Vision Based Flocking Control for Multi-agent Systems in 3-D Space

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Abstract: We consider the problem of designing distributed controllers for multiple mobile agents to achieve flocking motion in three-dimensional space based only on measured position. A flocking algorithm is proposed under which each agent has the important performance of cohesion maintenance and collision avoidance (CMCA) as basic control goals. In addition, velocity consensus will asymptotically attain. To achieve flocking motion, velocity information can be estimated by a velocity observer which only depends on position information, and using potential function which provides attractive/repulsive bounded force to ensure CMCA. The process of designed control law is presented by means of backstepping method. Simulation results emphasize the effectiveness of the proposed flocking control law.

Keywords: flocking, distribute control, potential function, velocity observer, vision-based.

1. INTRODUCTION

Making a group of mobile agents cooperate toward a common group objective is the multiple goals of cooperative control problem. This makes the design problem more complex and more challenging in view of the traditional control problem. In the area, various control methods, such as formation control (Nguyen and Do, 2006; Gustavi and Hu, 2008), rendezvous (Ganguli et al., 2009), swarmng (Gazi and Passino, 2002), and flocking control (Olfati-Saber, 2006; Do, 2011), have been proposed to address the problem. By making the agents self-organized and autonomous control to achieve velocity consensus and CMCA simultaneously, flocking control is more outstanding, especially in large-scale agents. Besides flocking has an extensive engineering application (Olfati-Saber, 2006), therefore it has attracted much more attention recently. In this paper, we address the flocking control problem based on multiple mobile agents systems in three-dimensional space, and both consensus and CMCA are guaranteed systematically in the case of lacking velocity measurements.

Flocking behavior is prevalent in nature in the form of flocking of birds, schooling of fish and so on. People are curious about how to generate this phenomenon, and begin to do related research. Dating back to 1987, Reynolds proposed three rules (i, Separation; ii, Alignment; iii, Cohesion) of flocking (Reynolds, 1987) by creating computer animation of flocking. Subsequently, many distributed control strategies are proposed for flocking control. To solve velocity consensus, graph theory (Lee and Spong, 2007), Lyapunov approach (Hong et al., 2007), and virtual leader (Su et al., 2009) are used. Artificial potential method (G.Wen et al., 2012) has been used to deal with the problem of CMCA. In (Cucker, 2010), velocity consensus is established by depending on conditions on the initial state of the system. By changing the topology of communication to guarantee velocity synchronization and collision avoidance in (Zavlanos et al., 2009). Other research works focus on studying flocking motion under some constraint conditions. For example, researched objective is from the point mass model to a real model, such as mobile robots (Dong, 2011), hovercrafts (Han and Ge, 2011). The researched space is from two-dimensional to three-dimensional space. The system is considered from full-system to nonholonomic systems system (Tanner et al., 2005). In (Su, 2012), flocking behavior is achieved with virtual leaders which only use position information. Nevertheless, most of these algorithms either just fulfilled one or two rules which previously mentioned, or two-dimensional space.

In this paper, we propose a control law allowing for flocking motion with multiple goals and using the multi-agent system with the dynamic form of a double integrator in three-dimensional space in the case of lacking velocity information. In order to achieve velocity consensus, we choose a velocity observer to estimate velocity information which only depends on position information. To achieve collision avoidance and cohesion maintenance, we develop

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a bounded potential function which can provide a bounded potential force to drive every agent to converge or to separate until the distance between the objective agent and other agents is kept in the desired range. The potential function is structured with respect to an error signal, which means that there has no constraint on the initial state of the system.

The remainder of the paper is organized as follows. In Section 2, we introduce our problem formulation. The main control law is designed in Section 3. We present simulation result in Section 4. And finally, Section 5 provides the ultimate conclusion.

Notations: \( \mathbb{R} \) and \( \mathbb{R}^+ \) are the sets of real numbers and non-negative real numbers, respectively. For \( q = [q_1, \cdots, q_n]^T \), \( \nabla_q \) is the del operator \( \nabla_q = \left[ \frac{\partial}{\partial q_1}, \cdots, \frac{\partial}{\partial q_n} \right]^T \); \( \| \cdot \| \) is the Euclidean norm of vectors.

2. PROBLEM FORMULATION

In this section, we introduce the multi-agent systems with the dynamic form of double integrator. Then we will determine the state of flocking, and state our flocking problem.

2.1 Second Order Particle Model

We consider a group of mobile agents labeled by numbers \( i = 1, \cdots, N \) which are moving in 3-D. The dynamics of each agent is described by the double integrator as follows:

\[
\begin{align*}
\dot{q}_i &= p_i \\
\dot{p}_i &= u_i, \quad i = 1, 2, \cdots, N
\end{align*}
\]

(1)

where \( q_i = [x_i, y_i, z_i]^T \) and \( p_i = [v_{x,i}, v_{y,i}, v_{z,i}]^T \) are the position and velocity vectors of \( i \)-th agent, respectively. \( u_i = [u_{x,i}, u_{y,i}, u_{z,i}]^T \) is the control law of agent \( i \).

Having the model (1), our next task is to formulate a flocking control problem. To this end, it is important to determine the target state to which we shall drive the system. Hence, we have the following subsection to determine the state of flocking.

2.2 The State of Flocking

In this subsection, we determine the desired flocking motion with mathematical rigor as control goal. Our objective is to use the same control algorithm to steer all agents to stable at the desired configuration, and velocity vector will be asymptotically consensus, and no collision occurs in this process.

At first, we consider the requirement of CMCA. In general, this property is reflected by the bounded distance between agents. Accordingly, we hope the distance between each agent stable at the desired range to ensure CMCA. Thus, we have the follow specification,

\[
r \leq r_{ij} \leq R \quad \forall \ i, j = 1, \cdots, N.
\]

(2)

where \( r \) and \( R \) are positive constants which can be designed \((R > r)\), and \( r_{ij} = \| q_i - q_j \| \) is the distance between agent \( i \) and agent \( j \) in three-dimensional space.

But the property of (2) can only solve the problem of CMCA at steady state, we have another problem is how to guarantee collision avoidance in the process of all agents enter steady state. Let us define an error signal \( e_{ij} \):

\[
e_{ij} = \begin{cases} 0 & r_{ij} \leq R \\ -(r_{ij} - r)^2 & r_{ij} < r \\ a & r_{ij} > R \end{cases}
\]

(3)

By means of the error signal \( e_{ij} \) to adjust the potential force to steer agents to converge or to separate, we can encode this property into a potential function \( U_{ij} \in \mathbb{R}^3: \mathbb{R}^3 \to \mathbb{R}^+ \). We choose the artificial potential energy function between agent \( i \) and agent \( j \) as a variation of the function used in (Liang and Lee, 2006; Delgado and Lopez, 2009), as shown in Fig. 1, and is:

\[
U_{ij} = K \cdot b^2 \ln \left( \cosh \left( \frac{e_{ij}}{b} \right) \right)
\]

(4)

where \( K \) is a gain that regulates the magnitude of the potential energy, and \( b \) will be the boundary of the saturation function of the force. The potential function has the following property:

\[
\lim_{e_{ij} \to +\infty} U_{ij} = +\infty, \quad \lim_{e_{ij} \to -\infty} U_{ij} = +\infty
\]

(5)

After defining the potential function between agent \( i \) and agent \( j \), we also define the potential function \( U_i \) of agent \( i \), and total potential function \( U \) of the whole system as follows:

\[
U_i = \sum_{j=1, j\neq i}^{N} U_{ij}(q_i, q_j)
\]

(6)

and

\[
U = \sum_{i=1}^{N} U_i = \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} U_{ij}(q_i, q_j)
\]

(7)

Let us define the potential force with respect to \( e_{ij} \) as follows:

\[
F_{ij} = \dot{U}_{ij} = K \cdot b \cdot \tanh \left( \frac{e_{ij}}{b} \right)
\]

(8)

From Fig. 2, we notice that the potential force is saturated by \( Kb \) and \(-Kb \). The force in agent \( i \) will be attractive if \( e_{ij} > 0 \), and will be repulsive if \( e_{ij} < 0 \).

We wish the desired configuration of stability as \( e_{ij} = 0 \), in other words, if we construct the potential function \( U \) reach its minimal value only at the desired configuration, thus we can achieve the desired result by minimizing \( U \). Accordingly, we have the following specification:

\[
\nabla_q U = 0, \forall \ i, j = 1, \cdots, N
\]

(9)

Then, for a steady collective motion, it is necessary to have a consensus on the velocity vector. Hence, we have the following requirement of velocity consensus:
\[ \lim_{t \to \infty} (p_i - p_j) = 0, \forall i, j = 1, \ldots, N \quad (10) \]

Summarizing the above consideration, we have following.

**Definition 1.** (State of flocking). Given a potential function \( U : \mathbb{R}^{3N} \to \mathbb{R}^+ \). The collective of multi-agent systems (1) is said to be a flocking motion with respect to \( U \) if such that the conditions: i) \( U \) remains bounded below and satisfies (9), and ii) velocity consensus in the sense of (10).

Having the Definition 1, our problem is to fulfill the conditions to develop our control law which can drive the system (1) to the target state of flocking. Before control design, we have the following subsection to formulate the flocking problem for the collective system (1).

### 2.3 Problem Statement

In this subsection, we formulate a flocking control problem whose objective is to drive the collective system (1) to the state of flocking defined in Definition 1. Our condition is that every agent can exchange information with each other, and only position information is available.

However, our current work is based only on the information of position. There is no information about velocity available to guarantee velocity consensus directly. The well-known method of solving this problem is to create a velocity observer. By means of the estimated value of velocity to compensate for the real velocity.

To achieve such objective, and consider Lyapunov-based control design method, we have the following flocking problem

**Problem 2.** (Flocking Control Problem). Developing a Lyapunov function \( V \) and a velocity observer, and design the control inputs \( u_i \) such that:

1. the velocity observation error converge to zero;
2. Lyapunov function \( \dot{V} \leq 0 \); and
3. Definition 1 is satisfied.

**Remark 3.** We shall solve the consensus problem by Lyapunov function method.

Different from traditional control problems such as stabilization where the target state is usually the origin, our control problem is a type of cooperative control problem. The main difficulty of flocking control is that its has multiple control goals. To overcome this difficulty, we shall present our control design in the next section.

### 3. CONTROL DESIGN

After knowing our control problem, in this section, we present a systematic design procedure to solve our flocking problem. The idea of our design is to solve CMCA by the potential function at first, than deal with velocity consensus by Lyapunov theory.

We consider the following Lyapunov function candidate

\[ V_1 = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \dot{r}_{ij}^T \nabla_{r_i} U_{ij} \quad (11) \]

The time derivative of \( V_1 \) is

\[ \dot{V}_1 = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \dot{r}_{ij}^T \nabla_{r_i} U_{ij} \]

\[ = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} (\dot{q}_i^T \nabla_{q_i} U_{ij} - \dot{q}_j^T \nabla_{q_j} U_{ij}) \]

\[ = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} (\dot{q}_i^T \nabla_{q_i} U_{ij} + \dot{q}_j^T \nabla_{q_j} U_{ij}) \]

\[ = \sum_{i=1}^{N} \dot{q}_i^T \nabla_{q_i} U_{ij} \quad (12) \]

\[ = \sum_{i=1}^{N} q_i^T \left( \sum_{j=1, j \neq i}^{N} \nabla_{q_i} U_{ij} \right) \]

\[ = \sum_{i=1}^{N} p_i^T \left( \sum_{j=1, j \neq i}^{N} \nabla_{q_i} U_{ij} \right) \]

\[ = \sum_{i=1}^{N} (p_i - \alpha_i + \alpha_i) \left( \sum_{j=1, j \neq i}^{N} \nabla_{q_i} U_{ij} \right) \]

Let us define

\[ \tilde{p}_i = p_i - \alpha_i \quad (13) \]

with \( \alpha_i \) as the virtual control. In order to drive \( \nabla_{q_i} U_{ij} = 0 \), we design

\[ \alpha_i = -c_1 \sum_{j=1, j \neq i}^{N} \nabla_{q_i} U_{ij} \quad (14) \]

where \( c_1 \) is a positive constant. Thus we obtain

\[ \dot{V}_1 = \sum_{i=1}^{N} (\tilde{p}_i)^T \left( \sum_{j=1, j \neq i}^{N} \nabla_{q_i} U_{ij} \right) \]

\[ - c_1 \sum_{i=1}^{N} \left\| \sum_{j=1, j \neq i}^{N} \nabla_{q_i} U_{ij} \right\|^2 \]

To ensure velocity consensus, we introduce another Lyapunov function,

\[ V_2 = V_1 + \frac{1}{2} \sum_{i=1}^{N} \tilde{p}_i^2 \quad (16) \]

The time derivative of \( V_2 \) is

\[ \dot{V}_2 = \dot{V}_1 + \sum_{i=1}^{N} \tilde{p}_i^T \tilde{p}_i \]

\[ = - c_1 \sum_{i=1}^{N} \left\| \sum_{j=1, j \neq i}^{N} \nabla_{q_i} U_{ij} \right\|^2 \]

\[ + \sum_{i=1}^{N} (\tilde{p}_i)^T \left( \sum_{j=1, j \neq i}^{N} \nabla_{q_i} U_{ij} \right) + \sum_{i=1}^{N} \tilde{p}_i^T \tilde{p}_i \quad (17) \]

\[ = - c_1 \sum_{i=1}^{N} \left\| \sum_{j=1, j \neq i}^{N} \nabla_{q_i} U_{ij} \right\|^2 \]

\[ + \sum_{i=1}^{N} (\tilde{p}_i)^T \left( \sum_{j=1, j \neq i}^{N} \nabla_{q_i} U_{ij} + \tilde{p}_i \right) \]

we design \( \hat{\tilde{p}}_i \) as follows:
\[ \dot{\tilde{p}}_i = -\sum_{j=1, j \neq i}^N \nabla_{q_i} U_{ij} + c_2 \sum_{j=1}^N (\tilde{p}_j - \tilde{p}_i) \]  

(18)

where \( c_2 \) is a positive constant. Thus

\[ \dot{V}_2 = -c_1 \sum_{i=1}^N \sum_{j=1, j \neq i}^N \nabla_{q_i} U_{ij}^2 \]

(19)

By means of Lyapunov theory, the system (1) can be stable. To obtain the control law \( u_i \), from (13) and (18), we have

\[ \dot{\tilde{p}}_i = \dot{\tilde{p}}_i - \dot{\alpha}_i = -\sum_{j=1, j \neq i}^N \nabla_{q_i} U_{ij} + c_2 \sum_{j=1}^N (\tilde{p}_j - \tilde{p}_i) \]

(20)

Finally, we can choose our control law as follows:

\[ u_i = -\sum_{j=1, j \neq i}^N \nabla_{q_i} U_{ij} + c_2 \sum_{j=1}^N (\tilde{p}_j - \tilde{p}_i) + \dot{\alpha}_i \]

(21)

**Theorem 4.** Under the control (21), the system (1) has the follow properties:

1. collision avoidance and cohesion maintenance;
2. the distance between each agent will stable at the desired range \([r, R]\); and
3. all agents consensus on velocity vector.

**Proof:** Due to our design method is based on Lyapunov function \( V_j \) and \( V_2 \), that means the control input \( u_i \) can force \( V_2 \leq 0 \). Hence, for (19), by LaSalle’s invariance principle, the trajectory of the closed-loop system converges to the set \( \Omega = \{(p_i, q_i) : V_2 = 0\} \). Thus, we have

\[ \lim_{t \to \infty} \nabla_{q_i} U_{ij} = 0 \]

(22)

\[ \lim_{t \to \infty} (\tilde{p}_i - \tilde{p}_j)^2 = 0 \]

(23)

As previous mentioned, as \( \nabla_{q_i} U_{ij} = 0 \), the system (1) can achieve CMCA by minimizing the potential function \( U_{ij} \), all agents will reach the desired configuration, i.e., \( r_{ij} \in [r, R] \).

We note that if the trajectory just enters the set \( \Omega \), according to Newton’s law, the velocity is not equal to zero. That means the converged rate of \( \nabla_{q_i} U_{ij} \) is faster than \( p_i \). Accordingly, we have the following lemma:

**Lemma 1.** For \( t > T_m = \max\{T_1, \cdots, T_n\} \), the control law \( u_i \) becomes

\[ u_i = \dot{\tilde{p}}_i = \sum_{j=1}^N (p_j - p_i) \]

(24)

Now we prove the velocity consensus, considering the Lyapunov function

\[ V_3 = \frac{1}{2} \sum_{i=1}^N p_i^2 \]

(25)

The time derivative of \( V_3 \) is

\[ \dot{V}_3 = \sum_{i=1}^N p_i \dot{p}_i \]

(26)

\[ = \sum_{i=1}^N p_i \sum_{j=1}^N (p_j - p_i) \]

\[ = -\sum_{i=1}^N p_i \sum_{j=1}^N (p_i - p_j) \]

\[ = -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N (p_i - p_j)^2 \]

Accordingly, velocity consensus is achieved.

This control law is proposed based on full information. If in the case of lacking velocity measurement, we cannot obtain the velocity information directly. To achieve our goal, we can create a velocity observer to estimate the value of the real velocity. Let \( \tilde{p}_i \) denote estimated value of velocity, choosing a velocity observer was used in (Guerra et al., 2007) as follows:

\[ \dot{\hat{q}}_i = \hat{p}_i + k_0 \hat{q}_i \]

\[ \dot{\hat{q}}_i = k_1 \cdot \sgn(\hat{q}_i) + k_2 \hat{q}_i \]

(27)

where \( k_0, k_1 \) and \( k_2 \) are designed parameters and \( \sgn(\cdot) \) is the signum function, \( \hat{q}_i \in \mathbb{R}^2 \) is the estimated value of position, and \( \hat{q}_i = q_i - q_i^0 \) is the estimate error.

For detailed observer see in (Guerra et al., 2007; Xian et al., 2004).

**4. SIMULATION**

In this section, in order to verify the effectiveness of the proposed flocking control law, we present the results of simulations of applying the control law to multiple mobile agent systems in three-dimensional space.

The related parameters are chosen as follows: \( r = 8; R = 9; K = 5; a = 0.1; b = 0.1; ts = 0.01; c_1 = 0.3; c_2 = 0.4; k_0 = 0.3; k_1 = 0.5; k_2 = 0.2 \), and consider a group of 3 agents, their initial status are selected at random.

![Fig. 3. The trajectories of 3-agents in 3-D](image-url)
Fig. 4. The distances between each agent

Fig. 5. The real velocities and the estimated velocities of 3-agents

Fig. 6. The velocities error between 3-agents in x,y,z-axis, respectively

Fig. 7. The inputs of 3-agents

5. CONCLUSION

In this paper, flocking problem is studied with an assumption of all-to-all communication. A distributed flocking control law is proposed to achieve flocking motion in three-dimensional space based only on measured position. There are no constraint on the initial state of the system, all agents self-propelled and stable at the desired configuration, and velocity vector will be asymptotically consensus, and no collision occurs in this process. Simulation results verify the effectiveness of the proposed method.

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