Seismic Attenuation Estimation From Instantaneous Frequency

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Abstract—An approach is proposed for quality (Q) factor estimation from the variation of envelope peak instantaneous frequency (EPIF). For a frequency-independent Q model, assuming that the propagating wavelet can be modeled by a Gaussian function with constant phase, an approximate analytic relation between Q and EPIF variation is derived. Synthetic tests show that the EPIF method has higher resolution and is less sensitive to noise and interference reflection than common methods. The field test of reflection seismic data indicates that the zone of lower Q-factors corresponds well to the gas reservoir.

Index Terms—Attenuation, instantaneous frequency (IF), quality (Q) factor.

I. INTRODUCTION

Seismic waves propagating through the Earth suffer absorption due to the anelasticity of media. Seismic attenuation is usually measured by the dimensionless quality factor Q, which is defined as the ratio of total energy restored and the energy loss in one cycle of wave propagation. The Q-factor has been estimated with various techniques and assumptions on the frequency dependence of attenuation and pulse spectral shape [1]–[3]. Most of these techniques begin with a constant Q-model, which assumes a linear frequency dependence of attenuation coefficient [3]. Generally, the amplitude variations of seismic signals are employed to estimate the Q-factor, such as the logarithm spectral ratio (LSR) method [1], [2]. The LSR method computes the LSR between the reference and target pulses and applies the least squares linear regression to the spectral ratio of two pulses within a selected frequency band, and then, the slope of a straight line fit to the logarithmic ratio of two spectra yields an attenuation coefficient defined by $\alpha_0 = \pi \Delta f / Q$, where $\Delta$ is the differential travel time between two pulses. Once the attenuation coefficient is known, the Q-factor can be calculated by $Q = \pi \Delta / \alpha_0$. Assuming that propagating seismic wavelets are Gaussian spectral shape, the centroid frequency shifting (CFS) method was used to estimate attenuation for the seismic waves [3] and radar waves [4]. The CFS method calculated the attenuation coefficient with $\alpha_0 = \Delta f_s \sigma_s^{-2}$, where $\sigma_s$ is the standard deviation of the reference pulse, $\Delta f_s$ is the variation in centroid frequencies between the reference and target pulses, and the centroid frequency is defined by the average frequency weighted by the Fourier amplitude spectrum. However, employing amplitude spectra sampled within a time window, these methods are very sensitive to interference reflection and noise. In this letter, we focus on Q-factor estimation by the variation of the envelope peak instantaneous frequency (IF) (EPIF). For a constant-Q model, assuming that the propagating wavelet can be modeled by a Gaussian function with constant phase, we derive an analytic relation between Q and EPIF variation. Finally, the proposed method is examined with both synthetic and field seismic data.

II. RELATION BETWEEN Q-FACTOR AND EPIF VARIATION

Considering a plane wave propagating through a viscoelastic medium for a linear frequency-attenuation model, if neglecting the effects of other factors such as geometrical spreading, instrument response, source/receiver coupling, radiation patterns, and reflection/transmission coefficients, the spectrum $R(\tau + \Delta\tau, \omega)$ of the received pulse can be expressed as [1]–[5]

$$R(\tau + \Delta\tau, \omega) = S(\tau, \omega) e^{i\omega \Delta\tau - \omega \Delta\tau^2 / (2Q)}$$

(1)

where $i$ is the imaginary unit, $Q$ is the Q-factor of the medium, $\omega$ is the angular frequency, $\tau$ is the total travel time with respect to source, and $S(\tau, \omega)$ is the spectrum of reference pulse at a reference wavefield point. Above all, we assume that the reference pulse can be well modeled by a Gaussian function with constant phase as

$$S(\tau, \omega) = A e^{-0.5(\omega - \sigma)^2 \delta^{-2} + i\phi}$$

(2)

where $\sigma$ is the modulated frequency, $A$ and $\phi$ are the amplitude and phase factor, respectively, and $\delta$ is the standard deviation or bandwidth of $S(\tau, \omega)$. For a constant-phase wavelet propagated for time $\Delta\tau$ through a homogenous viscoelastic medium, the EPIF is exactly equal to the average frequency weighted by its amplitude spectrum [5]

$$f_p(\tau + \Delta\tau) = \left[ \int_0^\infty f A(\tau + \Delta\tau, f) df \right]^{-1} \left[ \int_0^\infty A(\tau + \Delta\tau, f) df \right]$$

(3)

where frequencies $f = \omega / (2\pi)$ and $f_p(\tau + \Delta\tau)$, and $A(\tau + \Delta\tau, f)$ are the EPIF and amplitude spectrum after traveling time $\Delta\tau$, respectively. By applying (2) into (3), the EPIF $f_p(\tau)$ of a reference pulse (i.e., $\Delta\tau = 0$) is

$$f_p(\tau) = \sigma / (2\pi) + \Psi_0$$

(4)
For 1–1500 ms in different homogeneous viscoelastic media, the curves of both 10 · \log_{10} R and 10 · \log_{10} R are shown in (a) and (b), respectively [solid line] Q = 7, (circle) Q = 20, (dot) Q = 30, (triangle) Q = 50, and (cross) Q = 100.

where \( \Psi_0 = 0.25\pi^{-2} \delta^2 e^{-0.5\sigma^2-2} \left[ \int_0^\infty e^{-2\pi^2f^2} f^{-1} \right] \). Substituting (1) and (2) into (3), the EPIF \( f_p(\tau + \Delta \tau) \) of the received signal is

\[
f_p(\tau + \Delta \tau) = \sigma/2\pi - \delta^2\Delta\tau/(4\pi Q) + \Psi_1 \tag{5}
\]

where \( \Psi_1 = 0.25\pi^{-2} \delta^2 e^{-0.5\sigma^2-2} \sigma(2\pi\tau - \beta)^2 \left[ \int_0^\infty e^{-2\pi^2f^2} f^{-1} \right] \), and \( \beta = \delta\Delta\tau/(4\pi Q) \).

Letting \( \Psi = (2\pi)^{-1} \delta^2 e^{-2\pi^2\delta^2} \sigma(2\pi\tau - \beta)^2 \), we yield \( \Psi_0 < \Psi \) and \( \Psi_1 < \Psi \) by the probability integration formula. For convenience, let \( R \) be the ratio between \( \beta \) and \( \sigma/2\pi \) and \( R_1 \) be the ratio between \( \Psi \) and \( \Psi_1 \). In order to simplify (4) and (5), one needs to examine the magnitude of \( R \). First, the modulated frequency of a seismic wavelet \( \sigma/2\pi \) is commonly at 30–80 Hz. Second, \( Q \gg 1 \) exists for general viscoelastic media except for some extreme cases (e.g., \( Q \) is 5–50 for gas sandstone, 20–150 for sedimentary rock, and 75–100 for volcanic rock [6]), which may be restricted at 10–150. Third, the differential travel time \( \Delta\tau \) is usually on the order of \( 10^3 \)–\( 10^4 \) ms.

Averagely, \( \Delta\tau/(2Q) \) is on the order of \( 10^{-4} \)–\( 10^{-3} \), the magnitude of \( \beta \) is on the order of \( 10^{-2} \)–\( 10^{-1} \), and, therefore, \( \beta \) is usually much less than \( \sigma/2\pi \). For the aforementioned rocks, if the ratio between the modulated frequency \( \sigma/2\pi \) and bandwidth \( \delta \) is greater than 0.5 (e.g., 0.5–1.25), \( 2\pi^2\delta^2 \sigma/(2\pi - 3\beta) \) may be approximated by \( 0.5\sigma^2\delta^2 \), and its magnitude is greater than five (e.g., 5–30); then, the magnitude of \( \Psi \) (which may be approximated by \( (2\pi)^{-1} \delta^2 e^{-0.5\sigma^2-2} \)) is on the order of \( 10^{-10} \)–\( 10^{-3} \) due to rapid decay of exponential function term. In this way, the magnitude of \( R_1 \) is on the order of \( 10^{-8} \)–\( 10^{-2} \), and \( \Psi_1 \) is two to eight orders of magnitude lower than \( \beta \), i.e.,

\[
\Psi_1 \ll \beta. \tag{6}
\]

For example, to a 50-Hz Gaussian wavelet with \( \delta = 50 \) Hz that has propagated for 1–1500 ms in various homogeneous viscoelastic media, the magnitudes of 10 · \log_{10} R and 10 · \log_{10} R are shown in Fig. 1(a) and (b), respectively. For a quite small \( Q \)-factor such as \( Q = 7 \) in Fig. 1(a), (6) holds true when the differential travel time climbs to 1000 ms. In practice, for vertical seismic profiling (VSP) geometry, as shown in Fig. 2(a), when a seismic wave propagates through two adjacent geophones with a distance of 5–20 m at an apparent velocity of 3–8 km/s, the differential travel time is usually less than 20 ms. For such a small differential travel time, (6) exists for the rocks discussed earlier. Therefore, both \( \Psi_0 \) and \( \Psi_1 \) can be ignored with respect to \( \beta \). Subtracting (4) by (5) while ignoring \( \Psi_0 \) and \( \Psi_1 \) yields

\[
f_p(\tau + \Delta \tau) \approx \delta\Delta\tau/(4\pi Q). \tag{7}
\]

Therefore

\[
Q \approx \delta^2\Delta\tau/(4\pi Q f_p) \tag{8}
\]

where \( \Delta f_p = f_p(\tau) - f_p(\tau + \Delta \tau) \) is the EPIF variation.

As the ratio between \( \sigma/2\pi \) and \( \delta \) is less than 0.5, letting \( q_0 = 0.25\pi^2 e^{-0.5\sigma^2-2} \), \( g_1 = 0.25\pi^2 e^{-2\pi^2\delta^2} \), \( h_0 = \int_{-\infty}^{\infty} e^{-2\pi^2\delta^2} f^2 \), and \( h_1 = \int_{-\infty}^{\infty} e^{-2\pi^2\delta^2} f^2 \int_{-\infty}^{\infty} e^{-0.5\sigma^2-2} f^2 \), subtracting (4) by (5) gives

\[
\Delta f_p = \beta + g_0 h_0 - g_1 h_0/(1 - \gamma)^{-1} \tag{9}
\]

where \( \gamma = h_1 h_0^{-1} \). With \( \beta < \sigma/2\pi \) mentioned earlier, we have \( h_1 \ll h_0 \) and \( \gamma \ll 1 \); therefore, the third term on the right-hand side of (9) can be approximated by the first-order Taylor series

\[
\Delta f_p \approx \beta + 0.25\pi^2 e^{-0.5\sigma^2-2} [1 + (1 + \gamma) e^0 \delta^2 h_0^{-1} e^{-0.5\sigma^2-2} \tag{10}
\]

where \( \theta = 0.5\sigma \Delta\tau/Q - 0.5\delta^2 (0.5\Delta\tau/Q)^2 \). Usually, \( \theta \ll 1 \) and \( \theta \approx 0.5\sigma \Delta\tau/Q \) hold, so we have \( e^{\theta} \approx 1 + 1 + 0.5\sigma \Delta\tau/Q \) by the first-order approximation; neglecting \( \gamma \) simultaneously, (10) can be approximated by

\[
\Delta f_p \approx \beta \left[ 1 - (2\pi)^{-0.5} \int_{-\infty}^{\infty} e^{-0.5\sigma^2-2} f^{-1} (\sigma f^{-1}) \right] \beta \tag{11}
\]

where \( \phi(x) = (2\pi)^{-0.5} \int_{-\infty}^{\infty} e^{-0.5\sigma^2-2} f^{-1} (\sigma f^{-1}) = \lambda(\sigma f^{-1}) \lambda(\sigma f^{-1}) \) is the standard normal distribution function. Similarly, the translation of (11) achieves

\[
Q \approx \lambda(\sigma f^{-1})^{2}\Delta\tau/(4\pi Q f_p) \tag{12}
\]

where \( \lambda(x) = 1 - (2\pi)^{-0.5} e^{-0.5\sigma^2-2} (\phi^{-1}(\sigma f^{-1})) \) is called as a corrected term. The corrected term \( \lambda(\sigma f^{-1}) \) approaches to one when the ratio between \( \sigma/2\pi \) and \( \delta \) is greater than 0.5, and therefore, (12) is a general expression of (8). Both (8) and (12) imply that \( Q \)-factor can be estimated by EPIF variation, and they are basic formulas for \( Q \)-factor estimation by EPIF variation.

\( Q \)-factor measurement by (8) or (12) requires wavelet parameters \( \delta \) and \( \sigma \), differential travel time \( \Delta\tau \), and EPIF to be known. First, a reference pulse may be derived by a seismic
signal recorded by a geophone near the source, and then, modulated frequency and bandwidth can be estimated by [7]

\[ \sigma = \int_{0}^{\infty} |S(\tau, \omega)| d\omega \left[ \int_{0}^{\infty} |S(\tau, \omega)| d\omega \right]^{-1} \] (13)

\[ \delta^2 = \int_{0}^{\infty} (\omega - \sigma)^2 |S(\tau, \omega)| d\omega \left[ \int_{0}^{\infty} |S(\tau, \omega)| d\omega \right]^{-1} \] (14)

A modern source signature is controllable and repeatable, and therefore, it is in good coherence and could be recorded for analysis. In the case of no source information, a reference pulse may be derived by wavelet inversion. Second, differential travel time can be picked from the difference in the arrival time of the envelope peaks of two seismic waves considered. Finally, for time can be picked from the difference in the arrival time of the two seismic waves considered. Finally, for a real signal \( x(t) \), IF is usually calculated by the derivative of phase as [8], [9]

\[ f(t) = (2\pi)^{-1} \left[ x'(t)y(t) - x(t)y'(t) \right] e^{-2(t)} \] (15)

where \( y(t) \) is the Hilbert transform of \( x(t) \), and \( e^2(t) = x^2(t) + y^2(t) \). Of course, IF can also be computed by other approaches according to the signal class and signal-to-noise ratio (SNR) [8], [9]. From the IF curve of a received signal, EPIF can be picked at a point which corresponds to the envelope peak of instantaneous amplitude (IA). However, seismic data are usually contaminated by random noise, which may cause the IF derived by (15) to be unreliable. Large spikes in the IF occur when the denominator of (15) approaches zero more rapidly than the numerator. In order to remove the uninterested large spikes, the damped IF was defined as

\[ f(t) = (2\pi)^{-1} \left[ x'(t)y(t) - x(t)y'(t) \right] / \left[ e^2(t) + \varepsilon e_m \right] \] (16)

where \( e_m = \max \{ e^2(t) \} \), and \( \varepsilon \) is a damper coefficient with \( 0 < \varepsilon \ll 1 \). The damper factor can eliminate the spiky appearance of the IF in areas of low IA but does not significantly affect the value of IF where the IA is large. Moreover, for stability, IF may be further weighted by seismic trace amplitude envelope as [10]

\[ f_w(t) = \int_{t-T}^{t+T} f(t') W(t') dt' \left[ \int_{t-T}^{t+T} W(t') dt' \right]^{-1} \] (17)

where the weighting window is the squared IA, i.e., \( W(t) = e^2(t) \), and \( 2T + 1 \) is the length of the weighting window. The length of the weighting window should be carefully selected. It should be long enough to stabilize the results and yet short enough to preserve the detail in measurements. A variety of weighting window lengths ranging from three- to nine-point was examined, and the results showed that the variation of the weighting window length affects the absolute IF value to some degree, but the relative variation in EPIF was preserved.

In this letter, we call Q-factor estimation by (8) or (12) as EPIF method for short. The procedure of estimating attenuation with the EPIF method includes the following.

1) Calculate parameters \( \delta \) and \( \sigma \) of the reference pulse by (13) and (14), respectively.

2) Pick the differential travel time \( \Delta \tau \) from the arrival time of envelope peaks of seismic waves.

3) Compute EPIF variation between the reference and target pulses.

4) Calculate Q-factor by (8) or (12).

For the direct waves received by two geophones located in VSP geometry, as shown in Fig. 2(a), examining the EPIF variation of the two direct waves during the differential travel time, the Q-factor of the medium between the two geophones may be extracted. Similarly, for two reflected waves in reflection seismic geometry, as shown in Fig. 2(b), investigating the EPIF variation of the two reflected waves during the differential travel time, the Q-factor of the medium between two reflection interfaces may also be obtained.

III. EXAMPLES AND COMPARISONS

A. Synthetic Seismic Data 1

For a wedge model with 45° obliquity, using a 50-Hz Gaussian wavelet with \( \delta = 75 \) Hz, we forward the vertical reflection seismic data, as shown in Fig. 3(a) calculated by the CFS, LSR, and EPIF methods, as shown in Fig. 3(b). Some unrealistic and negative Q-factors occurred due to the wave interference at the interfaces, which makes the centroid frequency upshift but not downshift, as the EPIF predicted. The resolution is 25 m for the EPIF method and 45–50 m for two other methods. Thus, the EPIF method has higher resolution than the two other methods.

For a synthetic record of 50-m offset, as shown in Fig. 3(a), we test noise resistance by adding Gaussian white noises. For 500 repeated tests of various noise levels (i.e., \( SNR = 2–50 \) dB), Fig. 4(a) and (b) shows the relative error and standard deviation of Q-factors estimated by three methods. The result shows that the EPIF method is more noise resistant than the two other methods.

B. Synthetic Seismic Data 2

Fig. 5(a) shows the synthetic VSP data calculated at 100 geophones for a 50-Hz constant-phase source wavelet located at the Earth surface, in which the sampling rate is 2 ms
and the distance between two adjacent receivers is 5 m. For comparison, $Q$-factors are estimated by three methods [Fig. 5(b)]. Only the EPIF method can work well at 110–145 m and 245–305 m, where the direct waves are partially overlapped by the reflected waves. The results of $Q$-values estimated by three methods are improved using downward waves after upward waves were separated [see Fig. 5(c)]. This test result shows that the proposed method has less sensitivity to the interference of reflection waves and higher resolution than the two other methods.

C. Reflection Seismic Data

We also test the proposed method by real reflection seismic data. Fig. 6(a) shows a stacked seismic profile of a gas field, in which the seismic data are recorded by the digital geophones with a sampling rate of 1 ms. The distance between two arbitrary adjacent geophones is 25 m, all the geophones and source lay out in a line on the Earth surface, and the source is excited by the explosive at the Earth surface. The common shot point gathers are processed by resetting as common middle point gathers, normal move-out correction, stacking, and migration, which yield a profile as shown in Fig. 6(a). Two wells cross through this profile. Well 1 was drilled through 9.2-m gas sandstone with an open flow potential of $4.1481 \text{ m}^3/\text{d}$ at the gas test, but well 2 was drilled through 8-m sandstone without gas reservoir developed. For convenience, seismic waves within 900–1250 ms and 1250–1600 ms are considered as reference and target pulses, respectively. Fig. 6(b) shows the $Q$-factors estimated by the CFS, LSR, and EPIF methods. Real seismic data are always affected by many factors, which may lead different effects to each $Q$-estimation method; therefore, the $Q$-values estimated by three methods differ from each other. However, the $Q$-factors estimated by three methods share with a similar changing tendency on the whole. At the target reservoir (around at 1300–1350 ms), well 1 corresponds to lower $Q$-factors, while well 2 corresponds to larger $Q$-factors. The natural Gama (GR) and acoustic (AC) logs of (thick line) well 1 and (thin dashed–cross line) well 2 are given in (c) and (d), respectively. Usually, gas reservoir corresponds to a lower AC velocity and a lower GR value. At the same time, gas sandstone corresponds to a lower $Q$-factor, but dry sandstone corresponds to a larger one [11]. The results show that lower $Q$-factors correspond well to the gas sandstone and higher $Q$-factors correspond to dry sandstone.

IV. LIMITATIONS AND PRACTICAL CONSIDERATIONS

The approximation relation between $Q$ and EPIF variation is derived based on a hypothesis of a Gaussian impulse with constant phase. A constant-phase wavelet, being noncasual, is not physically realistic; however, a real causal reference wavelet can be converted to a constant-phase wavelet by an appropriate phase rotation. Theoretically, dispersion accompanies the attenuation of propagation wavelet, which causes the phase spectrum of the wavelet being a function of time, as well as frequency. Therefore, the phase-rotation operator should be a function of both time and frequency. However, the time dependence
is probably negligible in reflection seismology and may be ignored here. Moreover, wavelets derived by seismic-to-well correspondence often have a nearly constant phase. Correlated and undeconvolved vibroseis data should have nearly constant phase if the phase contributions of the geophones and recording system are small, and therefore, such data could be suitable for this analysis. The seismic to be analyzed should have a higher SNR without inverse-$Q$ filtering.

Furthermore, the error of the EPIF method is analyzed. Let the ratio $\eta = \sigma/(2\pi\delta)$ be $0.3–0.7$, $\Delta\tau$ be $10–100$ ms, and $Q$ be $5–150$. With the quantities of wavelet parameters, differential travel time, and EPIF variation estimated accurately, the error bar of $Q$-factors estimation by EPIF variation is shown in Fig. 7. The errors increase up to $14\%$ for the lowest $\eta$ and the biggest $\Delta\tau/Q$, which shows that errors increase for a lower $Q$ and a bigger $\Delta\tau$. A seismic wavelet with $\eta < 0.35$ will produce a poor approximation by a Gaussian wavelet, which conflicts with our assumption for the wavelet waveform. Thus, the error is bigger for $\eta < 0.35$. As $\eta > 0.35$, the errors are less than $6\%$, which are basically caused by the approximation in the deduction of (8) and (12).

Unfortunately, the estimation results of the EPIF method may be affected by the scattering and thin-bed tuning, which pollutes the IF calculation. Our further research will pay close attention to overcome this shortcoming, such as by using much upward/downward wave separation in seismic reflection prior to applying the EPIF method.

V. Conclusion

In this letter, we have proposed an EPIF method for $Q$-factor estimation, which avoids the subjective selection for time window and complicated iteration process. Synthetic test shows that the EPIF method has higher resolution, better noise resistance, and less sensitivity to interfering reflections. Synthetic and field examples have shown the validity of EPIF method for $Q$-factor estimation. The proposed method may become a helpful tool for gas reservoir characterization and attenuation estimation.

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