The Reliability Analysis of Turbine Blade Based on Strength Degradation

Jie Zhou, Hong-Zhong Huang*, Zhaochun Peng, Jun-Yu Guo, Cheng-Geng Huang
Center for System Reliability and Safety,
University of Electronic Science and Technology of China,
Chengdu, Sichuan, 611731, P.R. China

ABSTRACT
Fatigue failure is a process of crack initiation, crack propagation under continued cyclic loading, and finally leading to the fracture or failure. It can also be taken as a result of damage accumulation or strength degradation. In order to ensure the stability and reliability of aero engine, finite element analysis (FEA) is used to get the high stress regions, and strength degradation is considered as a dominated factor of integrated effect that leads to failure. The stress-strength interference (SSI) model is then used to assess the reliability of the high-pressure turbine blade.

KEYWORDS: Fatigue failure; Turbine blade; Strength degradation; Stress-strength interference

1 INTRODUCTION
The blade is one of the highest risk components in aero engine. According to the data of the failures of components in aircraft, 70% of those are ascribed to the blades [1]. The blades should have good operating characteristics under different working conditions. The turbine blade is often subjected to the high temperature and high pressure, and its life may be governed by a series of failure mechanisms, such as fatigue, creep, fracture, yielding, wear, corrosion, erosion, etc. If one of the turbine blades is broken, the aero engine system will fail, resulting in great economic loss and the death of passengers and air crew members. Therefore, it is essential to analyze the reliability of turbine blade.

Due to the complex nature of blade geometry, some aspects and invalid features that cause the singular element or influence the analysis of results should be simplified. During service operating, the rotational speed, working temperature, and aerodynamic should be known before analyzing the model of turbine blade. The ANSYS Workbench software is used to get the stress and critical regions. The mechanical properties of the material degrade progressively when the turbine blade is in service. The SSI model is used to analyze its reliability under a certain working condition.

2 DESCRIPTION OF HIGH-PRESSURE TURBINE BLADE

2.1 Blade Geometry
There are three critical regions [2-3] in the high-pressure turbine blade: the fir-tree mortise, the deep hole and the circularity transition root. Considering the complexity of structures and loadings, there are mainly two models for static analysis [4-7]. The blade body and fir-tree mortise are modeled separately to simplify the difficulties.

1) The blade is modeled to analyze the critical regions (deep hole and circularity transition root) in blade body, but it cannot get accurate results for the fir-tree mortise. The model A is shown in Figure 1.
2) The blade-disc coupled system is modeled to get the critical region of fir-tree mortise, which uses a nonlinear contact analysis technique that costs more time. In order to ensure the accuracy of the results and reduce the difficulties of meshing, a simplified version of the three-dimensional model for the 1/n high-pressure turbine disc (remain one fir-tree mortise) segment was created. The model B is shown in Figure 2.

2.2 Blade Material
The high-pressure turbine blade has good performances of corrosion resistance, high temperature and high strength. The chemical compositions and the material properties of Ni-base super alloy K403 can be found in [8], as listed in Table 1.

* Corresponding Author: hzhuang@uestc.edu.cn
Tel: +86-28-6181252; Fax: +86-28-61830227.
2.3 Working Condition

High-pressure turbine blades endure both mechanical and thermal loadings during service operation, which include centrifugal forces, thermal stresses, aerodynamic forces, vibratory stresses, and so on [1]. The centrifugal forces, thermal stresses, and aerodynamic forces have a significant effect for the static analysis of turbine blade. When analyzing the turbine blade by ANSYS Workbench, the vibratory stresses can be ignored.

(1) Centrifugal forces: high-pressure turbine blades work at a high rotating speed. Centrifugal forces are generated by the rotation;

(2) Thermal stresses: high-pressure turbine blades are surrounded by air with high temperatures. In order to obtain the stresses accurately, it is necessary to simulate the temperature fields. High temperature will decrease the material resistance. Based on the theories of metallic elastic and plastic deformation, the non-uniform distribution of temperature and different expansion deformation in radius direction will lead to great thermal stresses.

(3) Aerodynamic forces: high-pressure turbine blades are surrounded by air with pressures, which are distributed on the surface of blade.

2.4 Finite Element Analysis

The speed spectrum of high-pressure turbine blade is determined by flight mission. It can be divided into three parts according to the rotational speed, and the loading spectrum of 700 hours is shown in Table 2. The thermal stresses and aerodynamic forces are obtained from the experimental data. The temperature spectrum of turbine blade is loaded on the three-dimensional model for three kinds of working conditions. The aerodynamic forces for three kinds of working conditions are loaded on the surface of blades in ANSYS Workbench. From Figure 3 and Figure 4, the maximum speed is \( \omega = 17000 \text{rpm} \). By using a similar procedure, we can obtain the results for other working conditions.

Two models are used to calculate different critical regions, as stated in the Section 2.1. After a series of prepared work (input the properties of K403, meshed the model, set the loads and boundary conditions), we can get the results of stress analysis for model A and model B under three different working conditions, as shown in Figure 5, Figure 6 and Figure 7, respectively.

![Figure 1 The Blade](image1)

![Figure 2 The Blade-Disc Coupled System](image2)

![Figure 3](image3)

![Figure 4](image4)

![Figure 5](image5)

![Figure 6](image6)

![Figure 7](image7)

<table>
<thead>
<tr>
<th>Working condition</th>
<th>Number of cycles N</th>
<th>Rotational speed ( \omega ) (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>1200</td>
<td>0-17000-0</td>
</tr>
<tr>
<td>S2</td>
<td>1800</td>
<td>9200-17000-9200</td>
</tr>
<tr>
<td>S3</td>
<td>23000</td>
<td>16500-17000-16500</td>
</tr>
</tbody>
</table>

Two models are used to calculate different critical regions, as stated in the Section 2.1. After a series of prepared work (input the properties of K403, meshed the model, set the loads and boundary conditions), we can get the results of stress analysis for model A and model B under three different working conditions, as shown in Figure 5, Figure 6 and Figure 7, respectively.
The results for the working conditions are shown in Table 3. According to the results obtained, fir-tree mortise, deep hole and circularity transition root are the critical regions, and fir-tree mortise is the most critical region under the same working condition. It suggests that fir-tree mortise is the most dangerous area, where the crack propagation of blade always occurs.

### Table 3 The Results of FEA for Turbine Blade

<table>
<thead>
<tr>
<th>Critical area</th>
<th>Speed ( \omega ) (rpm)</th>
<th>circularity transition root (MPa)</th>
<th>deep hole (MPa)</th>
<th>fir-tree mortise (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9200</td>
<td>106.43</td>
<td>157.66</td>
<td>247.31</td>
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<tr>
<td></td>
<td>16500</td>
<td>346.33</td>
<td>501.34</td>
<td>795.56</td>
</tr>
<tr>
<td></td>
<td>17000</td>
<td>358.08</td>
<td>548.05</td>
<td>796.70</td>
</tr>
</tbody>
</table>

### 3 STRENGTH DEGRADATION

The fatigue damage evolution mechanism is complex for different structures and materials under cyclic loadings, and it is also the foundation of life prediction for engineering applications. Based on a lot of experimental data, many cumulative damage models [9] are proposed to describe the damage development of materials, which can be defined by strength degradation, stiffness degradation, and energy dissipation. The strength degradation evolution can be characterized by the linear or nonlinear model, and a nonlinear model is more suitable to reveal the degradation evolution.

Fatigue failure is mainly the process of strength degradation under continued cycle loadings, and it is related to the applied cycles and stresses, shown as

\[
\sigma_R(n) = f(n, \sigma)
\]

where \( \sigma_R(n) \) is the residual strength, \( n \) is the number of loading cycles, \( \sigma \) is the applied cyclic stress. And the boundary conditions are shown as follow: (1) \( \sigma_R(0) = \sigma_s \), the residual strength equals to the initial static strength; (2) \( \sigma_R(N) = \sigma \), it equals to the applied cyclic stress; when the number of loading cycles is \( N \), the material becomes “sudden death”; (3) \( \sigma_R(n) \) is a decreasing function with the loading cycles.

According to the degradation law based on the experimental investigation, several researchers [10] proposed a deterministic degradation model, that is,

\[
\frac{d\sigma_R(n)}{dn} = -AB\sigma_{\text{max}}^{1/A} \sigma^{1-1/A}
\]

where \( \sigma_{\text{max}} \) is the maximum cyclic stress, \( A \) and \( B \) are material constants that can be obtained from experiments.

Lu et al. [11] proposed a more simple equation based on the experimental data and S-N curve, as shown in Eq. (3).
\[
\frac{d\sigma_r(n)}{dn} = -\sigma^p \sigma_r^{-q}(n)
\]  
(3)

where \(\sigma_r(0) = \sigma_0\), and \(\sigma_0\) is the tensile strength and greater than \(\sigma_b\); \(p\) and \(q\) are material constants. By integrating Eq. (3), we obtain

\[
\sigma_r^{1+q}(n) = \sigma_0^{1+q} - (1 + q)\sigma_n^p n
\]  
(4)

Generally, \(\sigma\) is less than \(\sigma_0\), and \(\sigma^p\) is much smaller than \(\sigma_0^{1+q}\). Thus, Eq. (4) can be expressed as

\[
\sigma^p N = \frac{\sigma_0^{1+q}}{1+q}
\]  
(5)

Eq. (5) can be derived from the S-N curve, and \(p\) and \(q\) can be determined.

When dealing with variable amplitude or spectrum loading of the fatigue damage, the residual strength can be described as Eq. (6) for \(k\)-level fatigue loadings.

\[
\sigma_r^{1+q}(n_k) = \sigma_0^{1+q} - (1 + q)\sum_{i=1}^{k} \sigma_i^p n_i
\]  
(6)

The Eq. (6) indicates that the stiffness or strength degenerates under multi-level loading stresses, and the degradation rate can be expressed as

\[
A_i = \frac{\sigma_r(n_i)}{\sigma_r(n_{i-1})}
\]  
(7)

Combining Eq. (6) and Eq. (7), the residual strength [12] can be obtained as

\[
\sigma_r(n_k) = A_0 \times A_1 \times A_2 \dots A_k \times \sigma_0
\]

\[
= [1 - \frac{1}{\sigma_0^{1+q}} \sum_{i=1}^{k} \sigma_i^p n_i^{\frac{1}{1+q}}]^{1+q} \times \sigma_0
\]  
(8)

### 4 SSI MODEL

#### 4.1 Definition of Parameters

The high-pressure turbine blade in service is subjected to three-dimensional stress with variable loadings. According to the temperature of blades and the S-N curve [8], \(p\) and \(q\) can be obtained as

\[
p = 7.7665
\]

\[
q = 7.7969
\]

It is well known that the mean stress has an effect on the fatigue life and fatigue damage. Various models [13-18] have been proposed to predict the life by mean stress correction. The Goodman’s law is one of the main approaches for modifying the mean stress [19], as shown in Figure 8.

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**Figure 8 The Goodman’s law for mean stress correction**

Any loading cycle can be transformed into the symmetric cycle, which uses the Goodman’s law, that is,

\[
\sigma_{-1} = \frac{\sigma_u}{1 - \frac{1}{\sigma_m}}
\]  
(9)

where \(\sigma_{-1}\) is symmetry cyclic stress, \(\sigma_u\) is stress amplitude, and \(\sigma_m\) is the mean stress.

According to Table 2 and Table 3, the high-pressure turbine blades have three working conditions (S1, S2, S3). S3 has little effect on the fatigue damage when using the Miner rule [20], and S1 and S2 contribute a lot to the damage accumulation. So, the blades can be subjected to two-level loads, i.e. S1 and S2, as shown in Table 4. Since the fir-tree mortise is the most critical region, the stress of fir-tree mortise is considered as the maximum stress of blades, and the stresses for the other critical regions can be ignored.

<table>
<thead>
<tr>
<th>Table 4 The Two-Level Loads for Fir-Tree Mortise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational speed (rpm)</td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>0-17000-0</td>
</tr>
<tr>
<td>9200-17000-9200</td>
</tr>
</tbody>
</table>

#### 4.2 SSI Model for Reliability

The SSI model has been widely used to analyze the reliability of mechanical components, and the reliability is defined as the probability that the strength is less than the stress [21]. The components become unsafe because the stress is lower than the applied loads, as shown in Figure 9. The reliability of blades is estimated by the assumptions [22] that the stress and strength are independent.
According to the statistical data [23], the rotational speed of turbine blades can be treated as a random variable that is assumed to follow a normal distribution, so the stress also follows a normal distribution and $X \sim N(796.7, 37.5^2)$ . The mechanical properties for K403 is stable from 20°C to 800°C, treating the strength of blade as a normal distribution [24], that is $Y \sim N(\sigma_{0\mu}, \sigma_{\mu}^2)$ and that $\sigma_{0\mu} = 0.5\%\sigma_{\mu}$. The temperature of high-pressure turbine blades in service varies from 200°C-800°C [8]. Thus, we can obtain the fatigue strength coefficient $\sigma_f$, and the tensile breaking strength $\sigma_0$ is approximately equal to $\sigma_f$, that is $\sigma_0 \approx \sigma_f = 1180$MPa.

So, we can consider that $Y \sim N(1180, 59^2)$ .

The reliability of high-pressure turbine blade for one cycle $(n = 1)$ can be defined as

$$
R = P(Y - X > 0) = \int_0^{\infty} f_x(x) [\int_y^{\infty} f_y(y)] dy \ dx
$$

where $f_x(x)$ and $f_y(y)$ are the probability density function (PDF) of stress and strength, respectively.

The cumulative distribution function (CDF) of cyclic loads can be described as

$$
F_y(x) = F_y(x_1) \times F_y(x_2) \times \cdots \times F_y(x_i) = [F_y(x_i)]^i
$$

where $F_y(x_i)$ is the CDF of stress for the $i^{th}$-level loading.

The SSI model for high-pressure turbine blade can be modeled as

$$
R(n) = \int_0^{\infty} f_y(y,n) [\int_y^{\infty} f_x(x)]^{n-1} f_x(x) dx dy
$$

where $f_y(y,n)$ is the PDF of strength considering the strength degradation, which is related to loading cycles and cyclic loads. Because the mean of $F_y(x,y,n)$ decreases under cycle loadings, it is very difficult to calculate the reliability. For simplicity, we simplify the Eq. (12) by dividing the loading cycles into pieces, that is,

$$
R(n) = \prod_{i=1}^{n} R(n_i)
$$

where $n_1 + n_2 + \cdots + n_m = n$.

According to Eq. (13), the reliability of high-pressure turbine blade can be calculated, as shown in Table 5. In the Table, $N_1$ and $N_2$ are the loading cycles for the two-level loads.

From Table 5, the reliability of high-pressure turbine blades for working 1400 hours (2440+3600 cycles) is about 0.9865, and the reliability decreases quickly when the working hours are greater than 1400 hours.

5 CONCLUSIONS

In this paper, the FEA of the stresses, the strength degradation and SSI model are performed to solve the failure of high-pressure turbine blades. The FEA method can determine the critical regions. The strength degradation is considered under cyclic loadings, and the SSI model is also used to calculate the reliability. It provides the reasons for catastrophic failure and which region should be focused so as to reduce the stress concentration.

ACKNOWLEDGMENTS

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REFERENCE

Table 5 The Reliability of Turbine Blade

<table>
<thead>
<tr>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$n_i = N_1 + N_2$</th>
<th>$\sigma_r$</th>
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