Vulnerability analysis of complementary transportation systems with applications to railway and airline systems in China

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ABSTRACT

Most of existing studies on vulnerability analysis of multiple infrastructure systems mainly focus on negative effects of interdependencies, which mean that failures in one system can propagate to other systems and aggravate the initial damage. In reality, there also exist positive effects of interdependencies, which are shown in complementary systems and mean that if one system fails another system can provide alternative services to satisfy customers’ demands. Different types of transportation systems in a city or country are typical complementary systems. Taking railway and airline systems in China as an example, this paper proposes a network-based approach to model the vulnerability of complementary transportation systems, and based on this model, this paper further introduces a dynamic complementary strength metric, which can help decision makers design or select better complementary topologies from the vulnerability perspective. Also, based on a simple genetic algorithm, this paper analyzes whether critical components for single systems are still important when taking two systems as a whole for analysis. Results show that a protection strategy of hardening a few critical components is also good strategy for the combined system. In addition, the findings and several assumptions are further discussed to close the gap between theory and practice.

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1. Introduction

Transportation systems provide transport services for various types of customers and support the functioning of modern society. Due to their importance, their incapacity or interruption might bring many social problems [1,2]. Depending on various types of vehicles used for transport, there exist different types of transportation systems, such as railway systems, airline systems, road systems, which provide complementary transportation services from one location to another. Due to their complementary relationships, in the case of interruption or incapacity of one system, another one can provide alternative services to carry passengers and cargos to their desired location. A raised question is how strong the complementary relationships are for multiple types of transportation systems in a city or country. Also, whether critical components of single systems are still important when taking multiple complementary systems as a whole for analysis? This paper will address these problems by modeling and analyzing the vulnerability of complementary transportation systems, where the term vulnerability is usually defined as the performance drop under a disruptive event and the vulnerability analysis can provide answers to which components are the most critical and how single and multiple systems of concern perform under disruptive events in the case of considering or not considering the complementary relationships, and results from the latter question can further enable quantifying the complementary strength in a dynamic manner, which provides an initial step for designing or selecting optimum complementary topologies. In the literature, there are few vulnerability studies on complementary systems, but there exist many related studies on single systems and other types of interdependent infrastructure systems, and the approaches applied in these studies can be extended to analyze complementary system vulnerability.

For the vulnerability studies of single transportation systems, many scholars have applied network-based or topology-based approaches, where nodes represent the system components, such as railway stations, airports and road intersections, and links describe the relationships between components, such as railway lines, flight routes and road segments. Based on system topologies, it has been found that the vulnerability of a system is largely affected by its topological properties. One of them is the node degree distribution, which provides the fraction of the nodes that have their connections larger than any given value. It shows that some transportation systems have...
their node degree satisfying exponential distributions, which are robust to failures of randomly selected nodes or nodes with large connections, such as Chinese bus systems [3], India railway system [4], Urban street networks [5], Chinese railway system [6], while some systems have their node degree satisfying power-law distributions, which are robust to random failures of the nodes but vulnerable to failures of the nodes with large degree, such as India airline system [7], American airline system [8], worldwide cargo ship network [9,10]. These studies only considered the pure topologies of transportation systems for vulnerability analysis, while there exist some studies accounting for flow, such as trains, passengers, flights, upon the systems and analyzing their flow-based vulnerability, such as flow-based vulnerability analysis of Chinese railway system [11], road system in the capital city of the state of South Australia [12], European railway network [13] and Indian Railway network [4] under random failures and targeted attacks. Besides these two types of failures, many scholars have studied the vulnerability of transportation systems under natural hazards. Chang and Nojima [14] introduced a method to analyze the post-disaster performance of the railway system under earthquake scenarios, which was evaluated in terms of the total length of functional railway line and the total distance-based accessibility. Peeta et al. [15] assessed the vulnerability of highway networks subjected to earthquakes, and proposed a method to make post-disaster investment decisions for strengthening the highway network. Jenelius and Mattson [16] analyzed the vulnerability of road networks under area-covering disruptions, and the road network was covered using a grid of uniformly shaped and sized cells, where each cell represents the spatial extent of a disrupting event. Sohn [17] used an accessibility perspective to study the vulnerability of a highway network under flood damage by evaluating the significance of its links.

For the vulnerability studies of interdependent infrastructure systems, there exist many approaches in the literature, such as empirical approaches, agent based approaches, system dynamics based approaches, economic theory based approaches, network based approaches and others. A detailed review of these approaches is provided by one of the authors in Ref. [18]. The empirical approaches utilize the post-event incident reports and data to analyze interdependent system vulnerability, which enable to identify frequent and significant failure patterns, quantify interdependency strength metrics to inform decision making, make empirically-based risk analysis and provide alternatives to minimize the risk [19–23]. The agent based approaches use agents to computationally represent components in an infrastructure (such as electric transformers or generators) or some important players (such as government or weather) related to system operation, and take a set of rules to describe agents’ behaviors and their interactions with environment so that it can model all types of interdependencies among interdependent systems by discrete-event simulations, and provide scenario-based what-if analysis and the effectiveness assessment of different control strategies [24–28]. The system dynamics approaches use feedback loops, stocks, and flows to model the dynamic and evolutionary behavior of the interdependent systems by describing important causes and effects under disruptive scenarios, so that it can analyze the effects of policy and technical factors to reflect system evolution in the long term and provide investment recommendations, incorporate multi-attribute utility functions to compare alternative infrastructure protection strategies and help build consensus among stakeholders in a decision [29–32]. The economic theory based approaches regard each infrastructure system as an entity in a market of economy and then model their interdependencies by economic models, such as input-output models and computable general equilibrium models, which enable to analytically study how perturbations propagate among interconnected infrastructures and how to implement effective mitigation efforts [33–37]. The network based approaches model each involved system by a network and describe their interdependencies by interlinks, which enable to describe the topological features of interdependent systems and flow characteristics upon systems, identify the critical system components and provide suggestions on mitigation strategies at detailed topological levels [38–44]. However, these studies mainly considered negative effects of interdependencies, where failures in one system can propagate to other systems and aggravate the initial damage. In reality, there also exist positive effects of interdependencies, which are shown in complementary systems and in the case of interruption or incapacity of one system, another system can provide alternative services to satisfy customers’ demands. Different transportation systems in a city or country are typical complementary systems. This paper takes the railway and airline systems in China as an example, proposes a network-based approach to model and analyze the vulnerability of complementary transportation systems.

The rest of the paper is organized as follows: Section 2 provides network-based description of complementary transportation systems. Section 3 introduces their vulnerability model and illustrates its two applications, including quantification of dynamic complementary strength and identification of critical components. Section 4 takes the railway and airline systems in China an example to analyze their complementary strength and discuss whether critical components for single systems are still important for the combined system, and Section 5 discusses the findings and extensions while Section 6 provides conclusions and future research.

2. Network-based description of complementary transportation systems

Transportation systems have many types, such as railway systems, airline systems and road systems, which offer complementary transportation services to carry passengers and cargos from one location to another. These systems can be described by abstract networks. This paper takes railway system and airline system in China as an example to describe how to model them by networks and how to model their complementary relationships. For illustrative purpose, this paper only considers the complementary services for passenger transport, which means that if there is a disruption in one system, the other system can carry passengers to their desired locations.

The Chinese railway system plays a crucial role in the economy of China and supports the wellbeing of its citizens. In 2012, it transported around 1.89 billion passengers and approximately 3.89 billion metric tons of cargos. This paper mainly focuses on passenger transport for complementary analysis. According to the recent timetable “Chinese Railway Passenger Train Timetable” published in 2010 [45], there are more than 2000 passenger stations in total, among them this paper picks out those stations in the cities at prefecture level and above in China as well as some other stations which are railway hubs or have trains depart from or arrive at. Further, this paper combines multiple stations in a city into one station, which only changes the local connections and does not largely affect the overall topology so that the vulnerability will be less affected. Finally, the coarse-grained railway system has 399 stations, which are connected together by 500 railway links. A geographical representation of the rail layout is shown in Fig. 1 (a). Upon the rail layout, on a typical weekday there are 4196 trains transporting passengers between different cities. The initial departure and final arrival stations as well as the detailed routes of each train can be also obtained from the timetable. The railway network can be described by a network G=(N_R, E_R, F_R), where N_R denotes the set of railway stations, E_R is the set of railway links connecting these stations, and F_R records each train ID and its route.

The Chinese airline system has had a rapid development in recent years with the economy growth in China, and the number of airports has increased from 80 in the year 1980 to 160 in the
In the year 2012, it transported around 0.354 billion passengers, accounting for 1/9 passengers transported by airline systems around the world. As the railway system mainly transports passengers in domestic area, to consider the complementary effects from the airline system, only domestic flights are considered to construct the airline network. According to the “Statistical Data on Civil Aviation of China (in Chinese)” [46], there are 160 airports in total for passenger transport and two airports are connected by a link if there exists at least one flight between them on a typical weekday, and then there are a number of 1980 links. A geographical representation of the airline network is shown in Fig. 1(b). Similarly, the airline network can be described by a network $G_A=(N_A, E_A, F_A)$ where $N_A$ denotes the set of airports, $E_A$ is the set of links between any two airports if there exists at least one flight between them, and $F_A$ records each flight ID and its departure and arrival airports.

Based on the above description on railway and airline systems, the next step is to model their complementary relationships. An airline network node and a railway network node are connected by an interlink if these two nodes are located in the same city. By searching all pairs of airports and railway stations in the same city, there are finally 128 interlinks, which also means that there are 128 cities that have both railway stations and airports. Based on these data, the complementary relationships are then modeled by a two-column vector $C_{128 \times 2}$, each row of which includes two elements: one is the railway station ID and the other is the airport ID in the same city. By using similar approach, a two-column vector can be also constructed to model the complementary relationships for any two complementary systems.

### 3. Vulnerability models and analysis of complementary systems

This section will first introduce vulnerability models of complementary systems, and then provide its two applications, including quantification of complementary strength and identification of critical components.

#### 3.1. Vulnerability models of complementary systems

This paper defines the vulnerability of a system as its performance drop under a disruptive event. Based on this definition, to assess system vulnerability, it requires first defining a disruptive event and then specifying a performance metric. For the disruptive event, a system could be subjected to many types of events, and various types of disruptive events have different modeling of their impacts. In this paper, several typical types of disruptive events are used for vulnerability analysis, with their modeling details introduced in Section 4.

For the performance metric, this paper does not consider the flow patterns and then takes a flow-unrelated metric for vulnerability modeling and analysis. This metric is daily accessibility, which is the average fraction of nodes which can be reached from any selected node (excluding the selected node) by taking one or several types of transportation services on a typical day or within the subsequent 24 h immediately after the disruptive events. Denote $n$ as the number of total nodes of concern, and then in normal operation each node can reach other $n-1$ nodes by taking one or several types of transportation services on a typical day. Define $n_{de}$ as the number of nodes (excluding the $i$-th node itself) which can be reached from the $i$-th node within the subsequent 24 h after a disruptive event, then the daily accessibility after the event is defined as follow [11]:

$$A_{de} = \frac{1}{n} \sum_{i=1}^{n} \frac{n_{de}}{n-1}. \quad (1)$$

Note that this metric only quantifies the possibility of traveling from one location to another, without considering the flow-related overload issues or the possible panic issues due to the inoperability of several airports or railway stations. These issues would affect the feasibility or the need to travel and will be further discussed in Section 5. In normal operation, the daily accessibility $A_{no}=1$ and then the daily accessibility based vulnerability under that event are calculated as follow:

$$V = \frac{A_{no} - A_{de}}{A_{no}} = 1 - \frac{1}{n} \sum_{i=1}^{n} \frac{n_{de}}{n-1}. \quad (2)$$

Note that for the railway system and the airline system, the post-event daily accessibility metric is affected by the duration of the event. If the event lasts less than one day, the shorter the duration is, the more trains and flights that pass through the event location will not be affected, and then the post-event daily accessibility metric is higher; if the duration lasts for at least one day, affected trains and flights will keep fixed within the next 24 h immediately after the event, then the post-event accessibility metric and the corresponding vulnerability metric are independent with the event duration. For illustrative purpose, this paper assumes that all components’ damage states last longer than one day and then the post-event daily accessibility metric is independent to the event duration. After defining the disruptive events and specifying the performance metric, the next step for vulnerability analysis is to simulate system performance response under the event. Depending on different systems of concern, there exist various vulnerability models. This section next takes railway system and airline system in China as an example to
describe how to model the vulnerability of single systems and how to
model the vulnerability of multiple systems as a whole.

When only considering the airline system, a disruptive event initially
causes some airline component failures, where for a node failure,
the node and all its attached edges are removed from the airline
network; for an edge failure, then only the edge is removed
from the network. Based on the post-event network topology, the
number of nodes minus one in the connected cluster including the i-
th airport (the reason of minus one is to exclude the i-th airport) is
the value of \( n_{de} \), which can then give the accessibility-based airline
system vulnerability \( V_{de} \) under the event according to Eq. (2).

When only considering the railway system, a disruptive event
initially causes some railway component failures, and if a train \( k \)
has its route passing through one of these failed components, and
then the \( k \)-th train is assumed out of operation. After the event,
the new topology together with the remaining normal trains can
construct a space-of-changes network [47], where the nodes still
represent the 399 railway stations in the physical network and
two nodes are connected by a link if there exists at least one train
having its route including these two stations (note that the actual
physical network is taken to identify the affected trains, while this
space-of-changes network is used to compute system vulnerability
value according to those trains still in operation). Based on this
space-of-changes network, the value of \( n_{de} \) for the i-th station can be
computed as the number of nodes minus one in the connected
cluster including that station, and then the accessibility-based
railway system vulnerability \( V_{de} \) under the event can be computed
according to Eq. (2).

When considering the railway and airline systems together, the set
of all nodes of concern is \( N_R \cup N_A \). A disruptive event initially causes
some component failures, and then according to the performance
response modeling of railway system and airline system, the post-
event space-of-changes railway network and airline network can be
obtained respectively, and then based on the complementary relation-
ships, a combined network can be constructed according to the
following rule: all railway nodes keep their node IDs, while those
airline nodes which have complementary railway nodes have their
node IDs substituted by their complementary railway node IDs, and
other remaining airline nodes which do not have any complementary
railway nodes are reassigned by new IDs subsequently after the
final railway ID, and finally there are 399 + 160 = 128 – 431 nodes. For any
two of these nodes, they are connected if there exists a link between
them in the post-event space-of-changes railway network or the post-
event airline network. Based on the constructed railway and airline
combined network, the number of nodes minus one in the connected
cluster including the i-th node gives the value of \( n_{de} \), which can then
give the accessibility-based combined system vulnerability \( V_{de} \) according
to Eq. (2).

3.2. Quantification of complementary strength

According to the network-based description of single systems
and their complementary relationships in Section 2, the static
complementary strength \( SCS_{S2-S1} \) of one system \( S2 \) on the other
system \( S1 \) can be quantified by the following equation:

\[
SCS_{S2-S1} = \frac{\text{Number of components in } S2 \text{ which have complementary components in } S1}{\text{Number of components in } S2}
\]  

(3)

Taking the railway and airline systems in China as an example,
among 399 railway nodes and 160 airline nodes, 128 of them are
located in the same city and are complementary, then the static
complementary strength of airline system on railway system is
\( SCS_{A-R} = 128/399 = 0.32 \), and the static complementary strength of
railway system on airline system is \( SCS_{R-A} = 128/160 = 0.80 \). These
static complementary strength values provide an intuitive quantifica-
tion on how many nodes in one system have another type of
complementary nodes, but they cannot reflect how one system in
case of its disruption can be substituted by another system. To address
this issue, it requires quantifying complementary strength in a
dynamic manner with the consideration of system performance under
disruptive events. Based on the above vulnerability models of single
systems and both, this paper proposes a metric to quantify the
dynamic complementary strength between two systems.

Considering two systems \( S1 \) and \( S2 \) which are complementary,
under a disruptive event, assume that the vulnerability of the
system \( S1 \) is denoted by \( V_{S1} \), in the case of without considering the
complementary effects, and denoted by \( V_{S1S2-S1} \) in the case of
considering the complementary effects, then the dynamic com-
plementary strength \( DCS_{S2-S1} \) of system \( S2 \) on system \( S1 \) under
the event can be quantified by the following equation:

\[
DCS_{S2-S1} = \frac{V_{S1} - V_{S1S2-S1}}{V_{S1}}
\]  

(4)

Similarly, the dynamic complementary strength \( DCS_{S1-S2} \) of
system \( S1 \) on system \( S2 \) under the event can be quantified as follow:

\[
DCS_{S1-S2} = \frac{V_{S2} - V_{S1S2-S2}}{V_{S2}}
\]  

(5)

3.3. Identification of critical components

The proposed vulnerability models enable identifying a given
number of critical components, whose damage can cause the largest
system vulnerability among others. When only searching one or two
critical nodes, the results can be obtained by an enumeration
algorithm, which enumerates all scenarios of one or two node failures
to compute their vulnerability and then selects the scenario that has
the largest vulnerability. To identify more critical components, the
above enumeration algorithm is not suitable due to unaffordable
computational cost. Instead, this paper uses genetic algorithms, which
are power stochastic search algorithms that have been successfully
used in the literature to search the critical components [48]. A simple
form of genetic algorithm is used to search the set of components
whose removal can cause the maximum vulnerability or performance
drop, with the search procedures as follows:

1. Number system components (nodes or edges) from 1 to \( K \) ( \( K \)
   is the total number of system components) and express a set of
   components by a genotype, which is a string composed of a
   line of \( pK \) numbers randomly selected from 1 to \( K \). Note that
   the length of genotype divided by \( K \) is equal to the fraction of
   identified critical components \( p \). Genotypes of initial individu-
   als are randomly generated.

2. Compute the fitness value of each genotype by using linear
   normalization technique. For each genotype, which corresponds
to a set of components to be damaged, the post-event system
   performance after the removal of those components is com-
   puted. After calculating the post-event performance values for all
   produced genotypes, arrange them by the order starting with the
   smallest value (largest performance drop or vulnerability). The
   fitness value of the \( m \)-th genotype is defined by

   \[
f(m) = \max (a - bm, 1)
\]

   This paper sets \( a = 100 \) and \( b = 1.5 \).

3. Use the selection operator to choose superior genotypes according
to their fitness values by wheel selection technique: choose an
uniformly distributed arbitrary number between 0 to the total
sum of fitness values of all genotypes, make a cumulative sum of
all fitness values to get a summing fitness value sequence, and
then compare each element in this sequence with \( r \), and select the
genotype with its corresponding element in the summing fitness
value sequence first exceeding $\epsilon$. This process is repeated until enough genotypes for the next generation is reached.

(4) Use the crossover operator to produce new individuals (or descendants). For a pair of genotypes (or two parents), if a uniformly distributed random number is less than crossover probability, then the crossover operator is made according to the following rules: a cutting point which is a random integer number between 1 to the genotype length, is selected randomly; the first descendant inherits a longer substring from the first parent and replace the genes of shorter substring by genes appeared and randomly selected in the second parent but different with any gene in the longer substring. The second descendant inherits a longer substring from the second parent and replaces the genes of shorter substring by genes appeared and randomly selected in the first parent but different with any gene in the longer substring.

(5) Use the mutation operator to generate the next-generation individuals. For each descendant produced in forth step, a randomly selected gene is replaced at a mutation probability by a number within 1 to $K$ but not appeared in the genotype.

(6) Return to the second step until the maximum generation is reached.

Based on the above procedures, this paper sets the number of individuals in each generation as 100, the maximum generation as 200, the crossover probability as 0.5 and the mutation probability as 1.0. These values are set according to the reference [49] and are further adjusted according to the simulation results to ensure that the optimum results can be repeated or very close in several runs. The genotype in the final generation with the minimum post-event performance or the largest vulnerability corresponds to the set of critical components. Following the above procedures, a given number of critical components can be identified for each single system and the combined system.

4. Vulnerability analysis results

Taking the railway and the airline systems introduced in Section 2 as an example, this section analyzes their dynamic complementary strength and discusses whether critical components for single systems are still important for the combined system.

4.1. Complementary strength analysis

According to the vulnerability models and the complementary strength metrics in Eqs. (4) and (5), the dynamic complementary strength of airline system on railway system $DCS_{A,R}$ and railway system on airline system $DCS_{R,A}$ can be quantified respectively for any given disruptive event. This paper first considers two types of disruptive events, other types of events will be further studied in Section 5. One is random node failures, where a fraction $p$ of nodes is randomly selected and assumed failed. In the simulation, this paper sets railway node failure fraction $p_R$ and airline node failure fraction $p_A$ both in the range from 0 to 1.0 with a step of 0.05, and for each failure fraction, 10,000 damage scenarios have been generated. The other is degree based node attacks, where a fraction $p$ of nodes with the largest degree values is assumed failed. In the simulation, for a given $p$, there exists a possibility that the fraction of nodes with degree not less than $d$ is strictly larger than $p$ while the fraction of nodes with degree not less than $d + 1$ is strictly smaller than $p$, then which nodes with degree equal to $d$ are attacked is still uncertain, in this case, this paper runs the simulation 10,000 times to select different nodes with degree equal to $d$. Based on the above simulation settings, the following results are all averaged over 10,000 runs.

When the airline system has no damage with $p_A = 0$, the daily accessibility based railway vulnerability can be computed as a function of $p_R$ in the case of with and without considering the complementary airline system, with the results shown in Fig. 2(a) under random failures and degree-based attacks; similar results are also shown in Fig. 2(b) for the airline system vulnerability as a function of $p_A$ when the railway system has no damage with $p_R = 0$. From the figures, with the increase of the failure fraction of nodes, more and more links attached with these failed nodes are removed from the network, and then the vulnerability curves in different cases are all increasing functions. Compared to the random failures, the degree-based attacks can cause more links removed from the network, and then in different cases their vulnerability curves are always above those under the random failures. In addition, the railway system is more vulnerable than the airline system in different cases. This is because there are much more links in the airline network, which provide much more redundant paths among nodes and then make the network more robust.

When considering the complementary effects, as it is expected, the vulnerability curves in the case of considering complementary systems are always below the curves without considering them. In

Fig. 2. (a) Accessibility-based railway vulnerability as a function of railway node failure fraction in the cases of with and without considering the complementary airline system under random failures and degree-based attacks; (b) Accessibility-based airline vulnerability as a function of airline node failure fraction in the cases of with and without considering the complementary railway system under random failures and degree-based attacks.
the extreme case that all railway nodes are damaged, then only those cities or counties which have both railway stations and airports are mutually accessible, which means 128 railway nodes are still mutually accessible. According to Eq. (1), the daily accessibility of railway system is $A_{de}=1/399(0/398(399-128)+127/398*128)=0.1024$, which corresponds to a vulnerability value of 0.90 and is consistent with the result in Fig. 2(a). Similarly, for the airline system, when its nodes are all damaged, there still exist 128 airline nodes which are mutually accessible, and then the airline accessibility is $A_{de}=1/160(0/159*32+127/159*128))=0.64$, which corresponds to a vulnerability value of 0.361 and is also consistent with the result in Fig. 2(b).

Based on the railway and airline vulnerability curves in the case of with and without considering their complementary systems, the dynamic complementary strength DCS can be computed, with the results for $DCS_{A\rightarrow R}$ in Fig. 3(a) under random failures and in Fig. 4 (a) under degree-based attacks, and for $DCS_{R\rightarrow A}$ in Fig. 3(b) under random failures and in Fig. 4(b) under degree-based attacks. The figures also show the dynamic complementary strength curves when the complementary systems are subjected to more and more serious damage. For Figs. 3 and 4, it can be found that under various types of disruptive events the dynamic complementary strength values show largely different trends, which indicates that to design an optimum complementary topology that can maximize the complementary strength, it is important to model and analyze the hazards confronted by the systems of concern. In addition, for the random failures in Fig. 3, when the airline system has no damage, the complementary strength $DCS_{A\rightarrow R}$ reaches a maximum value close to 0.4 when the railway node failure fraction is around $p_R=0.15$, above which it decreases exponentially with the increase of $p_R$. If the airline system is subjected to more and more serious damage, $DCS_{A\rightarrow R}$ has the maximum value always at $p_R=0.15$. When the railway system has no damage, the complementary strength $DCS_{R\rightarrow A}$ is a decreasing function of $p_R$ but decreases very slowly with the value always above 0.64; if the railway system is subjected to more and more serious damage, $DCS_{R\rightarrow A}$ becomes smaller and smaller but keeps as decreasing curves. In addition, at any combination values of $p_A$ and $p_R$, the value of $DCS_{A\rightarrow R}$ is almost always not larger than the value of $DCS_{R\rightarrow A}$. This is because the static complementary strength of airline on railway is $SCS_{A\rightarrow R}=0.32$ while it is $SCS_{R\rightarrow A}=0.80$ for railway on airline, and higher static complementary strength also leads to larger dynamic complementary strength.

For the degree-based attacks in Fig. 4, when a fraction 5% of degree-based important railway nodes is damaged, it produces a strong complementary strength $DCS_{A\rightarrow R}=0.6$ of airline system on railway system. This is because among degree-based top 10, 20, 30, 40, 50 important railway nodes, there are respectively 9, 17, 20, 24, 27 nodes which have complementary airports. These data indicate that damaging a few degree-based important railway nodes will bring small vulnerability as many nodes have complementary airports, and when more nodes are attacked, more nodes which do not have complementary airports are damaged, and then the vulnerability increases quickly while the complementary strength decreases sharply. For the airline system, when around 20% degree-based important railway nodes are failed, it produces the largest complementary strength $DCS_{R\rightarrow A}=0.76$ when the railway system has no damage. When a few degree-based important railway nodes are damaged, railway system is seriously damaged so that the complementary strength of railway system on airline system decreases sharply.

### 4.2. Critical component analysis

By using enumeration algorithm, for single railway system, the most critical railway station is Weiwu station (whose failure causes a vulnerability $V_{R}=0.0963$) and the most two critical railway stations are Tongliao station and Shenyang station ($V_{R}=0.3358$); for single airline system, the most critical airport is Kunming airport or Wulumuqi airport ($V_{A}=0.0495$) while the most two critical airports are Guangzhou airport and Kunming airport ($V_{A}=0.1097$). For the railway and airline combined system, the most critical component is Qiqihar railway station ($V_{R}=0.0413$) and the most two critical components are both railway stations and have different combinations, including Xining and Qiqihar, Daermushi and Qiqihar, Wulanxian and Qiqihar, Tianlinxian and Qiqihar, Gangchaxian and Qiqihar, Tongliao and Qiqihar, Tonghua and Qiqihar, Guangzhou and Qiqihar ($V_{A}=0.0729$). It is interesting to find that the Qiqihar railway station is always included in different sets of the most two critical stations and its vulnerability is a half larger than the vulnerability of the most two critical components, which indicates that the Qiqihar station is very critical for the combined system.

Note that the failure of the most critical railway station Weiwu, and the failures of the most two critical railway stations Tongliao and Shengyang for single railway system, make the combined

![Fig. 3. (a) Vulnerability-based dynamic complementary strength curves of airline system on railway system when airline system is subjected to different levels of damage under random failures; (b) Vulnerability-based dynamic complementary strength curves of railway system on airline system when railway system is subjected to different levels of damage under random failures.](image-url)
system vulnerability $V_{RA}$ respectively as 0.0093 and 0.064, while the failure of the most critical airport Kunming (or Wulumuqi airport), and the failures of the most two critical airports Guangzhou and Kunming, make $V_{RA}$ respectively as 0.0093 (or 0.014), and 0.023. These values are both larger than the combined system vulnerability under the failures of randomly selected one and two nodes, which are respectively 0.0062 and 0.021. These results indicate that the critical one or two nodes for single systems are still important for the combined system. A raised question is whether this result still holds true for more critical components. To address this problem, the proposed genetic algorithm introduced in Section 3.3 is used to search a given number of critical nodes for single railway system, single airline system, and the combined system. The vulnerability of different systems in the case of those critical node failures is recorded, with the results shown in Fig. 5. Fig. 5(a) shows single railway vulnerability and the combined system vulnerability under failures of different numbers of critical railway nodes. Fig. 5(b) shows single airline vulnerability and the combined system vulnerability under failures of different numbers of critical airline nodes. Also, for comparison purpose, the combined system vulnerability under failures of randomly selected nodes is also shown in Fig. 5(a) and (b). Fig. 5(c) shows single railway vulnerability, single airline vulnerability and combined system vulnerability under failures of different numbers of critical components for the combined system. Note that critical component failures must cause larger vulnerability than node-based attacks when the number of failed nodes is identical, hence vulnerability comparison analysis between critical component failures and node-based attacks is not further discussed in this paper.

From the curves in Fig. 5(a), when different numbers of its critical railway components are damaged, the combined system always has larger vulnerability than that under failures of the same number of randomly selected nodes, which indicates that all critical railway stations are still important for the combined system. From the curves in Fig. 5(b), if the number of critical airline components is smaller than 15, the combined system has larger vulnerability than that under failures of the same number of randomly selected nodes; otherwise, the combined system has smaller vulnerability than the latter. These results indicate that when the number of critical airline nodes to be identified is large, these critical nodes will be less important than randomly selected nodes for the combined system, which also means a protection strategy of hardening a few critical components for single systems is still good strategy for the combined system. In addition, from the curves in Fig. 5(c), when the number of its critical components to be identified is less than 21, these critical components make large railway vulnerability but zero airline vulnerability. This is because these critical components for the combined system are all railway stations.

In addition, when the critical components of railway system and airline system are simultaneously subjected to damage, the complementary strength can be also computed. The results are similar to those under degree based attacks and then not shown in the paper. The complementary strength of airline system on railway system has the largest value at a few railway node failures, above which its value decreases sharply; while the complementary strength of railway system on airline system always keeps its value always above 0.64 when the railway system has no damage, but when the railway system has a few critical nodes damaged, the complementary strength decreases sharply.

5. Discussions

Sections 2 and 3 propose a network-based approach to model the vulnerability of complementary transportation systems, while Section 4 takes the railway and airline systems in China as an example to analyze their complementary strength and study whether critical components for single systems are still important for the combined system. However, the case study is only based on node failures, and then this section will further discuss the results under other types of disruptive events and discuss several limitations and extensions in the proposed approach to close the gap between theory and practice.

(1) Similar analysis is also made under edge failures, the results show that under random edge failures the dynamic complementary strength $DC_{SR_{-A}}$ reaches the maximum value at railway edge failure fraction $p_{R}=0.15$ when the airline system is subjected to different levels of damage, while the complementary strength $DC_{SR_{-A}}$ is an increasing function of airline edge failure fraction $p_{A}$ until $p_{A}=0.95$, above which $DC_{SR_{-A}}$ decreases a little. Also, all critical railway edges are still important for the combined system, and when the number of critical airline edges is less than 26, these critical airline edges are still important for the combined system, otherwise, they are less important than randomly selected edges in the combined system. These results as well as the findings in Section 4 suggest that a protection strategy of
(2) The proposed approach models the complementary systems only from the topological point of view without considering the flow pattern, hence, the daily accessibility metric only quantifies the possibility of travel among cities, without considering the travel feasibility due to flow-related issues, such as overload-induced unavailability to change the travel vehicles or unavailability to get travel tickets. When the flow-based data is available, by assigning all network links with capacity data, the proposed approach can be adapted to make a flow-based vulnerability analysis. In addition, the inoperability of several airports or railway stations might cause large-scale panic issues, which would affect the need to travel, and then some flow-based performance metric, such as the amount of transported customers will be further affected. In this case, additional psychological analysis can be integrated into the proposed framework to provide a more comprehensive vulnerability analysis.

(3) This paper assumes the disruptive event lasts for at least one day so that the daily accessibility metric is independent of the event duration. In fact, this assumption can be relaxed when the interruption times of all damaged components are given, and then the post-event daily accessibility metric and the corresponding vulnerability metric will be a function of the event duration. This time-dependent vulnerability metric can further help identify critical time interval during which a disruptive event occurs would lead to large vulnerability, and this critical time interval should be paid much more attention for system protection.

(4) This paper proposes a complementary strength metric between two systems, but does not further consider its applications. In fact, this metric can help decision makers design or select better complementary topologies from the vulnerability perspective. This can be realized by using a similar approach for the optimum interface design of multiple systems with negative interdependencies [50]. Given two complementary systems, when their main hazards can be modeled and their components’ fragility models under those hazards are provided, the expected complementary strength between two systems can be estimated, and then an optimum complementary topology which can maximize bi-directional complementary strength can be identified or selected.

6. Conclusions

In the literature, many scholars have analyzed the vulnerability of interdependent infrastructure systems. However, most of these studies focus on negative effects of interdependencies, which mean that in the case of damage on one system, failures can propagate to other systems and largely aggravate the initial damage. In reality, there also exist positive effects of interdependencies, which are shown in complementary systems and mean that in the case of damage on
one system, another system can provide alternative services to satisfy customers’ demands. Different transportation systems in a city or country are typical complementary systems. Taking the railway and airline systems in China as an example, this paper first proposes an approach to model the vulnerability of complementary transportation systems and then analyzes the dynamic complementary strength between the two systems and discusses whether critical components for single systems are still important for the combined system. Results show that the dynamic complementary strength for the railway system on the airline system is much larger than that for the airline system on the railway system, and a protection strategy of hardening a few critical components for single systems is also good strategy for the combined system. Whether this latter result still holds true for interdependent systems with negative effects of interdependencies will be further discussed in another work.

This paper also finds that in the case of the two systems are subjected to random failures, if 15% railway nodes are damaged, the dynamic supplementary strength of airline system on railway system reaches the largest value when the complementary airline system is subjected to different levels of damage, whether this magic number 15% is occasional or has some hidden reasons, it needs a more comprehensive vulnerability analysis on many other complementary systems, which is an interesting topic for future research. Also, this paper only takes two types of transportation systems under random failures and intentional attacks as an example for vulnerability analysis, integrating other types of transportation systems such as highway systems and considering other types of hazards such as hurricanes, earthquakes and floods to make a more comprehensive vulnerability analysis is a direction for future research. In addition, this paper only adopts a simple form of genetic algorithm to search critical components, but there still exist some other hybrid genetic algorithms and other evolution algorithms, which may be more efficient. Hence, proposing more efficient algorithms for critical component identification is also an interesting topic for future research. Moreover, there are many limitations and assumptions in this paper for the vulnerability assessment, such as time-independent performance metrics at daily scale, long event duration, no consideration of network congestion, relaxing these assumptions to analyze time-dependent vulnerability is also an interesting topic.

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References


