis}} + T_l}{b}\). These are unknown model parameters and may jump suddenly from one value to another. In other words, the parameters of the model cannot be accurately determined. Here, we make three assumptions for further discussion.

**Assumption 1.** \(\theta \in \Omega_0\), where \(\Omega_0\) refers to the parameter space \([\theta_{\text{min}}, \theta_{\text{max}}]\), \(i = 1, \ldots, 4,\) and \(\theta_{\text{min}}\) and \(\theta_{\text{max}}\) are known. In view of physical realizability, we have \(\theta_{\text{max}} > \theta_{\text{min}} > 0\).

**Assumption 2.** The disturbance \(d\) is bounded, i.e., \(|d| \leq \delta\), where \(\delta\) is known.

**Assumption 3.** The desired trajectory \(x_d\) is sufficiently smooth, which ensures that a second-order differential signal is available and bounded.

### III. DESIGN OF CONTROL STRATEGY

#### 3.1 Architecture of the servo system

In the tracking process of the control system with unknown parameters, chances are that the initial estimation of the parameters is far from the true values and that there may be abrupt jumps of parameters. In addition, the nonlinear disturbances add to the uncertainty of the system. Generally speaking, a higher adaptive gain could accelerate the convergence of the estimation of the unknown parameters. This, however, may result in performance deterioration due to high noise sensitivity of the steady state. Here a multiple-model control scheme with a small adaptive gain is proposed to improve the transient response of the system. The architecture of the scheme is shown in Fig. 1, and it consists of a controller based on a filter observer, fixed models, identified model, adaptive model, and switching rule.

As mentioned above, the nonlinear friction state \(z\) of the LuGre model is unmeasurable, but it can be evaluated by an observer. The adaptive controller is supposed to provide, with the output of \(u\) on the basis of the input \(x_d, \dot{x}_d, \ddot{x}_d\), the feedback state \(x_1, x_2,\) the observed state \(z,\) and the estimated parameters \(\hat{\theta}\).

The fixed models are determined by the average distribution of the interval \([\theta_{\text{min}}, \theta_{\text{max}}]\) with the fixed output \(\dot{\theta}_{f1}, \ldots, \dot{\theta}_{fn}\). The identified model produces the parameter estimate \(\hat{\theta}_{\text{ident}}\) through online identification. The adaptive model updates the output \(\dot{\hat{\theta}}_a\) through the adaptive law. Moreover, the adaptive output \(\dot{\hat{\theta}}_a\) can be reset to the optimal estimation.

The switching rule plays an important role in selecting the model that best fits the controlled plant. That is to say, an optimal estimate \(\hat{\theta}\) is determined to ensure stability and improve the control performance according to a certain index.

In the following sections, the design of all these parts will be described in detail.

#### 3.2 Adaptive controller based on observer

The system position tracking error is defined as:

$$e = x_d - x_1$$  \hspace{1cm} (8)

In order to design the controller for the servo system, the filtered tracking error is defined as:

$$\dot{y} = \dot{e} + \lambda e$$  \hspace{1cm} (9)

where \(\lambda\) is a positive gain. Now, suppose that \(E(s)\) and \(R(s)\) are the Laplace transform of \(e(t)\) and \(\gamma(t)\), respectively. Since \(G(s) = E(s)/R(s) = 1/(s + \lambda)\) is a stable transfer function, if \(\gamma(t)\) converges to zero or a small number exponentially and
given a proper $\lambda$, $e(t)$ will converge as well. Therefore, the problem of tracking $x_d$ is changed into minimizing $\gamma$.

To formulate the control input, taking the time-derivative of (9) and noting (7), we obtain

$$\dot{\gamma} = (\ddot{x}_d + \lambda \dot{e}) - \theta_1 u - \theta_2 x_2 - \theta_3 \frac{|x_2|}{g(x_2)} \gamma + \theta_4 z + d$$

(10)

Since the nonlinear friction state $z$ of the LuGre model is unmeasurable, it is evaluated by an observer and a dual filter. According to (3), the observer is designed as

$$\dot{\hat{z}} = x_2 - \frac{|x_2|}{g(x_2)} \hat{z}$$

(11)

where $\hat{z}$ is the estimation of $z$. Comparing (11) with (3), we obtain:

$$\dot{\tilde{z}} = - \frac{|x_2|}{g(x_2)} \tilde{z}$$

(12)

where $\tilde{z} = z - \hat{z}$ is the estimation error of $z$. A dual auxiliary filter is supposed to estimate the observer error $\tilde{z}$ to accelerate the convergence rate and reduce the influence of the observer error on the system, which is designed as:

$$\dot{\tilde{z}}_0 = - \frac{|x_2|}{g(x_2)} \tilde{z}_0 - k_0 \frac{|x_2|}{g(x_2)} \hat{z}$$

(13)

where $\tilde{z}_0$ and $\tilde{z}_1$ are the estimation of $z$, while $k_0$ and $k_1$ are adjustable filter gains.

Considering the open-loop system described as (10), the controller is designed as:

$$\begin{cases} u_1 = h \gamma \\ u_2 = \frac{1}{\theta_1} \left[ (\ddot{x}_d + \lambda \dot{e}) - \theta_2 \dot{x}_2 - \theta_3 \frac{|x_2|}{g(x_2)} (\ddot{z} + \tilde{z}_0) + \theta_4 (\ddot{z} + \tilde{z}_1) \right] \\ u = u_1 + u_2 \end{cases}$$

(15)

where $\theta_1$, $\theta_2$, $\theta_3$ and $\theta_4$ are the estimations of $\theta_1$, $\theta_2$, $\theta_3$ and $\theta_4$, respectively; $u_1$ is a proportional control term, $u_2$ is an adaptive control term, and $u$ is the composite output of the controller; and $h$ is the adjustable proportional gain, which is given by

$$h = \rho \frac{|x_2|}{g(x_2)} + h_0$$

(16)

where $\rho > 0$ and $h_0 > 0$. Here, let
\[
\begin{aligned}
\rho &= \frac{1}{\theta_{1\min}} \\
(\varphi^T \theta + d - \theta_{10})^T \gamma &\leq \varepsilon
\end{aligned}
\]  

(17)

where \(\varepsilon\) is a small positive parameter and \(\varphi\) is defined as \(\varphi = \left[ u_2, x_2, \frac{|x_2|}{g(x_2)} (\hat{z} + \zeta_0), - (\hat{z} + \zeta_1) \right]^T \).

From (10)–(15), we obtain the following closed-loop equation of the filtered tracking error:

\[
\dot{\varphi} = -\varphi^T \ddot{\theta} - \theta_1 \left[ \frac{|x_2|}{g(x_2)} (\hat{z} - \zeta_0) + \theta_4 (\hat{z} - \zeta_1) \right] - \theta_1 u_1 + d
\]

(18)

where \(\ddot{\theta} = \theta - \hat{\theta}\) is the estimate error of \(\theta\).

### 3.3 Determination of fixed models

In a multiple-model adaptive control system, fixed models have some advantages over adaptive models. From the perspective of computation, adaptive models are inefficient, as their parameters are adjusted dynamically in almost all of the work time. On the other hand, parameter variation with the time of the controlled plant means that the parameters of adaptive models have to be reset. It is possible to guarantee quick identification of the controlled plant only if the parameters are close to the true values. Fixed models do not have the above disadvantages; thus, they can be used in invariant or time-varying systems to improve control speed. Nevertheless, since fixed models can describe exactly only a finite number of environments, control precision cannot be guaranteed. Thus, the number and locations of fixed models are to be discussed.

Note that the unknown parameter spaces are closed and bounded, according to Assumption 1. The simplest way to determine the models is a uniform distribution over the parameter spaces. The parameter vector \(\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T\) is a 4-dimentional vector, in which every element has a minimum and a maximum value, denoting \(\theta_i \in [\theta_{i\min}, \theta_{i\max}], i = 1, \ldots, 4\). We define the width of \(\theta_i\) to be \(W_i = \theta_{i\max} - \theta_{i\min}\) and divide \(W_i\) into \(m_i(i = 1, \ldots, 4)\) equally spaced subintervals, where \(m_i\) is a positive integer. Hence, we take the dividing points of those subintervals as the candidate fixed models. For example, as we know the parameter \(\theta_1 \in [\theta_{1\min}, \theta_{1\max}]\) and the width \(W_1 = \theta_{1\max} - \theta_{1\min}\), set the number of the intervals as \(m_1 = 4\); thus, the size of the interval is \(\frac{1}{4} W_1\) and the intervening values are \(\theta_1 \epsilon [\theta_{1\min} + \frac{1}{4} W_1, \theta_{1\min} + \frac{2}{4} W_1, \theta_{1\min} + \frac{3}{4} W_1]\), indicating there are \(m_1 - 1 = 3\) fixed estimation values of the parameter \(\theta_1\) in its closed space. Similarly, the corresponding fixed values of the other parameters will be determined. The number of the fixed models is given by:

\[
n = \prod_{i=1}^{4} (m_i - 1)
\]

(19)

The space of the fixed models, denoted by \(\Omega_f\), is given by:

\[
\Omega_f = \left\{ \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_f \right\}
\]

\[
= \left\{ \theta_{1\min} + \frac{1}{m_1} W_1, \theta_{1\min} + \frac{2}{m_1} W_1, \ldots, \theta_{1\min} + \frac{m_1 - 1}{m_1} W_1 \right\} \times \left\{ \theta_{2\min} + \frac{1}{m_2} W_2, \theta_{2\min} + \frac{2}{m_2} W_2, \ldots, \theta_{2\min} + \frac{m_2 - 1}{m_2} W_2 \right\} \times \ldots \times \left\{ \theta_{4\min} + \frac{1}{m_4} W_4, \theta_{4\min} + \frac{2}{m_4} W_4, \ldots, \theta_{4\min} + \frac{m_4 - 1}{m_4} W_4 \right\}
\]

(20)

Now we obtain the number and locations of the fixed models. On the one hand, it is commonly accepted that the possibility of closing to the actual system model is greater if the intervals of every unknown parameter are small enough. On the other hand, the number of fixed models and the computational complexity will increase if the intervals are set to be smaller. In practical applications, the intervals should be chosen reasonably with experience.

### 3.4 Determination of identified model

As a special kind of fixed model, the identified model is equivalent to an infinite number of system models. In fact, the identified model effectively can detect the parameter jumps online and improve anti-jamming capacity. Based on the identifier, we can reduce the number of fixed models and improve the transient response of the system at the same time. The architecture of the identified model is shown in Fig. 2.

In order to obtain the identified model, we first construct a low-pass filter with state reset given by:

\[
\begin{aligned}
\Phi(t) &= 0 \\
\dot{\Phi}(t) + k \Phi(t) &= \Phi_0(t) &\text{if } t \neq t_i \\
\end{aligned}
\]

(21)

where \(\Phi_0 = \left[u_2, x_2, \frac{|x_2|}{g(x_2)} (\hat{z} + \zeta_0), - (\hat{z} + \zeta_1)\right]^T\) is the regressor input; \(\Phi = [\phi_1, \phi_2, \phi_3, \phi_4]^T\) is the filter output of \(\Phi_0\),

![Fig. 2. Architecture of the identified model.](https://example.com/fig2.png)
\[ t_0 = 0; \ t_i \text{ with } i = 0, 1, \ldots \text{ denote the switching time for the most appropriate model, at which the estimation state should be reset; and } k \text{ is a positive number to make sure that the filter output can track the corresponding input closely, which is given by} \]
\[ k = k_3 + k_4(t) \]  
(22)

where \( k_3 > 0, k_4(t) \) is a positive function. From (7) and (21), we know the following regression formula is established in the interval \([t_n, t_{n+1}]):\]
\[ \phi^T \theta - g_5 = g_6 \]  
(23)

where \( g_5 \) and \( g_6 \) are the filter outputs of \( d \) and \( \dot{x}_2 \), respectively, which are given by
\[
\begin{align*}
\dot{g}_5(t) &= d(t) & t = t_i & (i = 0, 1, \ldots) \\
\dot{g}_6(t) &= g_5(t) + k_5 g_6(t) & t = t_i & (i = 0, 1, \ldots)
\end{align*}
\]  
(24)

From (23) and (25), we know:
\[
\dot{g}_6 = \dot{x}_2 - k_6 g_2
\]  
(26)

Assume that the parameter vector \( \theta \) does not change in the interval \([t_i, t_{i+1}]), \exists i \in \{0, 1, \ldots\}. \) From (23), we know
\[
\int_{t_i}^{t_{i+1}} \phi \phi^T d\theta = \int_{t_i}^{t_{i+1}} \phi (g_5 + g_6) d\theta
\]  
(27)

Note that the lumped disturbance \( d \) is bounded according to Assumption 2; therefore, \( g_5 \) is bounded. Due to the uncertainty of disturbance \( d \), assume \( g_5 = 0 \). Define the identification matrices as:
\[
P(t) = \begin{cases} 
P_0 & t = t_0 \\
\int_{t_i}^{t} \phi \phi^T d\theta & t < t_i \leq t_{i+1} (i = 0, 1, \ldots)
\end{cases}
\]  
(28)

\[
Q(t) = \begin{cases} 
Q_0 & t = t_0 \\
\int_{t_i}^{t} \phi g_6 d\theta & t < t_i \leq t_{i+1} (i = 0, 1, \ldots)
\end{cases}
\]  
(29)

where \( P_0, Q_0 \in \Omega_\theta \). The identified model can be structured as:
\[
\dot{\theta}_{ident}(t) = P(t)^{-1}Q(t), \ t_i < t \leq t_{i+1} (i = 0, 1, \ldots)
\]  
(30)

where \( \dot{\theta}_{ident} \) is the parameter estimation of the identified model. As (30) implies, \( P(t) \) should be invertible. From (28), we know that \( P(t) \) is a positive semi-definite matrix, so all of its eigenvalues are non-negative. Hence, \( P(t) \) is invertible if and only if all of the eigenvalues are positive. As we know, the determinant of a matrix equals the product of all eigenvalues of the matrix. It is certain that \( P(t) \) is invertible if and only if the determinant is positive.

Here we define a small positive \( \varepsilon \) as the threshold of the determinant in order to avoid numerical problems. \( P(t) \) is invertible if the following condition is met
\[
det(P(t)) \geq \varepsilon_P \]  
(31)

where \( det(P(t)) \) is the determinant of \( P(t) \). In addition, to ensure the parameter estimate effectiveness, another condition is met simultaneously as follows:
\[
P(t)^{-1}Q(t) \in \Omega_\theta \]  
(32)

To sum up, if \( P(t) \) and \( Q(t) \) satisfy the two conditions (31) and (32) at the instant \( t_i \), they are reset to zero, i.e., \( P(t_i) = 0, Q(t_i) = 0 \). Here, we define a flag variable \( sw \) to mark the reset of \( P(t) \) and \( Q(t) \), which is given by:
\[
sw = \begin{cases} 
1, & \text{if } det(P(t)) \geq \varepsilon_P \text{ and } P(t)^{-1}Q(t) \in \Omega_\theta \\
0, & \text{otherwise}
\end{cases}
\]  
(33)

where \( t_{i+1} \) is the first instant when the two conditions are met after \( t_i \) By the reset of the identification matrices, the cumulative error before \( t_i \) is supposed to be eliminated effectively and the identified parameters are estimated using only the recent data. This is the reason this kind of identified model is effective when the parameters jump suddenly.

### 3.5 Determination of adaptive model

The fixed models improve the control speed, the adaptive model is supposed to improve the control accuracy, and the identified model ensures control stability. Thus, the multiple models of fixed models, identified model, and adaptive model simultaneously improve the control speed, accuracy, and stability.

As the discontinuous projection operator can assure that the estimate parameters change within the defined region, it is used as the adaptation law, which is denoted as
\[
Proj_{\theta_{cw}}(\Delta) = \left[ Proj_{\theta_{cw}}(\Delta_1), \ldots, Proj_{\theta_{cw}}(\Delta_i) \right]^T
\]

with
\[
Proj_{\theta_{cw}}(\Delta_i) = \begin{cases} 
0, & \text{if } \hat{\theta}_{ci} = \theta_{i max} \text{ and } \Delta_i > 0 \\
0, & \text{if } \hat{\theta}_{ci} = \theta_{i min} \text{ and } \Delta_i < 0 \\
\Delta_i, & \text{otherwise}
\end{cases}
\]  
(34)

where \( \Delta_i \) is the \( i-th \) component of the vector \( \Delta \).

When the parameter estimation of the identified model is optimal and the reset flag is effective, indicating that \( \dot{\theta}_{ident} \) is better than \( \dot{\theta}_{cw} \), we reset the adaption parameter \( \dot{\theta}_c \) to \( \dot{\theta}_{ident} \). When the parameter estimation of the fixed models is optimal, we reset the adaption parameter \( \dot{\theta}_f \) to \( \dot{\theta}_f \). Otherwise, the adaptation parameter \( \dot{\theta}_c \) is the best estimation. The adaptation law is given by:
\[ \hat{\theta}_a = \hat{\theta}_{\text{ident}} \text{ if } t = t_i, J^* = J(\hat{\theta}_{\text{ident}}) \text{ and } \text{sw} = 1 \]
\[ \hat{\theta}_a = \hat{\theta}^* \text{ if } t = t_i, J^* \neq J(\hat{\theta}_{\text{ident}}) \text{ and } J^* \neq J(\hat{\theta}_a) \]
\[ \hat{\theta}_a = \text{Proj}_{\hat{\theta}}(\Gamma t) \text{ otherwise} \]
\[ (35) \]

where \( \hat{\theta}^* \) represents the best choice according to a certain performance index, which is described in the following section in detail, and \( J^* \) denotes the corresponding performance index; \( J(\hat{\theta}_{\text{ident}}) \) and \( J(\hat{\theta}_a) \) denote the identification and adaptation performance indices, respectively; \( \tau = -\gamma_p \) is an adaptation function; and \( \Gamma > 0 \) is a matrix of learning factor.

### 3.6 Switching rule

Due to multiple models used in the adaptive control, the switching rule is introduced to choose which of the models best fits the controlled plant. The following performance index is expressed as:

\[ J(\Delta^T, t) = \mu_1 e(\Delta^T, t)^2 + \mu_2 \int_{t-t_e}^{t} e(\Delta^T, t)^2 \, dt \]
\[ (36) \]

where \( t_e \) is the length of the time interval for the performance index estimation, \( \mu_1 \) and \( \mu_2 \) are positive constants, and \( e(\Delta^T, t) \) is an estimation error defined as

\[ e(\Delta^T, t) = \Delta^T \phi(t) - g_\theta(t) \]
\[ (37) \]

The model with a small value of \( J \) is chosen as the optimal model \( J^* \), which means

\[ J^* = \min \{ J(\hat{\theta}_{f1}), J(\hat{\theta}_{f2}), \ldots, J(\hat{\theta}_{fn}), J(\hat{\theta}_{\text{ident}}), J(\hat{\theta}_a) \} \]
\[ (38) \]

where \( J(\hat{\theta}_{f1}), J(\hat{\theta}_{f2}), \ldots, J(\hat{\theta}_{fn}) \) are the performance indices of the fixed models, respectively, and \( J(\hat{\theta}_{\text{ident}}) \), \( J(\hat{\theta}_a) \) are the performance indices of the identified model and adaptive model respectively. Therefore, we see the optimal parameter estimation is

\[ \hat{\theta}(t) = \hat{\theta}^*(t) \text{ if } t = t_i \]
\[ \hat{\theta}(t) = \text{Proj}_{\hat{\theta}}(\Gamma t) \text{ if } t_i < t \leq t_{i+1} (i = 0, 1, \ldots) \]
\[ (39) \]

where \( \hat{\theta}^* \) is the corresponding parameter estimation of \( J^* \).

After describing all of the components in the adaptive control system, we show the algorithm flow diagram of the parameter estimation in Fig. 3.

### IV. STABILITY ANALYSIS

**Theorem 1.** If the multiple-model adaptive control scheme based on fixed, identified and adaptive models is applied for the controlled plant described as (7), then the parameter estimation satisfies \( \hat{\theta} \in \Omega_\theta, \forall t \geq 0 \).

From (20), we have:

\[ \Omega_{\theta} = \left\{ \hat{\theta}_{f1}, \hat{\theta}_{f2}, \ldots, \hat{\theta}_{fn} \right\} \subseteq \Omega_{\theta} \]
\[ (40) \]

Consequently, all the fixed parameter estimations vary in the given interval, i.e.

\[ \hat{\theta}_j \in \Omega_{\theta}, \forall t \geq 0, j = 1, 2, \ldots, n \]
\[ (41) \]

As for the adaptation estimation, from (35), we will explain in three cases.

**Case 1.** \( t = t_i, J^* = J(\hat{\theta}_{\text{ident}}), \) and \( \text{sw} = 1 \) in this case,

\[ \hat{\theta}_a = \hat{\theta}_{\text{ident}} \]
\[ (43) \]

From (42), we have \( \hat{\theta}_a \in \Omega_{\theta} \).

**Case 2.** \( t = t_i, \) and \( J^* \neq J(\hat{\theta}_{\text{ident}}), \) and \( J^* \neq J(\hat{\theta}_a) \) in this case,

\[ J^* = J(\hat{\theta}_{fj}), \hat{\theta}^* = \hat{\theta}_{fj}, j = 1, 2, \ldots, n \]
\[ (44) \]

This means one of the fixed parameter estimations is optimal at this instant. From (35), we have:

\[ \hat{\theta}_a = \hat{\theta}^* \]
\[ (45) \]

From (41) and (44), we have \( \hat{\theta}^* \in \Omega_{\theta} \).

**Case 3.** \( t \neq t_i, \) or \( J^* = J(\hat{\theta}_a), \) or \( \text{sw} = 0 \) with \( J^* = J(\hat{\theta}_{\text{ident}}) \) at \( t = t_i \) in this case, \( \hat{\theta}_a \) is updated by:

\[ \hat{\theta} = \text{Proj}_{\hat{\theta}}(\Gamma t) \]
\[ (46) \]

According to the characteristic of the projection operator, if \( \hat{\theta}_a(t_i) \in \Omega_{\theta} \) and the update law is \( \hat{\theta} = \text{Proj}_{\hat{\theta}}(\Gamma t) \) in the time interval \( [t_1, t_2] \), we have:

\[ \hat{\theta}_a(t) \in \Omega_{\theta}, \forall t \leq t \leq t_2 \]
\[ (47) \]

Hence, for all time in this case, we have \( \hat{\theta}_a \in \Omega_{\theta} \). From these three cases, we have:

\[ \hat{\theta}_a \in \Omega_{\theta}, \forall t \geq 0 \]
\[ (48) \]

From (41), (42), and (48), we have:

\[ \hat{\theta} \in \Omega_{\theta}, \forall t \geq 0 \]
\[ (49) \]

Therefore, Theorem 1 is proven.
Theorem 2. Assuming that \(d = 0\), only considering parameter changes, the controlled plant is described by (7), the adaptive controller is designed by (15), and the parameter estimation is updated by (39). Based on the multiple-model adaptive controller, the closed-loop system is asymptotically stable and the tracking error \(\lim_{t \to \infty} e(t) = 0\). The Lyapunov function is selected as:

\[
V_s = \frac{1}{2} \xi^2 + \frac{1}{2} \zeta^2 + \frac{\theta_1}{2k_0} (\zeta - \xi_0)^2 + \frac{\theta_2}{2k_1} (\zeta - \xi_1)^2
\]  

(50)
From (12)–(14) and (18), we have:
\[
\dot{V}_s = \gamma \dot{z} + \dot{z} \dot{z} + \frac{\theta_1}{k_0} (\dot{\zeta} - \dot{\zeta}_0) (\dot{\zeta} - \dot{\zeta}_0) + \frac{\theta_2}{k_1} (\dot{\zeta} - \dot{\zeta}_1) (\dot{\zeta} - \dot{\zeta}_1)
\]
\[
= -\left( \phi^T \dot{h} + \theta_1 h_0 \right) \gamma - \frac{\theta_1}{g(x_2)} \left[ \dot{x}_1^2 + \frac{\theta_2}{k_0} (\dot{\zeta} - \dot{\zeta}_0)^2 + \frac{\theta_3}{k_1} (\dot{\zeta} - \dot{\zeta}_1)^2 \right]
\]
(51)

As stated in Theorem 1, the parameter estimation \( \dot{\theta} \) changes within the region \( \Omega_\theta \) consistently, so \( \dot{\theta} \) is bounded such that \( \rho \) and \( h_0 \) satisfying (17) exist and can be accepted. From (15)–(17), we have:
\[
\dot{V}_s \leq - \frac{|x_1|}{g(x_2)} \gamma^2 \left[ \dot{x}_1^2 + \frac{\theta_2}{k_0} (\dot{\zeta} - \dot{\zeta}_0)^2 + \frac{\theta_3}{k_1} (\dot{\zeta} - \dot{\zeta}_1)^2 \right] - \left( \phi^T \dot{h} + \theta_1 h_0 \right) \gamma \leq - \frac{2|x_1|}{g(x_2)} V_s + \varepsilon
\]
(52)

Therefore, \( V_s \) is bounded, which means:
\[
0 \leq V_s \leq \exp(-\lambda_0 t)V_s(0) + \frac{\varepsilon}{\lambda_0} [1 - \exp(-\lambda_0 t)]
\]
(53)

where \( \lambda_0 = \frac{2|x_1|}{g(x_2)} \). Since \( V_s(t) \) is non-negative and \( \dot{V}_s \) is non-positive, the closed-loop system is asymptotically stable according to Lyapunov stability theorem. An assistant function is selected as:
\[
V_\theta = \frac{1}{2} \dot{\theta}^T \Gamma^{-1} \dot{\theta}
\]
(54)

The switching law of parameter estimation \( \dot{\theta} \) is shown in (39); thus, we have:
\[
\dot{V}_s = \dot{\theta}^T \Gamma^{-1} \dot{\theta} = -\theta^T \Gamma^{-1} \text{Proj}(\Gamma \dot{\theta})
\]
(55)

We define
\[
\dot{V} = V_s + V_\theta
\]
(56)

From (51) and (55), we have:
\[
\dot{V} = -\theta_1 u_1 \gamma - \frac{|x_1|}{g(x_2)} \left[ \dot{x}_1^2 + \frac{\theta_2}{k_0} (\dot{\zeta} - \dot{\zeta}_0)^2 + \frac{\theta_3}{k_1} (\dot{\zeta} - \dot{\zeta}_1)^2 \right]
\]
\[
= -\theta_1 \frac{|x_1|}{\theta_{1 \text{min}} g(x_2)} \gamma^2 - \frac{\theta_1}{\theta_{1 \text{min}} g(x_2)} \left[ \dot{x}_1^2 + \frac{\theta_2}{k_0} (\dot{\zeta} - \dot{\zeta}_0)^2 + \frac{\theta_3}{k_1} (\dot{\zeta} - \dot{\zeta}_1)^2 \right]
\]
(57)

Hence provided that \( h_0 > 0 \), which means \( h_0 \) may not satisfy the condition (17), we still have:
\[
\dot{V} \leq - \frac{|x_1|}{g(x_2)} \gamma^2 - \frac{|x_1|}{g(x_2)} \left[ \dot{x}_1^2 + \frac{\theta_2}{k_0} (\dot{\zeta} - \dot{\zeta}_0)^2 + \frac{\theta_3}{k_1} (\dot{\zeta} - \dot{\zeta}_1)^2 \right]
\]
\[
\leq - \frac{2|x_1|}{g(x_2)} V_s \leq 0
\]
(58)

Comparing (52) with (58), we know that, if \( h_0 > 0 \), the closed-loop system is convergent; if \( h_0 \) further satisfies the condition (17), the closed-loop system is exponentially convergent. From (58), we know that \( \gamma, \tau, \zeta_0 \) and \( \zeta_1 \) are uniformly bounded and we have \( \dot{V} \leq - \frac{2|x_1|}{g(x_2)} \gamma^2 \). It is apparent that \( \gamma \in L_2. \) From (10), we know \( u \) and \( x_2 \) are bounded because of the boundedness of \( \gamma \), so \( \phi, \phi_0 \), and \( \phi \) are bounded. From Assumption 1 and Theorem 1, we know \( \dot{\theta} \in L_\infty \). Barbalat’s lemma is cited to show:
\[
\lim_{t \to \infty} \gamma(t) = 0
\]
(59)

Then the standard linear control theory is cited to show:
\[
\lim_{t \to \infty} e(t) = 0
\]
(60)

Therefore, Theorem 2 is proven. In addition, if \( d \neq 0 \), from (27), we know the identified model is non-sensitive to disturbance. By the reset of the identification matrices, the cumulative error before \( t_i \) is supposed to be eliminated effectively and the identified parameters are estimated only using the recent data. Therefore, the identified model is effective when the parameters jump suddenly.

Fig. 4. Position tracking error of MMAC and CAC in simulation (step signal input).
In the process of simulation, we adopt Equation (1) as the model of the DC motor servo system and Equations (2)–(4) as the model of the friction torque of the system. Setting $a = 3.15, b = 0.35, B = 0.8, \sigma_0 = 11, \sigma_1 = 8.6, T_c = 3, T_s = 5, \dot{\theta}_s = 0.01$, we obtain the true value of $\theta$ as $\theta_1 = 2.86, \theta_2 = -35.86, \theta_3 = 24.57, \theta_4 = 31.43$. Assume the parameters always range within $\theta_1 \in [1, 5], \theta_2 \in [-37, -34], \theta_3 \in [22, 26], \theta_4 \in [30, 33]$. Dividing the ranges of each parameter into three equal parts and selecting the divisions as fixed models, we can get fixed models for each parameter as $\theta_1 = \{2.3, 3.6\}, \theta_2 = \{-36, -35\}, \theta_3 = \{23.3, 24.6\}, \theta_4 = \{31, 32\}$; thus, there are totally $2^4 = 16$ fixed models. Initialize the identification model and the adaptation model both with $\theta_1(0) = 1, \theta_2(0) = -37, \theta_3(0) = 26, \theta_4(0) = 30$, and state variables with $x_1(0) = x_2(0) = z(0) = \dot{z}(0) = \dot{\phi}_1(0)$. Set the controller parameters as $\lambda = 30, \rho = 1, h_0 = 10, k_0 = k_1 = 1, \tau_0 = \tau_1 = 0.1, c_0 = c_1 = 0.01, k = 0.001$, the adaptive learning factor $r = \text{diag}(15, 500, 6000, 3000)$, the invertible determining parameter of the identification matrix $p = 10^{-12}$, and the indicator function parameter $\mu_1 = 1, \mu_2 = 5$.

Assume that there exist transitions of the unknown parameters of the system as follows:

$$
\theta = \begin{cases} 
[2.4, -35, 25, 30.5]^T & t \leq 3 \\
[2.4, -35, 23, 32.5]^T & 3 < t \leq 6 \\
[2.4, -36, 25, 30.5]^T & 6 < t \leq 1
\end{cases}
$$

The third parameter jumps smaller and the fourth one jumps bigger at the instant $t = 3s$, while the first parameter jumps bigger and the second one jumps smaller at the instant $t = 6s$. We adopt ode4 and fixed step size 0.1ms during the simulation.

According to Figs. 4 and 5, when there exist sudden jumps of unknown parameters in the system, it is difficult for the traditional adaptive control to meet the requirements of fast convergence and good transient stability. The multiple-model-based adaptive method, however, is able to respond to the jump of parameters efficiently through

![Fig. 5.](image_url)  
**Fig. 5.** Position tracking error of MMAC and CAC in simulation (sine signal input).

![Fig. 6.](image_url)  
**Fig. 6.** Estimation of parameters.
switching rules, which helps to ensure good performance of transient convergence and steady accuracy.

We can see from Fig. 6 that, in traditional adaptive control, the convergence is not quick enough to adapt to the jump of parameters, while the estimation of MMAC could converge in a jumping style to the real values. Fig. 7 is the switching signals, which help to select the optimal model. Fig. 8 is the control signal, where we can see that, at the instant when the parameters jump, the signal is relatively smooth.

We then build a semi-physical simulation system based on Real-Time Workshop (RTW) and XPC target. This is a system connecting the MATLAB simulation loop on a computer with a real DC motor system. Real-Time Workshop is an important supplementary functional module of MATLAB/Simulink, where code is produced and a program under certain circumstances is generated according to the plant.

After repeated tuning of parameters, we obtain a set of ideal parameters as the control parameters being \( \lambda = 30, \rho = 1, h_0 = 10 \), the filter observer parameters being \( k_0 = k_1 = 1, r_0 = r_1 = 0.1, e_0 = e_1 = 0.01 \), indicator function parameters being \( \mu_1 = 1, \mu_2 = 2 \), the parameter learning matrix being \( \Gamma = \text{diag}(0.5, 0.7, 1, 0.3) \), the filter parameter being \( k = 0.001 \), and the identification matrix parameter \( e_p = 10^{-9} \).

We can see from Figs. 9 and 10 that the proposed approach based on multiple models can provide better performance.
transient performance as well as smaller errors compared with the traditional adaptive method, which is important to improve the low speed performance of the system.

VI. CONCLUSION

In this paper, we pay attention to external disturbances and parameter mutation of a servomechanism and propose an adaptive control method based on multiple models. Fixed models, the identification model, and an adaptive model are built according to the range of system uncertainty, and the model best matching the plant is selected through switching rules, which allow the model-based adaptive controller to respond to the external demand rapidly. Both software and semi-physical simulation results show that the control algorithm has better adaptive capacity, faster dynamic response speed and smaller steady state error. To sum up, the proposed method improves the performance of system at low speeds.

REFERENCES

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