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Optimal ordering and pricing policy with supplier quantity discounts and price-dependent stochastic demand

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We study a joint ordering and pricing problem for a retailer whose supplier provides all-unit quantity discount for the product. Both generalized disjunctive programming model and mixed integer nonlinear programming model are presented to formulate the problem. Some properties of the problem are analysed, based on which a solution algorithm is developed. Two numerical examples are presented to illustrate the problem, which are solved by our solution algorithm. Managerial analysis indicates that supplier quantity discount has much influence on the ordering and pricing policy of the retailer and more profit can be obtained when the supplier provides quantity discount.

Keywords: newsvendor; pricing; quantity discount; ordering; uncertain demand

AMS Subject Classifications: 90B05; 90C11; 90C30

1. Introduction

In retail business, retailers usually face two types of decisions: the marketing decision which is to determine a competitive selling price to attract enough customers, and the acquisition decision which is to order a proper quantity for the products. While determining the order for the products, retailers often need to consider the quantity discount provided by their suppliers. Retailers can procure products at a lower unit price if the ordering quantity is over a certain value – a price break point. However, it will increase the overstocking risk, especially when the demand is uncertain. Therefore, the joint ordering and pricing problem under uncertain environment becomes more challenging when quantity discount schemes are presented, as the retailers have to obtain a balance between ordering and selling.

This study investigates the joint ordering and pricing problem under uncertain demand and supplier discounts. The problem is to determine the optimal ordering quantity and selling price simultaneously, so as to maximize the retailer’s
expected profit. It can be viewed as a new extension to the classic newsvendor problem (NP), which is a combination of two other important extensions to NP: extension to considering supplier discount policy and extensions to considering newsvendor pricing policy.

It is an important extension to NP, which considers different newsvendor pricing policies [6]. Newsvendor pricing problem incorporates pricing decision into NP. In this scenario, the retailer stocks the products before the selling season, whose demand is stochastic and price-dependent, and he simultaneously determines the ordering quantity, \( q \), and the selling price, \( p \), to maximize the expected profit.

Petruzzi and Dada [12] present a comprehensive review and some meaningful extensions for the newsvendor pricing problem. Parlar and Weng [11] investigate the effects of coordinating pricing and production decisions on the improvement of a firm’s position in a price-competitive environment. The analyses illustrate that, by coordinating the pricing and production decisions, competing firms can increase their profitability, especially when conditions are unfavourable. Karakul [4] studies the joint pricing and procurement problem for fashion products in the existence of clearance markets. The expected demand is assumed to be linear price-dependent and the excess inventory in the end of the period is sold at a known discounted price in the clearance market. Granot and Yin [2] analyse the effect of price and order postponement in a decentralized newsvendor model with price-dependent and multiplicative random demand. Chen and Bell [1] investigate the influence of customer returns on the retailer’s joint price and inventory decision making. In the uncertain demand case, the single-period and multi-period problems are formulated and analytical solutions are presented. Pan et al. [9] construct a two-period newsvendor model to simultaneously determine the pricing and ordering decisions for a dominant retailer in a declining price environment.

Considering supplier discount policy is also an important extension to NP. Supplier quantity discount is an effective policy for suppliers to promote their products, which has been widely studied in operation research literature works. Jucker and Rosenblatt [3] present three types of quantity discounts: all-units quantity discounts, incremental quantity discounts and carload-lot discounts. Pantumsinchai and Knowles [10] formulate a single-period inventory problem with the consideration of standard container size discounts. Khouja [5] studies the NP that jointly considers the multiple progressive discounts employed by retailers to sell excess inventory and the quantity discounts provided by suppliers. Lin and Kroll [7] investigate the NP with quantity discounts and dual performance measure consideration. Zhang [15] investigates the multi-product NP with supplier quantity discount and capacity constraints.

Although the pricing policies for retailers and suppliers have been studied by many literature works on NP, few of them investigate the situation when both of them are considered in NP. Our study enriches the NP by integrating these two extensions in one model. The main contributions of this article are as follows. First, through the generalized disjunctive programming (GDP) technique, a GDP model is presented to simultaneously optimize the retailer’s ordering and pricing decisions under the consideration of supplier quantity discount. Second, a mixed integer nonlinear programming (MINLP) model is developed for the problem, which is compared with the GDP model. Third, the properties of problem are analysed and
the optimal solution is characterized. Further, the influence of supplier quantity
discount on the ordering and pricing policy is illustrated by numerical examples.

The rest of this article is organized as follows. In Section 2, the GDP formulation
and the MINLP formulation are presented for the problem, and some of its
properties are analysed. In Section 3, an optimal solution algorithm is developed to
solve the problem. Numerical examples and managerial analyses are reported in
Section 4. We finally conclude this article in Section 5.

2. Model formulation

In this section, the joint ordering and pricing problem is formulated as GDP and
MINLP models which are developed based on the following assumptions. The
supplier provides all-unit quantity discount to the retailer. The demand of the
product is assumed to be linear price-sensitive and additive stochastic, that is,
\( \bar{D}(p, u) = D(p) + u \), where \( D(p) = a - bp \) (where \( a > 0 \) and \( b > 0 \)) is the expected
demand as a function of the retail price \( p \) and \( u \) is a stochastic variable defined on the
range \([A, B]\) with a known distribution. This assumption for the demand has been
applied widely in revenue management and operations research literature works \([12]\).

2.1. Notations

The following notations are used in the formulation of the problem:

**Index:**

\( l = 1, \ldots, L \): index of discount segments offered by supplier.

**Parameters:**

- \( h \) per unit overstocking cost
- \( s \) per unit understocking cost
- \( c_l \) the unit acquisition price on discount segment \( l \)
- \( d_l^S \) the lower bound of the quantity on discount segment \( l \)
- \( d_l^H \) the upper bound of the quantity on discount segment \( l \)
- \( D(p) = a - bp \) the expected demand under price \( p \)

\( \bar{D}(p, u) = D(p) + u \) the price-independent stochastic demand

- \( u \) stochastic term defined on the range \([A, B]\) with mean \( \mu \) and
  standard deviation \( \sigma \)
- \( f(\cdot), F(\cdot) \) pdf and cdf of the distribution of \( u \).

**Variables:**

- \( p \) the retail price
- \( q \) the ordering quantity
- \( q_l \) the quantity ordered at discount segment \( l \).
- \( y_l \) 1 if the retailer buys products at discount segment \( l \); otherwise 0
- \( Y_l \) true if the retailer buys products at discount segment \( l \); otherwise false. \( Y_l \) is a logic variable.
Following Petruzzi and Dada [12], we also define:
\[ z = q - D(p). \]

### 2.2. Model

For all-unit quantity discount, the unit acquisition cost of the product, \( c \), is disjunctive, and can be described as follows:

\[
   c = \begin{cases} 
   c_1, & \text{if } d_1^L \leq q \leq d_1^H, \\
   c_2, & \text{if } d_2^L \leq q \leq d_2^H, \\
   \vdots & \vdots \\
   c_L, & \text{if } d_L^L \leq q \leq d_L^H,
   \end{cases}
\]

where \( q = a - bp + z \). For the details on all-unit quantity discounts, the reader is referred to Khouja [5] and Zhang [15]. Before the stochastic term \( u \) is realized, the retailer determines an ordering quantity, \( q \); and a retail price, \( p \); to obtain the maximum expected profit. The profit \( \Pi(q, p) \) can be modelled as

\[
   \Pi(q, p) = \begin{cases} 
   p\tilde{D}(p, u) - cq - h[q - \tilde{D}(p, u)], & \tilde{D}(p, u) \leq q, \\
   pq - cq - s[D(p) - q], & \tilde{D}(p, u) > q.
   \end{cases}
\]

A more convenient expression for this function is obtained by substituting \( z = q - D(p) \) into (2), then,

\[
   \Pi(z, p) = \begin{cases} 
   p[D(p) + u] - c[D(p) + z] - h[z - u], & u \leq z, \\
   p[D(p) + z] - c[D(p) + z] - s[u - z], & u > z.
   \end{cases}
\]

Thus, the expected profit is

\[
   E\Pi(z, p) = \int_A^B (p[D(p) + u] - h[z - u])f(u)du \\
   + \int_z^B (p[D(p) + z] - s[u - z])f(u)du - c[D(p) + z].
\]

Here we present a GDP formulation for the problem. GDP is proposed in Raman and Grossmann [13] and Turkay and Grossmann [14], which can efficiently model the discrete constraints, especially when binary and continuous variables are combined. In GDP, problems are modelled with Boolean and continuous variables with disjunctions and common constraints, and it has been widely applied in the synthesis and optimization of process systems. For more modelling techniques about GDP, the readers are referred to Raman and Grossmann [13]. The GDP model for the problem can be formulated as

\[
   \text{Max} \\
   E\Pi(z, p) = \int_A^z (p[D(p) + u] - h[z - u])f(u)du \\
   + \int_z^B (p[D(p) + z] - s[u - z])f(u)du - c[D(p) + z].
\]
subject to

\[ \bigvee_{l \in L} \begin{bmatrix} Y_l \\ c = c_l \\ d_l^S \leq D(p) + z \leq d_l^H \end{bmatrix}, \tag{5} \]

\[ A \leq z \leq B, \tag{6} \]

\[ p \geq 0, \tag{7} \]

\[ Y_l \in \{\text{true, false}\} \quad \forall l. \tag{8} \]

In the objective function, the first term represents the expected revenue minus the overstocking cost, the second term represents the expected revenue minus the understocking cost and the third term is the acquisition cost. Constraint (5) is the disjunctive constraints, where \( Y_l \) is the logic variable for discount segment \( l \). When \( Y_l = \text{true} \), the constraints in the same square bracket, \( c = c_l \) and \( d_l^S \leq D(p) + z \leq d_l^H \), are valid, and those two constraints ensure the quantity purchased at the price level positions in its corresponding discount segment. If \( Y_l = \text{false} \), the constraints in the square bracket are invalid. \( \vee \) is the logic operator that selects exactly one discount segment, that is, only one \( Y_l \) can be true for \( l = 1, \ldots, L \). Constraint (6) is the restriction for the value of \( z \). Constraint (7) is the nonnegative constraint for the selling price. Constraints (8) indicate logic variables.

We also present an MINLP formulation for the joint ordering and pricing problem.

Max

\[ E\Pi(z,p) = \int_A^B (p[D(p) + u] - h[z - u]) f(u) du + \int_A^B (p[D(p) + z] - s[u - z]) f(u) du - \sum_{l=1}^L c_l q_l, \tag{9} \]

subject to

\[ D(p) + z = \sum_{l=1}^L q_l, \tag{10} \]

\[ q_l \leq d_l^H y_l \quad \forall l, \tag{11} \]

\[ q_l \geq d_l^S y_l \quad \forall l, \tag{12} \]

\[ \sum_{l=1}^L y_l = 1, \tag{13} \]

\[ p \geq 0, \tag{7} \]
The objective function is to maximize the retailer's expected profit. Constraint (10) presents the relationship between the selling price and the ordering quantity. Constraints (11)–(13) are the quantity discount constraints: (11) and (12) ensure the amount ordered from the supplier at the price level positions in the discount interval offered. Constraint (13) ensures only one discount level is eventually applied to the amount if the retailer orders the product, which implies that only one of \( q_l \), \( l = 1, \ldots, L \), could be nonzero. Constraint (7) is the nonnegative constraint for the selling price. Constraint (14) is the value restriction for \( z \) and constraints (15) are the integral constraints.

Compared with the GDP model, we can see that \( Y_l \) and \( y_l \) are the variables applied to select discount segment. Constraint (13) shows that only one \( y_l \) can be set to 1. When \( y_{l^*} = 1 \), we know that \( d_{l^*}^S \leq q_{l^*} \leq d_{l^*}^H \) while other \( y_l \) and \( q_l \) are equal to zero. Thus, in this situation, the ordering quantity, \( q \), is equal to \( q_{l^*} \) and the acquisition cost \( c = c_{l^*} \). Similarly, we can analyse the GDP model when \( Y_l \) is set to true, and the same conclusion can be obtained. But the MINLP model consists of more variables and constraints, which allows the problem formulation to become more complex.

### 2.3. Properties

In the GDP model, the logic operator \( \lor \) in Constraint (5) ensures that only one discount segment can be selected. It implies that the solution to the problem must locate in one interval of \( L \) discount levels. Therefore, we can solve the problem by solving the \( L \) subproblems and each of them is associated with one price level. Then the best solution of the \( L \) subproblems is the optimal solution of the original problem.

For discount level \( l \), we can obtain a subproblem as follows.

Max

\[
ETI_l(z, p) = \int_A^z (p[D(p) + u] - h[z - u])f(u)du + \int_z^B (p[D(p) + z] - s[u - z])f(u)du - c_l[D(p) + z],
\]

subject to

\[
d_l^S \leq D(p) + z \leq d_l^H
\]

\[
A \leq z \leq B,
\]

\[
p \geq 0.
\]
We first introduce some properties of the objective function (16), which can help us to obtain the optimal solution for the subproblem without considering the discount interval constraint (17).

**Lemma 1** For any fixed $z$, the optimal selling price to maximize $E \Pi$ is determined uniquely as a function of $z$:

$$p^r = p'(z) = p^0 - \frac{\Theta(z)}{2b},$$

where $\Theta(z) = \int_z^B (u - z)f(u) du$ and $p^0 = \frac{a + b \gamma + \mu}{2b}$.

Lemma 1 has been illustrated in Mills [8] and Petruzzi and Dada [12].

Substituting $p^r = p'(z)$ into $E \Pi(z, p)$, the optimization of subproblem $E \Pi(z, p)$ becomes a maximization over the single variable $z$: Maximize $E \Pi(z, p(z))$. Petruzzi and Dada [12] present the sufficient conditions for the unimodality of the newsvendor pricing problem, which can be described as follows.

**Lemma 2** If $a - b(c_l - 2s) + A > 0$ and $F(z)$ is a distribution function satisfying the condition $2r(z)^2 + dr(z)/dz > 0$ for $A \leq z \leq B$, where $r(z) = f(z)/[1 - F(z)]$, then $E \Pi(z, p(z))$ is unimodal in $z$, and there is a unique $z^r$ in the region $[A, B]$ that satisfies $dE \Pi(z, p(z))/dz = 0$.

In this article, the parameters of our problem satisfy the two conditions in Lemma 2. $r(z)$ is the hazard rate, and Petruzzi and Dada [12] show that all nondecreasing hazard rate distributions satisfy $2r(z)^2 + dr(z)/dz > 0$. Thus, it is reasonable for many newsvendor type products to satisfy the two conditions in Lemma 2.

Lemmas 1 and 2 provide a way to obtain the optimal solution to maximize function $E \Pi(z, p)$, but it may violate constraint (10). Hence, we further investigate some properties of the subproblem.

From $z = q - D(p)$, we can obtain that, for a fixed $q$, $p_q = p_0(z) = (a + z - q)/b$.

**Proposition 1** For a fixed ordering quantity $q$, there exists a unique solution to maximize function $E \Pi(z, p)$, which is $p^{eq} = (a + z^{eq} - q)/b$, where $z^{eq}$ is the unique solution in the region $[A, B]$ that satisfies $dE \Pi(z, p_q(z))/dz = 0$.

In Proposition 1, $z^{eq}$ and $p^{eq}$ denote the optimal solution to maximize function $E \Pi(z, p)$ when the ordering quantity is a fixed $q$.

The proof of Proposition 1 is provided in the appendix.

Lemmas 1 and 2 indicate that function $E \Pi(z, p)$ is unimodal, and we can easily draw the following proposition:

**Proposition 2** The optimal solution for the subproblem is determined according to the following:

(a) $z^{le} = z^r$ and $p^{le} = p^r$, if $d^S \leq D(p^r) + z^r \leq d^H$, where $p^{le}$ and $z^{le}$ are the optimal solution for the subproblem, and $p^r$ and $z^r$ are the optimal solution to maximize function $E \Pi(z, p)$;

(b) $z^{le} = z^{ls}$ and $p^{le} = p^{ls}$, if $D(p^r) + z^r \leq d^S$, where $p^{ls}$ and $z^{ls}$ are the optimal solution to maximize function $E \Pi(z, p)$ when the ordering quantity is fixed to $d^S$;
(c) $z^{lc} = z^{lH}$ and $p^{lc} = p^{lH}$, if $D(p^r) + z^r \geq d^H_l$, where $p^{lH}$ and $z^{lH}$ are the optimal solution to maximize function $E\Pi_l[z, p]$ when the ordering quantity is fixed to $d^H_l$.

Propositions 1 and 2 provide a way to obtain the optimal solution for the subproblem.

It can be seen that functions $E\Pi_l$, for $l = 1, \ldots, L$, are produced by a series of discounts, thus they have the similar structure. Some properties are further investigated to illustrate their internal relationships.

**Proposition 3** The optimal solutions $p^h$ and $z^h$ to maximize function $E\Pi_l$, for $l = 1, \ldots, L$, satisfy $z^h < z^{l+1}$ and $D(p^r) + z^r < D(p^{l+1}) + z^{l+1}$.

**Proposition 4** The maximum values of functions $E\Pi_l$, $l = 1, \ldots, L$, satisfy $E\Pi_l < E\Pi_{l+1}$.

The proofs for Propositions 3 and 4 are provided in the appendix. Proposition 3 shows that the optimal ordering quantity for lower price level is larger than that for higher price level. Proposition 4 implies that we do not need to solve the subproblems with higher price levels if the solution to function $E\Pi_l$ at a lower price level is realizable.

### 3. Solution algorithm

Base on Lemmas 1 and 2 and Propositions 1–4, we can design the following optimal solution algorithm for the newsvendor pricing problem with supplier quantity discounts.

**Algorithm:**

**Step 0** Initialize the problem.

**Step 1** For $l = L$ to 1, obtain $z^h$ by Lemma 2 and obtain $p^h$ by Lemma 1,

if $d^i_l \leq D(p^h) + z^h \leq d^H_l$,
then $z^{lc} = z^h$, $p^{lc} = p^h$, $l = l$ and go to Step 2;
otherwise $z^{lc} = z^{ISr}$ and $p^{lc} = p^{ISr}$, where $p^{ISr}$ and $z^{ISr}$ are obtained by Proposition 2.

**Step 2**

$$\{z^*, p^*\} = \arg \max \{E\Pi_l(p^{lc}, z^{lc}) | l \geq l^*\},$$

where $z^*$ and $p^*$ are the optimal solution for the original problem $E\Pi$.

Step 1 calculates the optimal solution for function $E\Pi_l$ in descending order from $L$ to 1. Then checks if the optimal ordering quantity is realizable, i.e., $d^i_l \leq D(p^h) + z^h \leq d^H_l$. If so, according to Propositions 2 and 3, we do not need to calculate the optimal solutions to $E\Pi_l$ for $l < l^*$. Otherwise, by Proposition 2, we know that the optimal ordering quantity at this discount segment is the discount break point. Thus, we need to calculate the optimal solution to maximize function
$E \Pi_l[z, p]$ with $D(p^n_l) + z_l = d^n_l$, for $l > l^*$. In Step 2, the optimal solutions for all subproblems for $l > l^*$ are evaluated, and the best one is the optimum of the original problem. The algorithm is similar to the procedure used to solve the single item NP with quantity discount [7], but we incorporate the pricing decision into the problem.

4. Numerical results

In this section, two numerical examples are presented to illustrate the problem.

Example 1 In this example, the parameters are set as $a=9000$, $b=200$, $h=8$, $s=14$, and $z$ follows the normal distribution with mean of zero and standard deviation $\sigma=900$. The price schedule offered by the supplier is presented at Table 1.

Example 2 In this example, the parameters are set as $a=9000$, $b=300$, $h=8$, $s=14$, and $z$ follows the normal distribution with mean of zero and standard deviation $\sigma=600$. The price schedule offered by the supplier is the same with Example 1.

The two examples are solved by our solution algorithm, and the computational results are presented at Table 2.

4.1. Comparison of the results of discount case with that of nondiscount case

By Example 1, we analyse the influence of supplier quantity discount on the retailer’s ordering and pricing policy. In the nondiscount case, the supplier does not provide any discount, the acquisition price of the product is always $15.00 and other parameters are the same to that of Example 1. The nondiscount problem can be directly solved by Lemmas 1 and 2. For the nondiscount case, the optimal ordering quantity is 3268, the optimal selling price is 29.24 and the optimal profit is 26,638.18.
Comparing these results with the optimal solution for Example 1 with discount, we can find that the retailer would order more products and set a lower retail price for the product when the supplier provides quantity discount. Also, the retailer earns more profit in the discount case. In all, the quantity discount schemes do impact the retailer’s ordering and pricing policy.

5. Conclusions
In this article, we present an extension to the NP which incorporates the supplier quantity discounts and the retailer pricing policy. The problem was formulated both by GDP and MINLP. Some important properties of the problem are presented, which help to develop an optimal solution algorithm. Modelling experience shows that GDP has more advantage than MINLP when the discrete acquisition cost presents. Two numerical examples are proposed to illustrate the problem and solved by our solution algorithm. In comparison with the nondiscount case, we find that the retailer would order more products and provide a lower retail price for the product when the supplier provides quantity discount.

A meaningful extension to this article is to consider the multi-product acquisition and pricing problem with various capacity constraints. A Lagrangian relaxation approach can be utilized to solve the problem, and the algorithm efficiency for the cases of both single- and multi-product, especially for large scale instances, is to be studied. It is also very important to investigate the application of GDP in other operations management area, especially the optimization problem with both continuous and discrete constraints/variables. We consider the linearly price-dependent and additive stochastic demand form in this article, and considering other demand forms, such as nonlinearly price-dependent and multiplicative stochastic form, is another interesting extension for the problem.

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References
Appendix

Proof of Proposition 1: Substituting $p_q \equiv p_q(z) = (a + z - q)/b$ into $E\Pi(z,p)$, we can obtain

$$E\Pi[z, p_q(z)] = \int_A^z [(q - z + u)(a + z - q)/b - h(z - u)] f(u) du + \int_z^B (q(a + z - q)/b - s[u - z]) f(u) du - c_i q.$$ 

The first and second derivatives of $E\Pi[z, p_q(z)]$ are as follows:

$$\frac{dE\Pi[z, p_q(z)]}{dz} = \left[(q - a - 2z)/b - h - s\right] F(z) + \frac{1}{b} \int_A^z uf(u) du + (q/b + s),$$

and

$$\frac{d^2 E\Pi[z, p_q(z)]}{dz^2} = -2F(z)/b - [(a + z - q)/b + h + s] F(z) < 0.$$ 

Thus, the first derivative $dE\Pi[z, p_q(z)]/dz$ is a monotonically decreasing function of $z$. Here $q \ll B$,

$$\left.\frac{dE\Pi[z, p_q(z)]}{dz}\right|_{z=B} = \frac{2q - a - 2B + \mu}{b} - h - s < 0,$$

and

$$\left.\frac{dE\Pi[z, p_q(z)]}{dz}\right|_{z=A} = \frac{q}{b} + s > 0.$$ 

Thus, there is a unique $z^{opt}$ in region $[A, B]$ that satisfies $dE\Pi[z, p_q(z)]/dz = 0$. 

\begin{flushright}
\textbf{\textbullet}\end{flushright}
Proof of Proposition 3
Let
\[ R_l(z) = \frac{dE\Pi_l(z, p^l(z))}{dz} = -[c_l + s] + \left[ \frac{a + bc_l + \mu}{2b} + h + s - \frac{\Theta(z)}{2b} \right][1 - F(z)] \]
\[ = -c_l \frac{1 + F(z)}{2} - s + \left[ \frac{a + \mu}{2b} + h + s - \frac{\Theta(z)}{2b} \right][1 - F(z)], \]
for \( l = 1, \ldots, L. \)
As \( z^r \) and \( p^r \) are the optimal solutions to \( E\Pi_l \), for \( l = 1, \ldots, L \), from Lemma 2, we know that \( R_l(z_l^r) = 0 \).
Because \( R_l(z_l^{i+1}) - R_l(z_l^{i+1}) = -(c_l - c_{l+1})(1 + F(z_l^{i+1}))/2 < 0 \),
\[ R_l(z_l^{i+1}) < R_l(z_l^{i+1}) = 0. \]
Furthermore \( R_l(A) = [a - b(c_l - 2s) + A]/(2b) > 0 \).
Thus, the unique root \( z^r \) for \( R_l(z_l) = 0 \) belongs to range \( (A, z_l^{i+1}) \), that is, \( z^r < z_l^{i+1} \).
Let \( q^r = D(p^r) + z^r \), for \( l = 1, \ldots, L. \)
Substituting \( p^r = p^l(z_l^r) \) into \( q^r \), we obtain \( q^r = (a - bc_l - \mu)/2 + \Theta(z^r)/2 + z^r \).
Function \( q^r(z) = (a - bc_l - \mu)/2 + \Theta(z)/2 + z \) is a monotonically increasing function of \( z_i \), for \( i = 1, \ldots, L. \), as \( dq^r(z)/dz = [1 + F(z)]/2 > 0. \)

\[ z^r < z_l^{i+1}, \text{ thus } q^r(z^r) < q^r(z_l^{i+1}). \]
\[ q_l^{i+1}(z_l^{i+1}) - q^r(z_l^{i+1}) = b(c_l - c_{l+1})/2 > 0, \text{ therefore } q^r(z_l^{i+1}) < q_l^{i+1}(z_l^{i+1}). \]

We can get \( q^r(z^r) < q_l^{i+1}(z_l^{i+1}) \), that is, \( D(p^r) + z^r < D(p_l^{i+1}) + z_l^{i+1} \).

Proof of Proposition 4
\( E\Pi_l[z^r, p^r] \) is the maximum value of \( E\Pi_l[z, p] \) for \( l = 1, \ldots, L \), thus \( E\Pi_l[z^r, p^r] < E\Pi_{l+1}[z^r, p^r] \).
\[ E\Pi_l[z^r, p^r] - E\Pi_l[z^r, p^r] = (c_l - c_{l+1})[D(p^r) + z^r] > 0, \text{ therefore } E\Pi_l[z^r, p^r] < E\Pi_{l+1}[z_l^{i+1}, p_l^{i+1}]. \]