Modelling three-dimensional interfacial flow with sand dunes

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Abstract

Hyporheic zone is considered as a dynamic hydrologic ecotone that critical for maintaining the health of river systems. The hyporheic flux occurs generally in response to variations in river bedforms. In this study, we first generated several typical bedforms. The turbulent flow over the 3D dunes and hyporheic flow in the sediment are simulated through a computational fluid dynamics (CFD) approach. Turbulent flow in the water column is simulated by solving the Reynolds-averaged Navier-Stokes (RANS) equations with the k-\omega turbulence closure model, and a steady state groundwater flow model is applied for the underlying porous media. These two sets of equations are coupled through the pressure distribution at the sediment-water interface (SWI). Each case was subjected to different hydraulic conditions, i.e., increasing open channel Reynolds Numbers (Re). Results show that the pressure gradient along the SWI is highly controlled by the spatial structure of bedform, which consequently determines flow dynamics in the porous media. The interfacial flux is dominated by the pressure configuration over the SWI which is a function of Re via a power-law trend. The mean fluid residence time is related to Re by an inverse-power law relationship. This study has led to the basic understanding of hyporheic flow induced by more natural 3D dunes.

1. Introduction

The interaction between river water and groundwater, referred to as hyporheic exchange, has been considered as a key process that is associated with nutrient cycling, oxygen demanding as well as other organic matter transformation [1, 2]. Water flows over sand dunes would generate a net flow near the sand-water interface (SWI)
which is called as bedform-induced hyporheic flow. During the past several decades, the bedform-induced hyporheic flow has been widely studied through experimental and numerical approaches, while most of the research works to date have been concentrated on two dimensional bedform conditions, only a few studies are conducted in three-dimension. Tonina and Buffington [3] conducted experiments to analyze 3D hyporheic exchange through alternating bars with pools and riffles. New insight on 3D exchange through bars was gained, but their flow fields may be far from natural due to the confined width of the flume. Wörman et al. [4,5] followed Elliott and Brooks’ approach [6,7], prescribed pressure head on a flat SWI with a Fourier series translated from topographic data rather than a sine function. Recently, Käser et al.[8] have evaluated the preliminary characterization of hyporheic flux of three reaches on the River Leith located in the northwest of England by applying a 3D groundwater model based on high-resolution topography and stream elevation. Since the three-dimensional bedform has been proven to be more efficient in generating much more complex turbulent flow structures, which will result in an intricate pressure pattern over the SWI as well as the hyporheic process, understanding 3D bedform induced hyporheic is crucial for analyzing biogeochemical behaviors in a natural river.

In this paper, we first generate a group of sand dune shaped bedform based on the Rubin and Carter [9]. A coupled surface-subsurface modeling approach is then applied to solve the RANS equations with $k$-$\omega$ turbulence closure scheme for the surface water and Darcy’s flow for the groundwater flow. The flow structure and pressure pattern over the SWI, and consequently the hyporheic metrics (including the flux, residence time, and exchange depth/volume) are used to discuss the implications for understanding more naturally shaped bedform-induced hyporheic exchange.

2. Materials and methods

The geometry of sand dunes produced by changing and shifting of sediments during deposition is almost always complexly three-dimensional. The geometry of cross-bedding to the morphology and behavior of bedforms has been analyzed by Rubin and Carter [9] through computer modeling approaches. Figure 1 shows six different bedforms with similar wavelength ($\lambda=0.35$ m) and dune height ($H=0.04$ m) but different cross-bending shapes. From Fig.1(a) to Fig.1(f), the bedform gradually becomes more complex.

![Fig.1 Bedform generated based on Rubin and Carter [2005]](image)

All The turbulent flow over the 3D dunes and hyporheic flow in the sediment are simulated following the computational fluid dynamics (CFD) approach of Cardenas and Wilson [10] which has been shown to be robust. For an incompressible fluid, the steady state RANS equations read:

$$\frac{\partial u_i}{\partial x_i} = 0$$  

$$\rho U_i \frac{\partial u_i}{\partial x_i} = - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( 2 \mu S_{ij} - \rho u_i' u_j' \right)$$

where $i, j = 1, 2, 3$ are spatial indices corresponding to $x, y$ and $z$ directions, $\rho$ and $\mu$ are fluid density and dynamic viscosity (assumed standard for water), $U_i$ and $u_i'$ are time-averaged and fluctuating velocity components in $x_i$ directions, $P$ is the time-averaged pressure. $S_{ij}$ is the strain rate tensor defined as:

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

while $-\overline{u_i' u_j'} = \tau_{ij}/\rho$ is the mean strain rates related to Reynolds stresses ($\tau_{ij}$) by:
where $v_t = \mu t / \rho$ is the kinematic eddy viscosity, $\delta_{ij}$ is the Kronecker delta, and $k$ is turbulent kinetic energy. To solve the RANS equations, the Reynolds stresses, $\tau_{ij}$, must be modeled in order to close Equation (2). Since the $k-\omega$ turbulence closure scheme has been proven to work well for treating separated flows over dunes [11], it is applied here with the kinematic eddy viscosity:

$$v_t = \frac{k}{\omega}$$

where the specific dissipation $\omega$ is:

$$\omega = \frac{\varepsilon}{\beta^2 \kappa}$$

and $\beta$ is the turbulence dissipation rate. $\beta^*$ is a closure coefficient. The two groups of equations for $k$ and $\omega$ are:

$$\rho \frac{\partial (u_k)}{\partial x_j} = \rho \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[ (\mu + \mu_t \sigma_k) \frac{\partial k}{\partial x_j} \right]$$

$$\rho \frac{\partial (u_\omega)}{\partial x_j} = \alpha \frac{\rho \omega}{k} \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ (\mu + \mu_t \sigma_\omega) \frac{\partial \omega}{\partial x_j} \right]$$

The standard closure coefficients for the $k-\omega$ scheme are: $\alpha = 5/9$, $\beta = 3/40$, $\beta^* = 9/100$, and $\sigma_k = \sigma_\omega = 1/2$.

Three-dimensional porous flow through the sediment was modeled by solving the steady state groundwater flow equation and Darcy's law:

$$\frac{\partial}{\partial x_i} \left( -\kappa \frac{\partial \phi}{\mu} \frac{\partial x_i}{} \right) = 0$$

where $\kappa$ is the intrinsic permeability.

The Reynolds number $Re$ is defined in terms of the averaged dune height ($H$) as

$$Re = \frac{U_{ave} H}{v}$$

We normalized the effective hyporheic flux density ($q^*$) through the following expression:

$$q^* = \frac{q}{K A_s}$$

where $A_s$ is the surface area of the bedform, $K (= k \rho g / \mu)$ is the hydraulic conductivity of the porous media.

The mean residence times ($t_r$) of water flow through the interfacial exchanged zone (IEZ) is defined based on the exchanged flux and volume of IEZ as follows

$$t_r = \frac{V_z}{A_s q^*}$$

We express the dimensionless form as $t_r^* = t_r K / \lambda$ by dividing dimensionless volume ($V_z / A_s \lambda$) by $q^*$.

Boundary condition (BC) settings for the RANS domain and the porous domain are presented in Fig. 2(a) and Fig. 2(b), respectively. A spatially periodic pressure condition was prescribed on the upstream and downstream sides in both domains, with an additional pressure drop $dP$ between these two sides. Symmetry boundary types were set at the right and left sides (face to the downstream) in both domains. A symmetry boundary (i.e., no flux) was applied at the top face of the water column since in our simulations the water depth ($0.5m$) is much larger than the dune height $H$ ($0.04m$). Thus, it is reasonable to use the symmetry boundary to replace the free surface that generally handles in shallow water applications. A no-slip wall boundary was set at the bottom face of RANS domain.

Following the criteria in Cardenas and Wilson [10], the bottom pressure derived from RANS solutions was imposed as a Dirichlet boundary at the top face of the underlying Darcy's flow domain. This procedure linked the two domains through the pressure along the SWI. The bottom boundary of the sediment domain was set to be a no-flow boundary. The sediment thickness ($1m$) was adequately large that no longer had an effect on the flow field near the SWI.

The RANS simulations were solved using a commercial CFD code FLUENT. The meshing software package GAMBIT was used for grid generation. In order to get a small wall $y^+$ value (ideally less than 1) that suggesting adequate resolution of laminar sublayer at the wall, boundary layers mesh with higher density was applied nodes near the SWI. The number of structured grid nodes of the whole domain varies from ~1,300,000 to ~1,800,000 depending on the hydrodynamic conditions. To solve the groundwater flow equations in the porous media, we use the commercial Finite Element software ComsolMultiphysics. Tetrahedral mesh type was implemented in
ComsolMultiphysics with finer mesh in the vicinity of the SWI. The porous domain consists of more than 700,000 elements in total. For both domains, simulations are tested for mesh dependence.

3. Results

3.1. Pressure pattern over the SWI

Typical pressure distributions over the 6 bedform are shown in Fig.3. The pressure distribution is associated with the crest and trough of the sand dunes. Highest and lowest pressure zones over the SWI are the source and sink area for hyporheic flow. The higher pressure zone drives surface water into the sediment (downwelling) and out from the lower pressure zone (upwelling).

3.2. Hyporheic flow

The hyporheic flux increases with the water column $Re$ via power functions for both all the bedforms (Figure 4a). The power functions are fitted with high goodness-of-fit (see Table 1); Figure 4a shows that the volumetric fluxes of all the bedforms are more or less equal to each other under lower $Re$ (<3000). As the $Re$ increases, the bedform a, b and c drive more flux than bedform e, d and f. We expect that the bedform with more complex sand dunes would result in a higher hyporheic flux; however, the study shows that the hyporheic flux is not in a monotonic dependence on three-dimensionality of bedform.

Simulated IEZ depths (d) for all cases are plotted in Figure 4b. Here, d is defined as the vertical distance between the trough and deepest portion of the enveloping surface enclosing all the streamtubes originating from and returning to the interface. Under laminar water column conditions, the IEZ depths are unstable. This may be due to the absent of pronounced eddy under such a flow pattern.
The bulk fluid residence times for all bedforms are related to Re by inverse-power law relationships since the flux follow a power law dependence with Re. Figure 4c depicts this inverse behavior and shows a sharp decrease in residence time with an increase in Re. Differences in residence time between cases mainly exist in the lower Re flow conditions, when Re increases to 5000, there are only slight differences in residence times among all the bedforms.

**Fig.4 relationship of hyporheic flux (a), exchanged depth (b) and bulk residence times (c) vs. Re numbers**

**Table 1. The fitting parameters of the power law models**

<table>
<thead>
<tr>
<th>parameter relationship</th>
<th>Bedform #</th>
<th>y = mx^n</th>
<th>R^2</th>
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<tbody>
<tr>
<td>Hyporheic flux q^* vs. Re</td>
<td>a</td>
<td>1.12E-13</td>
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<tr>
<td></td>
<td>b</td>
<td>8.90E-13</td>
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<td></td>
<td>c</td>
<td>6.11E-14</td>
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<tr>
<td></td>
<td>d</td>
<td>1.29E-13</td>
<td>2.150</td>
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<tr>
<td></td>
<td>e</td>
<td>1.03E-12</td>
<td>1.599</td>
</tr>
<tr>
<td></td>
<td>f</td>
<td>5.99E-13</td>
<td>1.699</td>
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<tr>
<td>Residence time t^* vs. Re</td>
<td>a</td>
<td>5.81E+14</td>
<td>-1.872</td>
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<tr>
<td></td>
<td>b</td>
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<td></td>
<td>c</td>
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<td></td>
<td>f</td>
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<td>Exchanged depth d/λ vs. Re</td>
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<td>f</td>
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**4. Summary and conclusions**

The coupled modeling approach of surface-subsurface flow, which has been widely tested in two-dimensional flows, is applied in modeling three-dimensional bedform induced hyporheic flow. The hyporheic flux is dominated by the pressure configuration over the SWI which is a function of Re via a power-law trend. The hyporheic flux is not showing amonotonic dependence of three-dimensionality of bedform. The bulk fluid residence times are related to Re by an inverse-power law relationship.

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References