Simultaneous measurement of optical inhomogeneity and thickness variation by using dual-wavelength phase-shifting photorefractive holographic interferometry

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A method to measure thickness variation and optical inhomogeneity simultaneously with a dual-wavelength phase-shifting photorefractive holographic interferometer is proposed. This method has no special requirements on the sample surfaces, and additional operations such as coating refractive index matching liquid on surfaces is not needed. With the help of photorefractive holographic interferometry, the wavefront distortion caused by the interferometer system is compensated automatically. Compared with other methods, this method is simpler and more accurate. Computer simulation and optical experiment have verified its feasibility and reliability.

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1. Introduction

Optical inhomogeneity and thickness variation are two important parameters of optical components such as optical window, thin film [1], substrate [2] and crystals [3]. Due to its lower measurement uncertainty, higher precision and spatial resolution, optical interferometry has become the most common method of measuring the two parameters. A variety of interferometric methods to separately or simultaneously measure the two parameters have been reported [3–28]. In the separately measuring methods [6–20], one of the two parameters is usually a known quantity. While as for the simultaneously measuring methods, there are still some limitations [3,13,21–28]. For example, the method proposed by Park [3] needs as much as 13 interferograms and the data processing is time-consuming and cumbersome; in some methods, the front and rear surfaces of the sample must be parallel [21–24], or a specific wedge [25,26] is required to have convenient fringe spacing. Roberts and Langenbeck described a method [13] in which no specific wedge is required, but the sample must be initially coated on both surfaces and the coatings need to be successively removed between each two measurements; several alternatives [27,28] are limited to a point measurement of a small area, not the whole surface.

In this paper, to simultaneously measure both optical inhomogeneity and thickness variation, we propose a method based on dual-wavelength phase-shifting photorefractive holographic interferometer and the fact that under normal dispersion condition and in a relatively small wavelength range of visible light region the optical inhomogeneity for different wavelengths is approximately equal. In the present method, the specific wedge or parallelism of sample surfaces is not necessarily needed and the whole sample can be measured without additional operations, therefore, the range of its application is expanded; furthermore, the system errors are greatly reduced because of photorefractive holographic interferometry. We will first explain its principle and then give its verification by computer simulations and optical experiments.

2. Principle of measurement

The experimental setup of the proposed dual-wavelength phase-shifting photorefractive holographic interferometer is shown in Fig. 1, where M, BS, S, BE, and L denote reflection mirror, beam splitter, shutter, beam expander, and lens, respectively. It consists of two common optical configurations, one is the 4f Fourier transform system and the other is the Mach–Zehnder interferometer. The 4f Fourier transform system comprises lenses L1 and L2, the tested sample and CCD are placed on its input plane (object plane) and output plane (image plane), respectively, and the photorefractive crystal (PRC) used as the holographic recording...
media is located in the focal plane of L2 and L3. The Mach–Zehnder interferometer comprises the BS, M2, M3, and PRC. The light from Laser 1 (or Laser 2) is split into two beams by BS which is used as object wave and reference wave in photorefractive holographic recording processes, respectively. The object wave is collimated by BE and L1, illuminates the sample, and then is incident in the PRC. The reference wave is reflected by M3 attached to the PZT and then incident in the PRC. Finally, the two waves overlap and interfere in the PRC, and the volume hologram (grating) is formed via the photorefractive effect [29] in the PRC.

The recorded object wave can be reconstructed by the reference wave. When the object waves in different states or at different time are separately recorded and then simultaneously reconstructed, photorefractive holographic interferometry [30] is realized. The phase of the reference wave can be successively changed by moving M3 via the PZT controlled by computer and the corresponding phase-shifted holographic interferograms are recorded by CCD and processed in computer.

2.1. Relationship between phase difference and optical inhomogeneity and thickness variation

The object waves in the output plane in the case with and without the sample placed in the input plane are defined as signal wave \( U_1(x, y) \) and cavity wave \( U_2(x, y) \), respectively. Assuming the phase of the signal wave is denoted by \( \psi_1(x, y) \) and that of the cavity wave denoted by \( \psi_2(x, y) \), the phase difference between the two waves can be expressed as

\[
\Delta \psi(x, y) = \psi_1(x, y) - \psi_2(x, y) = (2\pi/\lambda)[n(x, y) - n_0]d(x, y),
\]

(1)

where \( \lambda \) is the light wavelength, \( n_0 \) is the refractive index of air and can be considered as a constant, \( n(x, y) \) and \( d(x, y) \) are the refractive index and thickness of the sample, respectively. In order to describe the optical inhomogeneity and thickness variation of the sample, the refractive index and thickness are denoted as the function of space coordinates, and can be expressed as

\[
n(x, y) = n_0 + \Delta n(x, y),
\]

(2)

and

\[
d(x, y) = d_0 + \Delta d(x, y),
\]

(3)

respectively. Where \( n_0 \) is the average refractive index, \( d_0 \) is the average thickness, \( \Delta n(x, y) \) and \( \Delta d(x, y) \) are the optical inhomogeneity and thickness variation to be measured, respectively. From Eqs. (1)–(3) we can obtain

\[
\Delta \psi(x, y) = \psi_1(x, y) - \psi_2(x, y) = (2\pi/\lambda)[n_0 - n_0 + \Delta n(x, y)][d_0 + \Delta d(x, y)],
\]

(4)

The refractive index of optical materials depends on the light wavelength. The wavelengths of Laser 1 and Laser 2 are denoted by \( \lambda_1 \) and \( \lambda_2 \), respectively. In the measurement procedure, the light beams from Laser 1 and Laser 2 are separately controlled by shutter \( S_1 \) and \( S_2 \), respectively. For Laser 1 and Laser 2, Eq. (4) can be separately expressed by the following two equations:

\[
\Delta \psi_1(x, y) = \psi_1(x, y) - \psi_2(x, y) = (2\pi/\lambda_1)[n_01 - n_0 + \Delta n_1(x, y)][d_0 + \Delta d(x, y)],
\]

(5)

\[
\Delta \psi_2(x, y) = \psi_1(x, y) - \psi_2(x, y) = (2\pi/\lambda_2)[n_02 - n_0 + \Delta n_2(x, y)][d_0 + \Delta d(x, y)],
\]

(6)

where \( n_{01}, n_{02}, n_0, d_0 \) are known quantities. According to the Lorentz–Lorenz dispersion equation [31], under normal dispersion condition and in a relatively small wavelength range of visible light region, the optical inhomogeneity for different wavelengths is approximately equal, that is, \( |\Delta n_1(x, y) - \Delta n_2(x, y)| \) is far less than \( \Delta n_1(x, y) \) or \( \Delta n_2(x, y) \), therefore the difference between \( \Delta n_1(x, y) \) and \( \Delta n_2(x, y) \) can be ignored. Hence we can let

\[
\Delta n_1(x, y) = \Delta n_2(x, y) = \Delta n(x, y).
\]

(7)

From Eqs. (5)–(7), the optical inhomogeneity \( \Delta n(x, y) \) and thickness variation \( \Delta d(x, y) \) of the sample can be obtained as

\[
\Delta n(x, y) = [\lambda_1\Delta \psi_1(n_01 - n_0 - \lambda_1\Delta \phi_1(n_01 - n_0))/\lambda_1\Delta \phi_1 - \lambda_2\Delta \phi_2),
\]

(8)

\[
\Delta d(x, y) = (\lambda_1\Delta \phi_1 - \lambda_2\Delta \phi_2)/[2\pi(n_01 - n_02)]; d_0,
\]

(9)

2.2. Measurement of phase difference \( \Delta \psi_1(x, y) \) and \( \Delta \psi_2(x, y) \)

In Eqs. (8) and (9), \( \Delta \psi_1(x, y) \) and \( \Delta \psi_2(x, y) \) can be obtained by experimental measurement with phase-shifting photorefractive holographic interferometry shown in Fig. 1. The experiment procedure is given as follows:

Stage 1: Let \( S_3 \) open and \( S_4 \) shut, that is, only laser 1 is used. First, no sample is placed on the input plane, and open \( S_3 \) and \( S_4 \) at the same time. In PRC, the object wave interferes with the reference wave, and the resulting intensity distribution produces via the photorefractive effect a volume hologram. After the object wave is recorded in PRC, let \( S_3 \) and \( S_4 \) shut simultaneously. This recorded object wave can be subsequently reconstructed by the reference wave. As mentioned before, in the case when no sample is placed on the input plane, the reconstructed object wave is defined as cavity wave and denoted by \( U_1(x, y) = A_1(x, y)\exp[i\varphi_{c1}(x, y)] \). Second, place the sample on the input plane, and then open \( S_3 \) and \( S_4 \) synchronously. In this case, two object waves are obtained on the output plane. One is the object signal wave which passes through the sample and PRC and then irradiates on CCD, it can be denoted by \( U_2(x, y) = A_2(x, y)\exp[i\varphi_{c2}(x, y)] \). The other is the reconstructed cavity wave \( U_2(x, y) \), which has been recorded in the PRC and is reconstructed by the present reference wave. The two waves, signal wave and cavity wave, interfere and form interferogram at the target plane of CCD. The interferogram is recorded by CCD and processed by computer. To realize phase-shifting interferometry, the phase of the reference wave as well as that of the reconstructed cavity wave can be successively changed by moving M3 attached to the PZT. Here the two-step generalized phase-shifting interferometry (GPSI) [32,33] is employed and the intensities of the interferograms can be written as

\[
l_{k1}(x, y) = A_{k1}^2 + A_{k1}^2 + 2A_{k1}A_{k1}^* \cos \{ \varphi_{c1}(x, y) + \varphi_{c1}(x, y) + (k-1)\delta_1 \} = A_{k1}^2 + A_{k1}^2 + 2A_{k1}A_{k1}^* \cos \{ \Delta \varphi(x, y) + (k-1)\delta_1 \},
\]

(10)

where \( k = 1, 2 \), respectively; and \( \delta_1 \) is the phase shift.

Stage 2: Let \( S_2 \) open and \( S_1 \) shut, that is, only laser 2 is used. The following procedure in this stage is similar as in stage 1. The only difference is that after stage 1 the position of the
As a result of the using of photorefractive crystal, the phase differences \( \phi_1(x, y) \) and \( \phi_c(x, y) \) can be reconstructed directly with phase-shifting photorefractive holographic interferometry. Otherwise, the phase \( \phi_1(x, y) \) and \( \phi_c(x, y) \) of signal and cavity wave have to be retrieved separately, and then the phase differences can be calculated. Thus, the wavefront distortion caused by the interferometer system is eliminated and the system errors are reduced greatly.

3. Computer simulations and optical experiments

The computer simulations and optical experiments have been carried out to demonstrate the present method.

In computer simulation experiment, the wavelengths of the two light sources are \( \lambda_1 = 632.8 \) and \( \lambda_2 = 532 \) nm. A virtual optical plate with a wedge angle of \( 8 \times 10^{-4} \) rad is used as the tested sample. Assume that its average refractive indices for \( \lambda_1 \) and \( \lambda_2 \) are \( n_{10} = 1.4570 \) and \( n_{20} = 1.4607 \), respectively, and its average thickness is \( d_0 = 12.7 \) mm. The optical inhomogeneity of the sample, \( \Delta n(x, y) \), is represented by the peaks function in MATLAB, which is expressed as

\[
\Delta n(x, y) = 3(1-x)^2 \exp[-x^2-(y+1)^2] - 10(x/5-x^2-y^2) \exp[-x^2-y^2] - (1/3) \exp[-(x+1)^2-y^2].
\]

Its three-dimensional distribution is shown in Fig. 2(a) and its PV value (difference between the maximum and minimum value) is

\[
\Delta = \text{PV value}.
\]
set as $1.8082 \times 10^{-6}$. Without loss of generality, the surface shape variation of the sample is represented by function

$$w(x, y) = 5x(x^2 + y^2) - x/(4R) - y,$$

(15)

where $R = 100$ cm. Its three-dimensional distribution is shown in Fig. 3(a) and its PV value is set as $1.6850 \times 10^{-6}$ m.

Two groups of interferograms are generated for $\lambda_1$ and $\lambda_2$. Fig. 4(a) and (b) are two interferograms for $\lambda_1$ corresponding to the case before and after phase-shifting, respectively, and Fig. 4(c) and (d) are that for $\lambda_2$. The phase shift between adjacent interferograms is set as $\pi/2$. In practice, the actual phase shifts deviate from the preset ones due to the factor as disturbance and mechanical vibration. Therefore, two-step GPSI algorithms [32] are utilized to extract actual phase shifts and reconstruct the phase differences $\Delta \varphi_1$ and $\Delta \varphi_2$.

The thickness variation depends on the wedge angle and surface shape variation. However, in three-dimensional distribution of the thickness variation, only an inclined plane can be seen due to the effect of the wedge angle and we cannot distinguish whether the calculated result is in accord with the preset one. To intuitively compare the preset and calculated results of thickness variation, the effect of the wedge angle is eliminated by spatial filtering. Figs. 2(b) and 3(b) are the measured three-dimensional distributions of optical inhomogeneity and thickness variation (without the effect of wedge angle), respectively. Actually we cannot find any difference between Figs. 2(a) and (b) and 3(a) and (b). The PV and RMS (root mean square) values of optical inhomogeneity are calculated and listed in Table 1, and that of thickness variation are calculated and listed in Table 2. For optical inhomogeneity, the preset PV value is in full accord with the measured value, the relative error between preset and measured PV value is 0.004%. For thickness variation, the relative error between preset and measured PV value is 0.04%, the preset RMS value is in full accord with the measured value.

In optical experiments, He–Ne laser of wavelength $\lambda_1 = 632.8$ nm and frequency doubled Nd:YAG laser of wavelength $\lambda_2 = 532$ nm are used as light sources. The tested sample is an optical flat (NF48–129, Edmund Optics Ltd.) with the diameter of 1 in. and the average thickness of 0.5 in. The optical inhomogeneity of the sample is at the order of magnitude of $10^{-7}$. The accuracy of each surface of the sample is $1/20 \lambda_1$ (PV value for one surface shape variation). The average refractive indices of the sample for $\lambda_1$ and $\lambda_2$ are $n_1 = 1.4570$ and $n_2 = 1.4607$, respectively. The Ce:SBN crystal with size of $0.75 \text{ cm} \times 0.55 \text{ cm} \times 0.45 \text{ cm}$ is used as holographic recording media, and its optical axis is in the horizontal direction and parallel to its front surface. The CCD camera (EO-1918M, Edmund Optics Ltd.) has $1600 \times 1200$ pixels with a pixel size of $4.4 \times 4.4 \mu \text{m}$ and pixel depth of 8-bit (256 Gy levels).

Two interferograms are recorded for each wavelength. Fig. 5(a) and (b) are two interferograms for $\lambda_1$ and Fig. 5(c) and (d) are that for $\lambda_2$. The phase shift between adjacent interferograms is set as $\pi/2$. The phase difference between signal and cavity waves for two wavelengths, $\Delta \varphi_1$ and $\Delta \varphi_2$, are retrieved with two-step GPSI.

### Table 1

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<th>Thickness variation</th>
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<tr>
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<td>Measured value</td>
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### Table 2

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<td></td>
<td>PV ($10^{-6}$)</td>
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<tr>
<td>Given value</td>
<td>1.8082</td>
</tr>
<tr>
<td>Measured value</td>
<td>1.8082</td>
</tr>
</tbody>
</table>

Fig. 4. (a) and (b) Two interferograms in computer simulation when the wavelength is $\lambda_1$. (c) and (d) Two interferograms when the wavelength is $\lambda_2$. 
algorithm [32]. And the optical inhomogeneity of the sample and its thickness variation, $\Delta n(x,y)$ and $\Delta d(x,y)$, can be calculated by using Eqs. (8) and (9), which are shown in Figs. 6 and 7, respectively. The measured PV value of optical inhomogeneity with our method is $3.2255 \times 10^{-7}$, which is in accord with the actual order of magnitude of the optical inhomogeneity ($10^{-7}$) given by the manufacturer.

In practice, the two surfaces of an optical flat cannot be absolutely parallel and there is always a small wedge angle. The thickness variation of the sample depends on its surface shape variation and wedge angle. The equidistant parallel fringes in the interferograms shown in Fig. 5 are caused by the wedge angle of the tested sample.

The accurate wedge angle of the sample and its detailed thickness information are not given by the manufacturer except the parameters about the accuracy of two surfaces, therefore in order to evaluate our method we can eliminate the effect of wedge angle by spatial filtering and estimate the PV value of thickness variation of the sample from its given accuracy of surfaces, and then compare it with the measured PV value. The calculated thickness variation shown in Fig. 7 has eliminated the effect of wedge angle by spatial filtering, and it is merely caused by the surface shape variation. The given accuracy of one surface of the sample is $1/20\lambda_1 (3.164 \times 10^{-8} \text{ m})$, so the estimated PV value of thickness variation resulted from its two surfaces should be less than $1/10\lambda_1 (6.328 \times 10^{-8} \text{ m})$. The measured PV value of the

Fig. 5. (a) and (b) Two interferograms in optical experiment when the wavelength $\lambda_1=0.6328 \mu\text{m}$; (c) and (d) Two interferograms when the wavelength $\lambda_1=0.532 \mu\text{m}$.

Fig. 6. Measured distribution of optical inhomogeneity in optical experiment.

Fig. 7. Measured distribution of thickness variation in optical experiment.
sample with our method is $1.0008 \times 10^{-8}$ m and it is indeed in the range of $1/10\lambda$.

4. Conclusion

In summary, a novel method to measure the thickness variation and optical inhomogeneity simultaneously is proposed, which is realized by using of dual-wavelength phase-shifting photoreflective holographic interferometry and based on the fact that under normal dispersion condition and in a relatively small wavelength range of visible light region the optical inhomogeneity for different wavelength is approximately equal. Computer simulations and optical experiments have shown that this method could measure thickness variation and optical inhomogeneity simply and accurately. Different from other methods, the sample is not needed to be adjusted and coated with refractive index matching liquid in measurement, and requirements such as special wedge or parallelism of sample surfaces are no longer necessary. Besides, wavefront distortion caused by the interferometer system can be automatically compensated due to the use of photoreflective holographic interferometry.

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