To assess on dynamic property with drainage of a pile in a saturated soil by BEM of Green’s functions

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SUMMARY. It sometime is harmful for engineering pile that drainage takes place under dynamic load. However, the drainage is not taken into account in the investigation of dynamic behavior of pile in the saturated soil so far. In this paper, Using Green's function of the solid and fluid phases for a two-phase saturated medium in the axisymmetric coordinates, applying Lamb’s integral equations with drainage, associating with the continuous conditions and the boundary conditions of pile-soil interaction, the authors via BEM to obtain solutions of the dynamic displacement and the pore pressure as well as the flux of drainage for a single pile with drainage in the saturated soil. Some figures of the dynamic parameters under drained or/and undrained versus dimensionless frequency are plotted respectively. That these results accord with correspond reality justifies that BEM is an effective numerical method used to solve the issue of engineering problem.

INTRODUCTION

BEM maintains the special advantage on computation of soil dynamics, for the reason that dimension reduction greatly effect allay the number of the algebra equations. What is more, the integral equations are the accurate solution for problem, as numerical quadrature is directly applied to the integral equation, which can obtain a precise result. Finally, it is important that the singular solution of integral equations submits to radiate condition of infinite long automatically. BEM needs no hypothesis and discussion such as artificial boundary. Yet the approach of Green’s function is very valuable to BEM on the soil dynamics. In fact Green’s integral only relies on area, rather than the boundary of definite problems. Once the Green’s function on certain area is attained, all the Dirichlet’s problems in that area can be solved whatever the boundary condition is. Therefore, some problem that do not resolve obtain the better settlement also, e.g. the computation for a pile with drainage under dynamic load in the saturated soil.

The pile is one of important appliance for the foundations in today. Since the 1920s investigation of pile was begun, the theory and technology of pile has acquired a great development, so that achievements are too vast to review most of the relevant literatures in this paper. The main approaches of investigation on properties of pile have formed, the computation methods adopted have also become an important part of the investigation of pile, including FEM, SplineFEM, BEM and hybrid method. the works of Ellison[1], Kuhlemeyer[2], Faruque[3], Roesset[4], Sen[5], Rajapakse[6], Mammoon[7] and Liyanapathirana[8] can be accessed.

Butterfield[9] put forward that Green's function can be used to analyse dynamic property of pile in elastic half-space. Hereafter, based on Green's function in the homogeneous elastic...
half-space, Chapel[10] analysed dynamic response of a single pile subject to oblique incident seismic waves by BEM. Some approximate solutions with Green’s functions of soil dynamics were presented by Beskos[11] and Dominguez[12], about the content of pile had been set forth. The research achievements have shown that BEM is an effective numerical method used to solve the issue of pile-soil interaction.

We should be noted that drainage is not taken into account the investigation of dynamic behavior of pile in the saturated soil so far. Actually, the sand blasting and drainage will always happen in situ saturated sand when a shock takes place. These phenomena are the same as happened in pile driving. In fact, the drainage of the saturated sand under earthquake bring about a harmful influence on the engineering properties of the pile, as Cubrinovski et al.[13] and Harada et al.[14] have already pointed. In this paper, based on Green's function of the solid and fluid phases in

\[ \begin{align*}
\mu \nabla \cdot \mathbf{u} + \nabla \left[ (\lambda + \mu) \nabla \cdot \mathbf{u} + \alpha M \nabla \cdot \mathbf{w} \right] &= \rho \ddot{\mathbf{u}} + \rho \ddot{\mathbf{w}} \\
\nabla (\alpha \nabla \cdot \mathbf{u} + M \nabla \cdot \mathbf{w}) &= \rho_f \dddot{\mathbf{u}} + \gamma(\omega) \ddot{\mathbf{w}}
\end{align*} \]

where \( \lambda \) and \( \mu \) are Lame's coefficients; \( \alpha \) and \( M \) are the parameters derived by Biot in his investigation of a two-phase saturated medium; \( \mathbf{u} \) and \( \mathbf{w} \) are the vectors describing displacement of the solid phase and the average displacement of the fluid phase relative to the solid phase, respectively, \( \dddot{\mathbf{u}} = \frac{d^3 \mathbf{u}}{dt^3} \), \( \dddot{\mathbf{w}} = \frac{d^3 \mathbf{w}}{dt^3} \). \( \rho \) and \( \rho_f \) are the density of a two-phase saturated medium and fluid phase, respectively. \( \dot{\lambda}, \mu, \alpha, M \) can be regarded as 4 independent elastic constants.

The solution of dynamic equation

The dynamic Biot’s equation[17-19] for a saturated porous medium can be written as follows:

\[ \begin{align*}
\mathbf{G}(R, \omega) &= \frac{1}{4 \pi \omega^2} \left[ \frac{1}{\rho - \rho_f} \left[ \nabla \times \nabla \times (1 \frac{e^{-ik_{n}R}}{R}) \right] - \frac{\lambda}{\rho + \rho_f} \xi_1 \left[ \nabla \cdot (1 \frac{e^{-ik_{n}R}}{R}) \right] + \frac{\lambda_2}{\rho + \rho_f} \xi_2 \left[ \nabla \cdot (1 \frac{e^{-ik_{n}R}}{R}) \right] \right],
\end{align*} \]

where

\[ \begin{align*}
\lambda_n &= (1 + \xi_n) / (\xi_1 - \xi_2), \quad \xi_n = \frac{\lambda + 2\mu + \alpha^2 M - \rho(\alpha_n)^2}{\rho_f(\alpha_n)^2 - \alpha M} \quad (n=1,2),
\end{align*} \]

\( K_{n_1}, K_{n_2}, \) and \( K_p \) are wave numbers of the fast and slow dilational, and distortional wave respectively. \( \alpha_1, \alpha_2 \) and \( \beta \) are the velocities of the fast, slow dilational and the distortional
waves respectively. \( i \) is the imaginary unit, \( I \) is the unit matrix of second-order. \( R \) is the distance from the field point to the source point. Eq.(2) can also be rewritten as:

\[
G(R, \omega) = \frac{1}{4\pi \omega} \left[ \eta_1 \left( \nabla \times \nabla \times I(e^{-i\omega \cdot k} / R) \right) - \eta_2 \left( \nabla \nabla \cdot I(e^{-i\omega \cdot k} / R) \right) + \eta_3 \left( \nabla \nabla \cdot I(e^{-i\omega \cdot k} / R) \right) \right],
\]

(4)

where \( \eta_1 = \lambda_1 / (\rho + \rho \xi) \),
\( \eta_2 = \lambda_2 / (\rho + \rho \xi) \) and \( \eta_3 = \lambda_3 / (\rho + \rho \xi) \);
which represents the coefficients of distributive mass for the fast and slow dilational and the distortional waves in the dynamic process respectively.

**The Green function in axisymmetric coordinates**

\[
G_{ij}(R / \omega) = \begin{pmatrix}
G_{rr} & G_{ro} & G_{rz} \\
G_{ro} & \rho G_{o\theta} & \rho G_{r\theta} \\
G_{rz} & \rho G_{r\theta} & \rho G_{zz}
\end{pmatrix},
\]

(5)

The Green’s functions including the solid phase and fluid phase in the cylindrical and axisymmetric coordinates respectively are

\[
G(R / \omega) = \begin{pmatrix}
G_{rr} & G_{ro} & G_{rz} \\
G_{ro} & \rho G_{o\theta} & \rho G_{r\theta} \\
G_{rz} & \rho G_{r\theta} & \rho G_{zz}
\end{pmatrix}, \quad G(R / \omega) = \begin{pmatrix}
G_{rr} & 0 & G_{r\theta} \\
0 & 0 & 0 \\
G_{r\theta} & 0 & G_{\theta\theta}
\end{pmatrix}.
\]

(6)

The Green’s functions of the fluid phase are \( G_{4r}, G_{4o}, G_{r4}, G_{z4}, G_{44} \) in the axisymmetric coordinates. \( G_s \) is defined as the pore pressure at the field point due to a unit impulse acting at the source point in the \( r \) direction. We can obtain it as same as authors performed in reference paper. \( G_{r4} \) is defined as the displacement at the field point in the \( r \) direction due to the injection of a unit volume of fluid impulse into the pore at the source point.

We can present it by integration \( G_{r4} \) with respect to time according to reason of Betti’s theorem (reciprocity) \([25,20]\). \( G_{44} \) is define as the pore pressure at the field point due to the injection of a unit volume of fluid impulse into the pore at the source point. Associating with \( G_{4r} \) and \( G_{r4} \), \( G_{44} \) can be expressed. \([20]\) These deductions are obtained via the axisymmetric coordinate transformation and the Sommerfeld’s integral presented in reference. \([20,24]\) They are as follows:

\[
G_r = \frac{1}{4\pi \omega} \int_0^\infty \left\{ -\eta_1 \frac{b}{r} e^{-kr} + \eta_2 \left( a_\theta - a_r \frac{k^2}{a_r} \right) e^{-kr} \right\} J_0(kr)dk,
\]

(7)

\[
G_{rr} = G_{rr} = \frac{1}{4\pi \omega} \int_0^\infty \left\{ \frac{k}{r} e^{-kr} + \eta_3 \left( a_\theta - a_r \frac{k^2}{a_r} \right) e^{-kr} \right\} J_1(kr)dk,
\]

(8)

\[
G_{eo} = \frac{1}{4\pi \omega} \int_0^\infty \left\{ \eta_1 \frac{k^2}{rb} e^{-kr} + \eta_1 \left( a_\theta - a_e \frac{k^2}{a_e} \right) e^{-kr} \right\} J_0(kr)dk + \int_0^\infty \left\{ \eta_2 \frac{k}{rb} e^{-kr} + \eta_3 \frac{k}{rb} e^{-kr} \right\} J_1(kr)dk,
\]

(9)

\[
G_{oe} = \frac{1}{4\pi \omega} \left\{ M \xi \left( a_\theta + a_e \right) \left( a_\theta - a_e \frac{k^2}{a_e} \right) e^{-kr} \right\} J_0(kr/r - kJ_0(kr))dk,
\]

(10)
\[ G_{ix} = \frac{1}{4\pi\omega^3} \int_0^\infty \left\{ \left[ (\alpha + \xi) M \eta_\alpha (a^2 - k^2) e^{-\omega t} \right] + \frac{1}{i\omega} \left[ M \xi_\alpha \eta_\alpha (a^2 - k^2) e^{-\omega t} \right] \right\} J_0(kr) kdk, \]  
\[ G_{ix} = \frac{i}{4\pi\omega^3} \int_0^\infty \left\{ M \xi_\alpha \eta_\alpha (a^2 - k^2) e^{-\omega t} \right\} J_0(kr) kdk, \]  
\[ G_{ix} = \frac{1}{4\pi\omega^3} \int_0^\infty \left\{ \left[ (\alpha + \xi) M \eta_\alpha (a^2 - k^2) e^{-\omega t} \right] - \frac{1}{i\omega} \left[ M \xi_\alpha \eta_\alpha (a^2 - k^2) e^{-\omega t} \right] \right\} J_0(kr) kdk, \]  
\[ G_{ix} = \frac{M}{4\pi\omega^3} \int_0^\infty \left\{ \eta_\alpha \left\{ \left[ (\alpha + \xi) \frac{\xi}{\omega^2 T^2} + \left[ (\alpha + \xi) \frac{\xi}{\omega^2 T^2} + \frac{\xi}{\omega^2 T} \right] \right] e^{-\omega t} \right\} \right\} J_0(kr) kdk, \]  
where \( T \) is the period between arrival time of the fast dilational wave and the slow dilational wave.

The governing dynamic equation and Lamb' s integral  
As shown in Fig. 1, the governing dynamic equations of a pile described by Sen\(^5\) are as follows:

\[ d^4 \bar{w}_r / dz^4 + m \omega^2 \bar{w}_r / E_p A_p = - \pi a \bar{\phi}_r / E_p A_p \]  
\[ d^4 \bar{w}_z / dx^4 - m \omega^2 \bar{w}_z / E_p I_p = - a \bar{\phi}_r / E_p I_p, \]

where \( m, E_p, A_p \) and \( a \) are the mass density, the modulus of elasticity, the cross-sectional area and the diameter of the pile respectively. \( \bar{w}_r \) and \( \bar{w}_z \) are own displacement of pile in radial and axial direction, respectively. \( \bar{\phi}_r \) is a trail density function of pile and soil in the \( j \) th direction.\(^5\)  
The fundamental solutions of Eqs. (15) and (16) are\(^5\) as:

\[ \bar{w}_z = A_1 \sin (k_z z) + A_2 \cos (k_z z) \]  
\[ \bar{w}_r = B_1 \sin (k_z z) + B_2 \cos (k_z z) + B_3 \sin (k_z z) + B_4 \cos (k_z z), \]

where \( k_z = \sqrt{m \omega^2 / (E_p A_p)}, k_z = \sqrt{m \omega^2 / (E_p I_p)}. \) \( I_p \) is the sectional inertia moment of pile, \( A_1, A_2, B_1, B_2, B_3 \) and \( B_4 \) are the undetermined coefficients.  
The solutions of Eqs. (17) and (18) reveal the vibration displacements of pile in the axial and radial direction respectively. The total displacement of pile in the \( i \) th direction \( U_i \) is

\[ U_i = \bar{w}_i + u_i^*, \]

where \( u_i^* \) denotes the vibration displacement of soil in the \( i \) th direction. For only computation of axial direction we can assume:

\[ t_i(\xi, \tau) \bar{G}_y(x/\xi, t-\tau) \neq - \sigma_y(x, \tau) u_i(\xi, t-\tau) = \bar{\phi}_r(r_0) \bar{G}_y(r-r_0). \]

The vibration displacement of soil \( u_i^* \) can be obtained by Lamb’ s integral of Somigliana’ s type as follows:\(^{15,25}\)

\[ 0.5 u_i^* = \int_S \left[ \phi_i(r_0) \bar{G}_y(r-r_0) + p(r_0) \xi_\alpha \bar{G}_{ik}^i(r-r_0) - \bar{G}_y(r-r_0) \xi_\alpha u_k(r_0) \right] dS \]

\[ - \int_S \left[ \xi_\alpha u_{ij}(r_0) \bar{G}_{ik}(r-r_0) - \xi_\alpha \bar{G}_{(ik)i}(r-r_0) p(r_0) \right] dV. \]

The pore pressure is:\(^{15,25}\)

\[ 0.5 P = \int_S \left[ \xi_\alpha u_{ij}(r_0) \bar{G}_{ik}(r-r_0) \right] dV \]

\[ + \int_S \left[ \phi_i(r_0) \bar{G}_y(r-r_0) + p(r_0) \xi_\alpha \bar{G}_{ik}(r-r_0) - \bar{G}_y(r-r_0) \xi_\alpha u_k(r_0) \right] dS . \]
The flux of drainage is:

\[
\bar{q}_j^r (r - r_o, \omega) = \int \left[ \phi_i (r_o, \omega) \bar{g}_j (r - r_o; \omega) - \bar{h}_j (r - r_o; \omega) u_i (r_o, \omega) \right] \mathrm{d} s - \int \left[ p(r_o, \omega) \bar{g}_j (r - r_o; \omega) - \bar{h}_j (r - r_o; \omega) q(r_o, \omega) \right] \mathrm{d} s
\]

(23)

where

\[
\bar{g}_j = \beta_i (G_{kk,j} / \rho_j + i \omega G_{kk,j}) \bar{h}_j = \beta_r \left[ i G_{kk,j} / (\omega \rho_j) - G_{kk,j} \right]
\]

(24)

\[
\bar{h}_j = \beta_r \left[ \lambda G_{kk,j} + \mu (G_{kk,k} + G_{kk,j}) n_j - \alpha G_{kk,j} n_j \right] / \rho_j + i \omega \left[ \lambda G_{kk,j} n_j + \mu (G_{kk,k} + G_{kk,j}) n_j - \alpha G_{kk,j} n_j \right]
\]

(25)

\[
\bar{g}_j = (\beta_j / i \omega \rho_j)^2 \left[ (G_{kk,k} - \omega^2 \rho_j G_{kk,j}) n_j \right] - (i \beta_j / \omega \rho_j) \left[ (G_{kk,k} - \omega^2 \rho_j G_{kk,j}) n_j \right]
\]

(26)

We should pay attention to Einstein summation convention in Eqs. (21) - (26). The subscript \( k=1,2 \) in \( G_{kk,j} \) correspond to the fast and the slow dilatational waves respectively. 0.5 in the Eqs. (34) and (35) is a coefficient of Lamb's integral denoted point in the boundary of integral domain. Green's functions are shown as Eqs. (7) - (15).

**The continuous conditions and boundary conditions**

If the subscript \( s-P \) stand for soil and pile respectively, the continuous condition of the trail in the interface of pile and soil is\[5]:

\[
\left\{ \phi \right\}_w = \left\{ -\phi_r \right\}_w.
\]

(27)

The constitutive equation of pile-soil system is\[5]:

\[
\left\{ P \right\} = [St] \left\{ U_r \right\},
\]

(28)

where \( St = K + ic \), \( K \) is stiffness, \( c \) is damping. The displacement of pile \( U_r \) is composed \( U_z \) and \( U_r \).

If a total displacement of pile in the axial direction is concerned with only, \( U_r = \{ U_r \} \).

\( St \) is the complex stiffness of pile-soil system in the axial direction. The undetermined coefficients \( A_1, A_2, B_1, B_2, B_3 \) and \( B_4 \) in \( \sigma_z (\infty,z) = \sigma_r (\infty,z) = \tau_{z \infty} (\infty,z) = 0 \).

(29)

(2) The normal stress at free surface is zero, \( \sigma_z (r,0) = 0 \).

(3) If the end of pile in soil is an elastic support, then the boundary conditions of the governing equation Eq. (16) satisfy:

\[
\left. \left( m \omega^2 \dddot{w}_z + E_p A_p \ddot{w}_z / dz \right) \right|_{z=H} = 0, \left. \left( \ddot{w}_z / dz \right) \right|_{z=0} = P(t) / (E_p A_p).
\]

(30)

For damping \( C \) the solution of Eq. (16) is:

\[
\ddot{w}_z = \left[ A_1 \sin (k_1 z) + A_2 \cos (k_1 z) \right] e^{-c_i \omega}.
\]

(31)

Then \( A_1, A_2 \) can be determined:

\[
A_1 = \frac{P(t)}{(E_p A_p) k_1}, A_2 = \frac{P(t)}{(E_p A_p) k_1} \left[ m \omega^2 \sin(k_1 H) + E_p A_p k_1 \cos(k_1 H) \right]
\]

(32)

Substituting Eq. (32) into Eq. (31), we can obtain the displacement at the top of pile and dynamic stiffness respectively.

![Fig. 1 The mechanical model of pile in saturated soil](image-url)
\[ \bar{w}_1 |_{z=0} = \frac{1}{(E_pA_p)k_1} \left[ m\omega^2 \sin(k_1H) + E_pA_p k_1 \cos(k_1H) \right] e^{c_1 z} = \left. \frac{m\omega^2 \sin(k_1H) - m\omega^2 \cos(k_1H) k_1}{E_pA_p k_1 (E_pA_p k_1 - m\omega^2 k_1)} \right|_{z=0} e^{c_1 z} \]

\[ \bar{K}_d = \left( E_pA_p k_1 \right) \left( E_pA_p k_1 - m\omega^2 k_1 \right) \left( m\omega^2 \sin(k_1H) + E_pA_p k_1 \cos(k_1H) \right) e^{c_1 z} \]

The numerical results of BEM

For computations of the soil displacement \( u^*_1 \), the porous pressure \( p \) and the flux of drainage \( \phi^*_1 \) we use BEM in Lamb’s integral equations, where the integral area is discrete. The boundary is divided into \( N \) elements, and there are \( N_N \) and \( N_T \) elements in coordinate and abscissa respectively (it can be simplified for axisymmetric coordinates). The displacement of a element \( \{ \delta_N^e \} = [\delta_1, \delta_2, \ldots, \delta_N]^T \) volume in Eq. (21) had been transformed to the plane integral respect with to the area by the Green’s formula. And

\[ 0.5 \{ \delta_N^e \} = \sum_{i=1}^{N_N} \sum_{j=1}^{N_T} \left[ \begin{array}{c} G_{11}^e \{ \phi_j \} + \sum_{k=1}^{N_T} \left[ \left( \delta_1 G_{12}^e \right) + \left( \delta_2 G_{22}^e \right) \right] \right] \{ p_i \} + \sum_{k=1}^{N_T} \left[ \left( \delta_1 G_{12}^e \right) + \left( \delta_2 G_{22}^e \right) \right] \{ p \}
- \sum_{i=1}^{N_N} \left[ \left( \delta_1 G_{11}^e \right) + \left( \delta_2 G_{21}^e \right) \right] \{ G_i \} + \sum_{i=1}^{N_N} \left[ \left( \delta_1 G_{21}^e \right) + \left( \delta_2 G_{11}^e \right) \right] \{ G_i \}, \]

where \( \{ \delta_N^e \} \) is the displacement in \( e \) th direction at source point corresponding to the element No. \( N \). The space integral respect with to the

\[ \left[ G_{11}^e \right] = \sum_{i=1}^{N_N} \sum_{j=1}^{N_T} \left[ \{ G_{11}^e \} \{ M \} J_i \right] |J_i| |J_i| \Delta \eta \Delta \chi \]
\[ \left[ G_{22}^e \right] = \sum_{i=1}^{N_N} \sum_{j=1}^{N_T} \left[ \{ G_{21}^e \} \{ M \} J_i \right] |J_i| |J_i| \Delta \eta \Delta \chi \]
\[ \left[ G_{12}^e \right] = \sum_{i=1}^{N_N} \sum_{j=1}^{N_T} \left[ \{ G_{12}^e \} \{ M \} J_i \right] |J_i| |J_i| \Delta \eta \Delta \chi \]

where \( \Delta \chi \) and \( \Delta \eta \) are the discrete differential components of displacement and stress respectively; \( |J| \) are the Jacobi’s determinants of displacement and stress respectively. The individual equations become entire equations \( \{ \delta_N^e \} = [L] \{ U_N \}, \{ \delta_N^e \} \{ p \} = [L] \{ S_N \} \), where \( [L] \) is a Bull matrix composite of 0 and 1. Substitution of it into (35) yields

\[ \sum_{i=1}^{N_N} \sum_{j=1}^{N_T} \left[ \{ G_{11}^e \} \{ M \} J_i \right] |J_i| |J_i| \Delta \eta \Delta \chi \]

Finally, we can discretize the Lamb’s integral equation (22) to a algebra equations of BEM as follows:
\[
\sum_{i=1}^{N_K} [H_i][U_{i1}] + \sum_{i=1}^{N_R} [H_i][U_{i2}] + \sum_{i=1}^{N_L} Z_{ei} [R_i][S_i] = 0,
\]

where \( Z_{ei} \) is a parameter that describes the relationship of \( \phi_s^e \) and \( p \) for simplified computation, and
\[
[H_i] = \left[ 2 \sum_{i=1}^{N_K} \left[ [\xi_iG_{ik}] \right] \right] |[L]| |[L]|, \quad [R_i] = \left[ 2 \sum_{i=1}^{N_R} \left[ [\xi_iG_{is}] \right] \right] |[L]|,
\]
\[
{\{U_{i1}\}} = \left\{ U_{(z+i)} \right\}, \quad \{U_{i2}\} = \left\{ U_{(z+i)} \right\}.
\]

Similarly, we can also write discrete the equation for Eq. (23).

For convenience of computation we let \([M]|/[|l|] = 2\pi ; \Delta \chi = \Delta H = 0.01, \ N_\chi = 2000 \) (for side of pile); \( N_\eta = 0.15, \ N_s = 100 \) (for end of pile). The parameters are as: \( H = 20 \ E_p = 20GPa \ m, m = 2500kg/m^3 \) (for pile); \( \alpha = 0.55 \) \([25]\); the permeability \( K = 10^{-10} (m^2) \), the viscosity coefficient \( \eta = 1.02 \times 10^{-3}Ns/m^2 \), the porosity \( \beta_s = 0.45 \); the shear modulus \( \mu = 1.0 \times 10^7 \) Pa, the Poisson's ratio \( \nu = 0.333 \); the bulks modulus of two-phase saturated medium, fluid and solid are respectively as: \( K_s = K_s \left[ 1+\beta_s \left( K_s / K_f - 1 \right) \right] = 311.4 \) GPa, \( K_f = 2.0 \times 10^7 \) Pa, \( K_s = 3.60 \times 10^{10} \) Pa. And the density of the fluid, solid and saturated soil are respectively as: \( \rho = \rho_s \rho_f + (1-\rho_s) \rho_f = 1935kg/m^3 \), \( \rho_f = 1000kg/m^3 \), \( \rho_s = 2700kg/m^3 \); the dynamic parameters \( \xi_i = 0.5549, \xi_s = 1.3183 \), \( \lambda_i = -0.2376, \lambda_s = -1.2376 \). Substituting them into corresponding formulas to calculate, we can show the results in Figs 2-11, respectively. In Fig. 2 and Figs. 4-7 the unit of displacement is \( \text{m} \), the related numerical unit in Figs. 8 and 9 is \( \text{Mpa} \). The unit for flux of drainage is \( \text{itre}/s(\phi m^3)/s \). The dimensionless frequency is \( a_0 = \omega H / V_s \). The standard velocity of distortional wave is \( V_s = \sqrt{\mu / \rho} \).

Fig. 2 shows the vibration displacement \( \bar{w}_s \) at the top of pile in vertical direction with drainage versus the dimensionless frequency \( a_0 \). Fig. 3 shows the comparing results of the computational stiffness by current paper (drained, solid line) and that by Li’s paper (LI Qiang, et. al, 2004) (undrained, dash line), the decrease in dynamic stiffness of pile with drainage can be discovered easily. Fig. 4 shows the previous results (LI Qiang, et. al, 2004), the undrained displacement of soil \( u_i^* \) at different field points (the distance \( R \) is 0.1, 0.2 and 0.6 times pile diameter away from the source) versus the dimensionless frequency \( a_0 \). We should pay attention to Fig. 5. Fig. 5 shows the soil displacement with drainage \( u_i^* \) at different points (the distance \( R \) is 0.1, 0.2 and 0.6 times pile diameter away from source) versus the dimensionless frequency \( a_0 \). The shapes of curves in Figs. 4 and 5 are similar to each other, but the magnitude is different. The attenuation degree of displacement in drained condition is greater than that of the undrained condition. The similar phenomenon has been observed in the seismic records in liquefied soil sites (Cubrinovski et. al., 2004, Harada et. al, 2004), but no computations have been proposed so far except that in current paper. Comparing Fig. 2 and Figs. 4, 5; we can find that as the dimensionless frequency \( a_0 \) increases, the more attenuation takes place in the displacement of pile \( \bar{w}_s \) than that of soil \( u_i^* \). And the maximum displacement of the pile is much less than that of the soil. Fig. 6 shows the previous results (LI Qiang, et. al, 2004), the undrained displacement of soil \( u_i^* \) for different slenderness \( H / a = 0.15, 20 \) versus the dimensionless frequency \( a_0 \). Fig. 7 shows the drained soil
displacement $u^*$ for different slenderness $(H/a = 10, 15, 20)$ versus the dimensionless frequency $a_0$. We can discover that the drained displacement intensity of is weaker than undrained that. Fig. 8 shows the porous pressure $p$ at different field points (the distance $R$ is 10, 20 and 40 times diameter of the pile) in undrained condition versus the dimensionless frequency $a_0$. The dissipation of the porous pressure in undrained condition is weak regardless its distance away from the source. Fig. 9 shows the porous pressure $p$ with drainage at different field point (the distance $R$ is 10, 20, and 40 times diameter of the pile) versus the dimensionless frequency $a_0$. The dissipation of the drained porous pressure $p$ is intense regardless of the distance away from the source at the low range of frequency. Fig. 10 shows the flux of drainage versus the dimensionless $a_0$ at a field point ($R=10$ times pile diameter) for different slenderness (the ratio of pile length $H$ and diameter $a$ is 10, 15, 20 respectively.), and we can find that drainage would almost not happen at the higher range of frequency and increases with an increase of slenderness. Fig. 20 shows that an appropriate maximum flux around pile is about 4.2 litre/s. Fig. 11 is plotted across the flux of drainage for the dimensionless frequency $a_0$. Finally, we can find that the results above-mentioned accord with the correspond reality.

CONCLUSION
By Green’s function of the solid and fluid phases for a two-phase saturated medium in the axisymmetric coordinates, Lamb’s integral equation with drainage and the continuous conditions and boundary conditions of pile-soil interaction, the dynamic displacement and the pore pressure for a single pile with drainage in the saturated soil, as well as the flux of drainage have been obtained via BEM. These results above-mentioned accord with correspond reality, so following conclusions can be drawn:
(1) It justifies that BEM is an effective numerical method used to solve the issue of pile.
(2) The performances by BEM above-mentioned may benefit to further investigation regarding the aseismic property of pile, dynamic response and dynamic interaction of pile-soil in the saturated soil.

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Fig. 4 The undrained displacement of soil $u_z^-$ at different slenderness vs dimensionless frequency

Fig. 5 The drained displacement of soil $u_z^-$ at different slenderness vs dimensionless frequency

Fig. 6 The undrained displacement of soil $u_z^-$ for different distance versus dimensionless frequency

Fig. 7 The displacement of $s$ of soil $u_z^-$ with drainage at different distance versus dimensionless frequency

Fig. 8 The undrained porous pressure $p$ for different slenderness versus dimensionless frequency

Fig. 9 The drained porous pressure $p$ for different slenderness versus dimensionless frequency

Fig. 10 The flux of drainage around pile versus time

Fig. 11 The flux of drainage around pile versus dimensionless frequency
REFERENCES


25. Cheng AH, Badmus T, Beskos DE. Integral equation for dynamic poroelasticity in frequency domain with